Assignment 10

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Abstract—**This is a simple document explaining how to determine vectors are linear independent or not in a vector space.**

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1 PROBLEM

Are the vector:

$$\alpha_1 = (1, 1, 2, 4) \qquad (1.0.1)$$
$$\alpha_2 = (2, -1, -5, 2) \qquad (1.0.2)$$
$$\alpha_3 = (1, -1, -4, 0) \qquad (1.0.3)$$
$$\alpha_4 = (2, 1, 1, 6) \qquad (1.0.4)$$

linear independent in R^4 ?

2 EXPLANATION

Now,

$$C_1\alpha_1 + C_2\alpha_2 + C_3\alpha_3 + C_4\alpha_4 = 0 \qquad (2.0.1)$$

So,

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (2.0.2)$$

$$\implies A\mathbf{C} = \mathbf{0} \qquad (2.0.3)$$

Now, considering the coefficient matrix A:

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \xrightarrow[R_4 \leftarrow R_4 - 4R_1]{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & -2 \end{pmatrix}$$

$$\xrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3/(-3)} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & 3 & 2 & 1 \\ 0 & -6 & -4 & -2 \end{pmatrix} \xleftarrow[R_4 \leftarrow R_4/(-2)]{R_2 \leftarrow R_2/(-1)}$$

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 \end{pmatrix} \xrightarrow[R_4 \leftarrow R_4 - R_2]{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[R_1 \leftarrow R_1 - R_2]{R_2 \leftarrow 2R_2/3} \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 2 & \frac{4}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (2.0.4)$$

Now,

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (2.0.5)$$

$$\implies \mathbf{C} = C_3\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix} + C_4\begin{pmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0} \qquad (2.0.6)$$

So, the vectors are linearly dependent in the vector space R^4.