Assignment 18

Jayati Dutta

Abstract

This is a simple document explaining about the properties of linear operator **T** on the vector space **V** especially the characteristic values and characteristic vectors.

Download all and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/ncert/geometry/figs
```

1 Problem

Let **V** be the space of $n \times n$ matrices over F. Let **A** be a fixed $n \times n$ matrix over F. Let **T** be the linear operator 'left multiplication by **A**' on **V**. Is it true that **A** and **T** have the same characteristic values?

2 Solution

Given	**V** is the space of $n \times n$ matrices over F **A** is a fixed $n \times n$ matrix over F **T** be the linear operator on **V** such that $\mathbf{T}(\mathbf{B}) = \mathbf{AB}$
To prove	**A** and **T** have the same characteristic values
Theorem	Let λ be a characteristic value of **T** and $\lambda \in F$ and **v** is the corresponding characteristic vector, then if **T** is a linear operator on a finite dimensional space **V**, it must be $\det(T - \lambda I) = 0$ and for $(\mathbf{T} - \lambda\mathbf{I})\mathbf{v} = \theta$, $\mathbf{v} \neq \theta$
Proof	As per the problem statement, $\mathbf{Tv} = \lambda\mathbf{v}$ **or,** $\mathbf{Av} = \lambda\mathbf{v}$, as $\mathbf{Tv} = \mathbf{Av}$ **or,** $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \theta$ From here 2 cases can be arrived.
Case 1:	$\det((\mathbf{A} - \lambda\mathbf{I})) = 0$, where $\mathbf{v} \neq \theta$ $\implies \lambda$ is characteristic value of **A**.
Case 2:	$\det((\mathbf{A} - \lambda\mathbf{I})) \neq 0$ $\implies (\mathbf{A} - \lambda\mathbf{I})$ is invertible and $\mathbf{v} = \theta$

	so, for $(\mathbf{T} - \lambda\mathbf{I})\mathbf{v} = \theta$ and $(\mathbf{T} - \lambda\mathbf{I}) \neq \theta$ $\implies$ $\mathbf{v}$ is not a charcteristic vector of $\mathbf{T}$ which is a contradiction. So, case 2 is not possible.
Conclusion	So, from the above 2 cases and from the theorem, it can be concluded that $\mathbf{A}$ and $\mathbf{T}$ have the same characteristic values

TABLE 0: **Solution summary**