Assignment 18

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Abstract

This is a simple document explaining about the properties of linear operator **T** on the vector space **V** especially the characteristic values and characteristic vectors.

Download all and latex-tikz codes from

svn co https://github.com/gadepall/school/trunk/ncert/geometry/figs

1 Problem

Let **V** be the space of $n \times n$ matrices over F. Let **A** be a fixed $n \times n$ matrix over F. Let **T** be the linear operator 'left multiplication by **A**' on **V**. Is it true that **A** and **T** have the same characteristic values?

2 Solution

Given	**V** is the space of $n \times n$ matrices over F **A** is a fixed $n \times n$ matrix over F **T** be the linear operator on **V** such that $\mathbf{T}(\mathbf{B}) = \mathbf{AB}$
To prove	**A** and **T** have the same characteristic values
Theorem	Let λ be a characteristic value of **T** and $\lambda \in F$,then if **T** is a linear operator on a finite dimensional space **V**, it must be $\mathbf{TB} = \lambda\mathbf{B}$ for all **B** where **B** is any $n \times n$ matrix in **V** So, $(\mathbf{T} - \lambda\mathbf{I})\mathbf{B} = \mathbf{0}, \implies \det(\mathbf{T} - \lambda\mathbf{I}) = 0$
Proof	As per the problem statement, $\mathbf{TB} = \lambda\mathbf{B}$ **or,** $\mathbf{AB} = \lambda\mathbf{B}$, as $\mathbf{TB} = \mathbf{AB}$ **or,** $(\mathbf{A} - \lambda\mathbf{I})\mathbf{B} = \mathbf{0}$ From here 2 cases can be arrived.
Case 1:	$\det((\mathbf{A} - \lambda\mathbf{I})) = 0$, where $\mathbf{B} \neq \mathbf{0}$ $\implies \lambda$ is characteristic value of **A**.
Case 2:	$\det((\mathbf{A} - \lambda\mathbf{I})) \neq 0$

	$\implies (\mathbf{A}-\lambda\mathbf{I})$ is invertible and $\mathbf{B}=\mathbf{0}$ so, for $(\mathbf{T}-\lambda\mathbf{I})\mathbf{B}=\mathbf{0}$ and $(\mathbf{T}-\lambda\mathbf{I})\neq\mathbf{0}$ $\implies \det(\mathbf{T}-\lambda\mathbf{I})\neq 0$ which is a contradiction. So, case 2 is not possible.
Conclusion	So, from the above 2 cases and from the theorem,it can be concluded that **A** and **T** have the same characteristic values

TABLE 0: **Solution summary**