Assignment 16

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***Abstract*—This is a simple document explaining how to determine whether a set of polynomials are linearly independent or not.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Let **S** be a set of non-zero polynomials over a field F. If no two elements of **S** have the same degree, show that **S** is an independent set in **F**[**x**].

2 Solution

Given	**S** be a set of non-zero polynomial over a field F No two elements of **S** have the same degree
To prove	**S** is an independent set in **F**[**x**]
Linear Independency	Let f_1,f_2,...,f_n are the polynomials and they will be linearly independent if $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ for $a_1 = a_2 = ... = a_n = 0$ where a_1,a_2,...,a_n are scalars from field F

Hence, it is proved that **S** is an independent set in **F**[**x**].

Proof	Let the degrees of f_1,f_2,...,f_n are d_1,d_2,...,d_n respectively such that the degree of $f_i = d_i \neq d_j$ for j=1,2,....,n and $i \neq j$ and $d_1 < d_2 < .... < d_n$ so d_n is the largest degree
	Now, let $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ where $f_1 = \sum_{i=0}^{d_1} k_{1i}x^i$ $f_2 = \sum_{i=0}^{d_2} k_{2i}x^i$ **or,**$f_2 = \sum_{i=0}^{d_1} k_{2i}x^i + \sum_{i=d_1+1}^{d_2} k_{2i}x^i$ Similarly, $f_{n-1} = \sum_{i=0}^{d_{n-1}} k_{(n-1)i}x^i$ $\implies f_{n-1} = \sum_{i=0}^{d_{n-2}} k_{(n-1)i}x^i + \sum_{i=d_{n-2}+1}^{d_{n-1}} k_{(n-1)i}x^i$ and $f_n = \sum_{i=0}^{d_{n-1}} k_{ni}x^i + \sum_{i=d_{n-1}+1}^{d_n} k_{ni}x^i$
	Now, $a_1f_1 + a_2f_2 + ... + a_nf_n$ $= a_1 \sum_{i=0}^{d_1} k_{1i}x^i + a_2(\sum_{i=0}^{d_1} k_{2i}x^i + \sum_{i=d_1+1}^{d_2} k_{2i}x^i)$ $+... + a_n(\sum_{i=0}^{d_1} k_{ni}x^i + \sum_{i=d_1+1}^{d_2} k_{ni}x^i + ..$ $.. + \sum_{i=d_{n-1}+1}^{d_n} k_{ni}x^i)$ $= \sum_{i=0}^{d_1}(a_1k_{1i} + a_2k_{2i} + .. + a_nk_{ni})x^i$ $+ \sum_{i=d_1+1}^{d_2}(a_2k_{2i} + .. + a_nk_{ni})x^i$ $+.. + \sum_{i=d_{n-1}+1}^{d_n} a_nk_{ni}x^i$ Now, as $a_1f_1 + a_2f_2 + ... + a_nf_n = \theta$ for $d_{n-1} + 1 \leq i \leq d_n$, $k_{ni} \neq 0$ so a_n must be 0
	Now, discarding a_n associated term, we get $\sum_{i=0}^{d_1}(a_1k_{1i} + a_2k_{2i} + .. + a_nk_{ni})x^i$ $+ \sum_{i=d_1+1}^{d_2}(a_2k_{2i} + .. + a_nk_{ni})x^i$ $+.. + \sum_{i=d_{n-2}+1}^{d_{n-1}} a_{n-1}k_{(n-1)i}x^i = 0$ so, for $d_{n-2} + 1 \leq i \leq d_{n-1}$, $k_{(n-1)i} \neq 0$ $\implies a_{n-1} = 0$

Proof	Similarly, for $d_1 + 1 \leq i \leq d_2$ $\sum_{i=0}^{d_1}(a_1 k_{1i} + a_2 k_{2i})x^i + \sum_{i=d_1+1}^{d_2} a_2 k_{2i} x^i$ $\implies a_2 = 0$ as $k_{2i} \neq 0$ $\implies \sum_{i=0}^{d_1} a_1 k_{1i} x^i = 0$ $\implies a_1 = 0$ as $k_{1i} \neq 0$
	In this way, it can be proved that $a_1 = a_2 = ... = a_{n-1} = a_n = 0$ for $a_1 f_1 + a_2 f_2 + ... + a_n f_n = \theta$ $\implies f_1, f_2, ..., f_n$ are linearly independent