Assignment 13

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***Abstract*—This is a simple document explaining how a linear operator of a vector space depends upon an ordered basis of that vector space.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let V be a two-dimensional vector space over the field F and let B be an ordered basis for V. If T is a linear operator on V and

$$[T]_B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{1.0.1}$$

Prove that

$$T^2 - (a+d)T + (ad-bc)I = 0 \tag{1.0.2}$$

2 Explanation

Here T is a linear operator on V and B is an ordered basis of V. Let us consider $[T]_B = A$, so $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Now, the characteristic equation of A is:

$$\left|A - \lambda I\right| = 0 \tag{2.0.1}$$

$$\implies \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 \tag{2.0.2}$$

$$\implies \lambda^2 - (a+d)\lambda + (ad-bc) = 0 \tag{2.0.3}$$

According to the Cayley-Hamilton's Theorem, every square matrix satisfies its own characteristic equation. Here A is a 2x2 square matrix, so it should also satisfy its characteristic equation.

Now,

$$\lambda^2 - (a+d)\lambda + (ad-bc) = 0 \tag{2.0.4}$$

$$\implies A^2 - (a+d)A + (ad-bc)I = 0 \tag{2.0.5}$$

We can also write the equation 2.0.5 as:

$$[T]_B^2 - (a+d)[T]_B + (ad-bc)I = 0 \tag{2.0.6}$$

$$\textbf{or,} T^2 - (a+d)T + (ad-bc)I = 0 \tag{2.0.7}$$

Hence, the above problem statement is proved.