Assignment 10

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***Abstract*—This is a simple document explaining how to determine vectors are linear independent or not in a vector space.**

Download all and latex-tikz codes from

svn co https://github.com/gadepall/school/trunk/ncert/geometry/figs

1 Problem

Are the vector:

$$\alpha_{\mathbf{1}} = (1,1,2,4) \tag{1.0.1}$$
$$\alpha_{\mathbf{2}} = (2,-1,-5,2) \tag{1.0.2}$$
$$\alpha_{\mathbf{3}} = (1,-1,-4,0) \tag{1.0.3}$$
$$\alpha_{\mathbf{4}} = (2,1,1,6) \tag{1.0.4}$$

linear independent in R^4 ?

2 Explanation

Now,

$$C_1\alpha_{\mathbf{1}} + C_2\alpha_{\mathbf{2}} + C_3\alpha_{\mathbf{3}} + C_4\alpha_{\mathbf{4}} = \mathbf{0} \tag{2.0.1}$$
$$\implies C_1 + 2C_2 + C_3 + 2C_4 = 0 \tag{2.0.2}$$
$$C_1 - C_2 - C_3 + C_4 = 0 \tag{2.0.3}$$
$$2C_1 - 5C_2 - 4C_3 + C_4 = 0 \tag{2.0.4}$$
$$4C_1 + 2C_2 + 0C_3 + 6C_4 = 0 \tag{2.0.5}$$

So,

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$
$$\implies A\mathbf{C} = \mathbf{0} \tag{2.0.7}$$

Now, considering the coefficient matrix A:

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 - 4R_1]{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & -2 \end{pmatrix}$$
$$\xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3/(-3)} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & 3 & 2 & 1 \\ 0 & -6 & -4 & -2 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4/(-2)]{R_2 \leftarrow R_2/(-1)}$$
$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xleftrightarrow[R_1 \leftarrow R_1 - R_2]{R_2 \leftarrow 2R_2/3} \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 2 & \frac{4}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2/2}$$
$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.8}$$

Now,

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.9}$$
$$\implies \mathbf{C} = C_3\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix} + C_4\begin{pmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 0 \\ 1 \end{pmatrix} \tag{2.0.10}$$

As

$$C_1, C_2, C_3, C_4 \neq 0 \tag{2.0.11}$$

for $C_1\alpha_{\mathbf{1}} + C_2\alpha_{\mathbf{2}} + C_3\alpha_{\mathbf{3}} + C_4\alpha_{\mathbf{4}} = \mathbf{0}$, so, the vectors are linearly dependent in the vector space R^4.