Assignment 15

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***Abstract*—This is a simple document explaining how to determine whether a functional is linear or not on a vector space using trace of matrices.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let V be the vector space of $n \times n$ matrices over the field F. If $\mathbf{B}$ is a fixed $n \times n$ matrix, define a function f_B on V by $f_B(\mathbf{A}) = tr(\mathbf{B}^t\mathbf{A})$. Show that, f_B is a linear functional on V.

2 Solution

Hence, it can be said that f_B is a linear functional on V.

Given	V is a vector space of $n \times n$ matrices over the field F $\mathbf{B}$ is a fixed $n \times n$ matrix A function f_B on V is defined such that $f_B(\mathbf{A}) = tr(\mathbf{B}^t\mathbf{A})$
To prove	f_B is a linear functional on V
Properties of Trace of Matrix	1.$tr(\mathbf{M}+\mathbf{N}) = tr(\mathbf{M}) + tr(\mathbf{N})$ 2. $tr(c\mathbf{M}) = c(tr(\mathbf{M}))$ Where $\mathbf{M}$ and $\mathbf{N}$ are two square matrices
Proof	As $f_B(\mathbf{A}) = tr(\mathbf{B}^t\mathbf{A})$ $\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = tr(\mathbf{B}^t(c\mathbf{A_1} + \mathbf{A_2}))$ $\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = tr(c\mathbf{B}^t\mathbf{A_1}) + tr(\mathbf{B}^t\mathbf{A_2})$ $\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = c(tr(\mathbf{B}^t\mathbf{A_1})) + tr(\mathbf{B}^t\mathbf{A_2})$ $\implies f_B(c\mathbf{A_1} + \mathbf{A_2}) = cf_B(\mathbf{A_1}) + f_B(\mathbf{A_2})$