

Assignment 10

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Abstract—This is a simple document explaining how to determine vectors are linear independent or not in a vector space.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

Are the vector:

$$\alpha_1 = (1, 1, 2, 4) \quad (1.0.1)$$

$$\alpha_2 = (2, -1, -5, 2) \quad (1.0.2)$$

$$\alpha_3 = (1, -1, -4, 0) \quad (1.0.3)$$

$$\alpha_4 = (2, 1, 1, 6) \quad (1.0.4)$$

linear independent in R^4 ?

2 EXPLANATION

Now,

$$C_1\alpha_1 + C_2\alpha_2 + C_3\alpha_3 + C_4\alpha_4 = \mathbf{0} \quad (2.0.1)$$

$$\Rightarrow C_1 + 2C_2 + C_3 + 2C_4 = 0 \quad (2.0.2)$$

$$C_1 - C_2 - C_3 + C_4 = 0 \quad (2.0.3)$$

$$2C_1 - 5C_2 - 4C_3 + C_4 = 0 \quad (2.0.4)$$

$$4C_1 + 2C_2 + 0C_3 + 6C_4 = 0 \quad (2.0.5)$$

So,

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$\Rightarrow AC = \mathbf{0} \quad (2.0.7)$$

Now, considering the coefficient matrix A:

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 - 4R_1]{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & -2 \end{pmatrix}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 / (-3)} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & 3 & 2 & 1 \\ 0 & -6 & -4 & -2 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 / (-2)]{R_2 \leftarrow R_2 / (-1)}$$

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - R_2]{R_2 \leftarrow 2R_2 / 3} \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 2 & \frac{4}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 / 2}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.8)$$

Now,

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = C_3 \begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix} + C_4 \begin{pmatrix} -\frac{4}{3} \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

As

$$C_1, C_2, C_3, C_4 \neq 0 \quad (2.0.11)$$

for $C_1\alpha_1 + C_2\alpha_2 + C_3\alpha_3 + C_4\alpha_4 = \mathbf{0}$, so, the vectors are linearly dependent in the vector space R^4 .