Assignment 16

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***Abstract*—This is a simple document explaining how to determine whether a set of polynomials are linearly independent or not.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let $\mathbf{S}$ be a set of non-zero polynomials over a field F. If no two elements of $\mathbf{S}$ have the same degree, show that $\mathbf{S}$ is an independent set in $\mathbf{F}[\mathbf{x}]$.

2 Solution

Hence, it is proved that $\mathbf{S}$ is an independent set in $\mathbf{F}[\mathbf{x}]$.

Given	**S** be a set of non-zero polynomial over a field F No two elements of **S** have the same degree
To prove	**S** is an independent set in **F[x]**
Linear Independency	Let f_1,f_2,...,f_n are the polynomials and they will be linearly independent if $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ for $a_1 = a_2 = ... = a_n = 0$ where a_1,a_2,...,a_n are scalars from field F
Proof	Let the degrees of f_1,f_2,...,f_n are d_1,d_2,...,d_n respectively such that the degree of $f_i = d_i \neq d_j$ for j=1,2,....,n and $i \neq j$ Now, let $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ and the the degree of $f_i = d_i = d$ which is the largest degree among all the polynomials and f_i has the co-efficient a_i then if $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ and $d_i = d \neq d_j$ for j=1,2,....,n and $i \neq j$ then it must be $a_i = 0$ Now, consider the second largest degree d_k and it is the degree of the polynomial f_k a_k is the coefficient of f_k $d_k \neq d_j$ for j=1,2,....,n and $k \neq j$, so similarly $a_k = 0$ In this way, it can be proved that $a_1 = a_2 = ... = a_n = 0$ for $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ $\implies$ f_1,f_2,....,f_n are linearly independent