Assignment 12

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***Abstract*—This is a simple document explaining how to get the basis of a vector space when vectors from another vector space are given and the vector spaces are in isomorphic relationship.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let V be a vector space over the field of complex numbers and suppose there is an isomorphism T of V onto C^3. Let α_1, α_2, α_3 and α_4 be the vectors in V such that:

$$T\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix} \tag{1.0.1}$$

$$T\alpha_2 = \begin{pmatrix} -2 \\ 1+i \\ 0 \end{pmatrix} \tag{1.0.2}$$

$$T\alpha_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \tag{1.0.3}$$

$$T\alpha_4 = \begin{pmatrix} \sqrt{2} \\ i \\ 3 \end{pmatrix} \tag{1.0.4}$$

Find a basis for the subspace of V spanned by the 4 vectors α_i.

2 Explanation

V is a vector space and V is isomorphic to C^3 via isomorphism T which implies that C^3 is also isomorphic to V via isomorphism T^{-1}.

As V is isomorphic to C^3, so

$$dim(V) = dim(C^3) = 3 \tag{2.0.1}$$

Now,

$$\begin{pmatrix} 1 & 0 & i \\ -2 & 1+i & 0 \\ -1 & 1 & 1 \\ \sqrt{2} & i & 3 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow \sqrt{2}R_4]{R_3 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & 0 & i \\ -2 & 1+i & 0 \\ 0 & 1 & 1+i \\ 2 & i\sqrt{2} & 3\sqrt{2} \end{pmatrix}$$
$$\xleftrightarrow{R_4 \leftarrow R_4 + R_2} \begin{pmatrix} 1 & 0 & i \\ -2 & 1+i & 0 \\ 0 & 1 & 1+i \\ 0 & 1+i(1+\sqrt{2}) & 3\sqrt{2} \end{pmatrix}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix} 1 & 0 & i \\ 0 & 1+i & 2i \\ 0 & 1 & 1+i \\ 0 & 1+i(1+\sqrt{2}) & 3\sqrt{2} \end{pmatrix}$$
$$\xleftrightarrow{R_3 \leftarrow (1+i)R_3} \begin{pmatrix} 1 & 0 & i \\ 0 & 1+i & 2i \\ 0 & 1+i & 2i \\ 0 & 1+i(1+\sqrt{2}) & 3\sqrt{2} \end{pmatrix}$$
$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & i \\ 0 & 1+i & 2i \\ 0 & 0 & 0 \\ 0 & 1+i(1+\sqrt{2}) & 3\sqrt{2} \end{pmatrix} \tag{2.0.2}$$

From here we can get that $T\alpha_3$ is dependent vector while $T\alpha_1$, $T\alpha_2$ and $T\alpha_4$ are independent vector. These $T\alpha_1$, $T\alpha_2$ and $T\alpha_4$ also span the vector space C^3, so these 3 vectors are the basis of C^3.

As $dim(V) = 3$, so it must have 3 basis and as V and C^3 are isomorphic so α_1, α_2 and α_4 are the basis of V.