Assignment 17

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Abstract

This is a simple document explaining how to determine the upper and lower limits of the dimension of a vector space which is the intersection of 3 subspaces of a vector space and also to check whether the former vector space is a subspace of that vector space or not.

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/ncert/geometry/figs
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1 Problem

Let $\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$ be 3 distinct subspaces of $\mathbf{R}^{10}$ such that each $\mathbf{W_i}$ has dimension of 9. Let $\mathbf{W} = \mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}$. Then we can conclude that
1. $\mathbf{W}$ may not be a subspace of $\mathbf{R}^{10}$
2. dim $\mathbf{W} \leq 8$
3. dim $\mathbf{W} \geq 7$
4. dim $\mathbf{W} \leq 3$

2 Solution

Given	$\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$ are 3 distinct subspaces of $\mathbf{R}^{10}$ Each $\mathbf{W_i}$ has dimension 9 $\mathbf{W} = \mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}$
Statement1	$\mathbf{W}$ may not be a subspace of $\mathbf{R}^{10}$
Explanation	As $\mathbf{W} = \mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}$ and $\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$ are subspaces of $\mathbf{W}$,then $\mathbf{W}$ must be a subspace of $\mathbf{R}^{10}$. So the first option is false.
Statement2	dim $\mathbf{W} \leq 8$
Explanation	As $\mathbf{W}$ be a subspace of a finite dimension vector space $\mathbf{R}^{10}$ and dim $\mathbf{R}^{10}$= 10, so $\mathbf{W}$ is finite dimension and

	dim $\mathbf{W} \leq 10$
Theorem	dim $(\mathbf{W_1} \cap \mathbf{W_2})$ $=$ dim$(\mathbf{W_1})$+dim$(\mathbf{W_2})$-dim$(\mathbf{W_1} + \mathbf{W_2})$ and $\mathbf{W_1} \cap \mathbf{W_2}$ is also a subspace of $\mathbf{R}^{10}$
Proof	The minimum dimension of $\mathbf{W} = \mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}$
Explanation	Let us consider $\mathbf{V} = \mathbf{R}^{10}$ and $dim(\mathbf{V}) = 10$ and $\mathbf{U} = \mathbf{W_1} \cap \mathbf{W_2}$ So, $dim(\mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}) = dim(\mathbf{U})$ $+dim(\mathbf{W_3})$ - $dim(\mathbf{U} + \mathbf{W_3})$ **or,** $dim(\mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3}) = dim(\mathbf{W_1})$ $+dim(\mathbf{W_2})+dim(\mathbf{W_3})$ - $dim(\mathbf{W_1} + \mathbf{W_1})$ $-dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3})$
	Now, $(\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3} \subseteq \mathbf{V}$ $\implies dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3}) \leq dim(\mathbf{V})$ $\implies -dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3}) \geq -dim(\mathbf{V})$ Similarly, $(\mathbf{W_1} + \mathbf{W_2}) \subseteq \mathbf{V}$ $\implies dim(\mathbf{W_1} + \mathbf{W_2}) \leq dim(\mathbf{V})$ $\implies -dim(\mathbf{W_1} + \mathbf{W_2}) \geq -dim(\mathbf{V})$
	Considering these two inequations, $-dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3})-dim(\mathbf{W_1} + \mathbf{W_2})$ $\geq -2dim(\mathbf{V})$ **or,** $dim(\mathbf{W_1}) + dim(\mathbf{W_2}) + dim(\mathbf{W_3})$ $-dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3})-dim(\mathbf{W_1} + \mathbf{W_2})$ $\geq dim(\mathbf{W_1}) + dim(\mathbf{W_2}) + dim(\mathbf{W_3}) - 2dim(\mathbf{V})$ **or,** $dim(\mathbf{W_1} \cap \mathbf{W_2} \cap \mathbf{W_3})$ $\geq dim(\mathbf{W_1}) + dim(\mathbf{W_2}) + dim(\mathbf{W_3}) - 2dim(\mathbf{V})$ $\implies dim(\mathbf{W}) \geq dim(\mathbf{W_1}) + dim(\mathbf{W_2})$ $+dim(\mathbf{W_3}) - 2dim(\mathbf{V})$ $\implies dim(\mathbf{W_1}) + dim(\mathbf{W_2}) + dim(\mathbf{W_3})$ $-dim((\mathbf{W_1} \cap \mathbf{W_2}) + \mathbf{W_3})-dim(\mathbf{W_1} + \mathbf{W_2})$
Statement 3	dim $\mathbf{W} \geq 7$
Explanation	As $dim(\mathbf{W}) \geq dim(\mathbf{W_1}) + dim(\mathbf{W_2})$ $+dim(\mathbf{W_3}) - 2dim(\mathbf{V})$ $\implies dim(\mathbf{W}) \geq (9+9+9) - (2\times10)$

	$\implies dim(\mathbf{W}) \geq 7$
Answer	$7 \leq dim(\mathbf{W}) \leq 10$

TABLE 0: **Solution summary**

Hence, we can conclude that $dim(\mathbf{W}) \geq 7$.