

# Challenge Problem 2

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Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

## 1 PROBLEM

The center of the conic section:

$$y^2 - 2\sqrt{3}xy + 3x^2 + 6x - 4y + 5 = 0 \quad (1.0.1)$$

## 2 EXPLANATION

$$\mathbf{V} = \begin{pmatrix} 3 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \quad (2.0.1)$$

and

$$\mathbf{u} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (2.0.2)$$

$$f = 5 \quad (2.0.3)$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \quad (2.0.4)$$

and

$$\mathbf{p}_2 = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \quad (2.0.5)$$

Now,  $\mathbf{P}$  should be chosen in such a way that  $\mathbf{P}^T = \mathbf{P}^{-1}$  and  $\mathbf{P}^T \mathbf{V} \mathbf{P} = \mathbf{D}$ . but here it is not satisfying these equations. Here,  $\mathbf{P}^{-1} = \frac{1}{4} \mathbf{P}^T$ . So,  $\mathbf{P}$  should be selected as :

$$\mathbf{P} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad (2.0.6)$$

If I did it so, then  $|\mathbf{P}|=1$  and  $\mathbf{P}^{-1}=\mathbf{P}^T$ . So,

$$\mathbf{p}_1 = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.7)$$

and

$$\mathbf{p}_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \quad (2.0.8)$$

Now,  $\eta$  can be calculated as :

$$\eta = 2\mathbf{p}_1^T \mathbf{u} \quad (2.0.9)$$

$$\eta = 2 \left( \frac{1}{2} \quad \frac{\sqrt{3}}{2} \right) \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (2.0.10)$$

$$\eta = 3 - 2\sqrt{3} \quad (2.0.11)$$

Now,

$$\mathbf{u}^T + \eta \mathbf{p}_1^T = \begin{pmatrix} 2.76 & -2.4 \end{pmatrix} \quad (2.0.12)$$

$$\eta \mathbf{p}_1 - \mathbf{u} = (3 - 2\sqrt{3}) \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (2.0.13)$$

$$\eta \mathbf{p}_1 - \mathbf{u} = \begin{pmatrix} \frac{-3-2\sqrt{3}}{2} \\ \frac{-2+3\sqrt{3}}{2} \end{pmatrix} \quad (2.0.14)$$

Now considering the augmented matrix,

$$\begin{pmatrix} 2.76 & -2.4 & -5 \\ 3 & -\sqrt{3} & \frac{-3-2\sqrt{3}}{2} \\ -\sqrt{3} & 1 & \frac{-2+3\sqrt{3}}{2} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 / \sqrt{3}} \quad (2.0.15)$$

$$\begin{pmatrix} 2.76 & -2.4 & -5 \\ \sqrt{3} & -1 & \frac{-\sqrt{3}-2}{2} \\ -\sqrt{3} & 1 & \frac{-2+3\sqrt{3}}{2} \end{pmatrix} \quad (2.0.16)$$

$$(2.0.17)$$