Assignment 15

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***Abstract*—This is a simple document explaining how to determine whether a functional is linear or not on a vector space using trace of matrices.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let V be the vector space of $n \times n$ matrices over the field F. If $\mathbf{B}$ is a fixed $n \times n$ matrix, define a function f_B on V by $f_B(\mathbf{A}) = trace(\mathbf{B}^t\mathbf{A})$. Show that, f_B is a linear functional on V.

2 Explanation

From the problem statement we get that,

$$f_B(\mathbf{A}) = trace(\mathbf{B}^t\mathbf{A}) \tag{2.0.1}$$

Now, to prove if f_B is a linear functional on V or not, we have to check the condition:

$$f_B(c\mathbf{A_1} + \mathbf{A_2}) = cf_B(\mathbf{A_1}) + f_B(\mathbf{A_2}) \tag{2.0.2}$$

If this above condition is satisfied then we can say that f_B is a linear functional on V.

3 Solution

Two important properties of trace of matrices are:

$$tr(\mathbf{M} + \mathbf{N}) = tr(\mathbf{M}) + tr(\mathbf{N}) \tag{3.0.1}$$

$$tr(c\mathbf{M}) = ctr(\mathbf{M}) \tag{3.0.2}$$

Where $\mathbf{M}$ and $\mathbf{N}$ are two square matrices. As,

$$f_B(\mathbf{A}) = trace(\mathbf{B}^t\mathbf{A}) \tag{3.0.3}$$

So,

$$f_B(c\mathbf{A_1} + \mathbf{A_2}) = trace(\mathbf{B}^t(c\mathbf{A_1} + \mathbf{A_2})) \tag{3.0.4}$$

$$\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = trace(c\mathbf{B}^t\mathbf{A_1}) + trace(\mathbf{B}^t\mathbf{A_2}) \tag{3.0.5}$$

$$\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = c(trace(\mathbf{B}^t\mathbf{A_1})) + trace(\mathbf{B}^t\mathbf{A_2}) \tag{3.0.6}$$

$$\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = cf_B(\mathbf{A_1}) + f_B(\mathbf{A_2}) \tag{3.0.7}$$

Hence, it can be said that f_B is a linear functional on V.