Assignment 18

Jayati Dutta

Abstract

This is a simple document explaining about the properties of linear operator **T** on the vector space **V** especially the characteristic values and characteristic vectors.

Download all and latex-tikz codes from

svn co https://github.com/gadepall/school/trunk/ncert/geometry/figs

1 Problem

Let $\mathbf{V}$ be the space of $n \times n$ matrices over F. Let $\mathbf{A}$ be a fixed $n \times n$ matrix over F. Let $\mathbf{T}$ be the linear operator 'left multiplication by $\mathbf{A}$' on $\mathbf{V}$. Is it true that $\mathbf{A}$ and $\mathbf{T}$ have the same characteristic values?

2 Solution

Given	$\mathbf{V}$ is the space of $n \times n$ matrices over F $\mathbf{A}$ is a fixed $n \times n$ matrix over F $\mathbf{T}$ be the linear operator on $\mathbf{V}$ such that $\mathbf{T}(\mathbf{B}) = \mathbf{AB}$
To prove	$\mathbf{A}$ and $\mathbf{T}$ have the same characteristic values
Theorem	Let λ be a characteristic value of $\mathbf{T}$ and $\lambda \in F$ and $\mathbf{v}$ is the corresponding characteristic vector which is a $n \times n$ matrix, then if $\mathbf{T}$ is a linear operator on a finite dimensional space $\mathbf{V}$, it must be $\det(T - \lambda I) = 0$ and for $(\mathbf{T} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$
Proof	As per the problem statement, $\mathbf{Tv} = \lambda\mathbf{v}$ **or,** $\mathbf{Av} = \lambda\mathbf{v}$, as $\mathbf{Tv} = \mathbf{Av}$ **or,** $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$ From here 2 cases can be arrived.
Case 1:	$\det((\mathbf{A} - \lambda\mathbf{I})) = 0$, where $\mathbf{v} \neq \mathbf{0}$ $\implies \lambda$ is characteristic value of $\mathbf{A}$.
Case 2:	$\det((\mathbf{A} - \lambda\mathbf{I})) \neq 0$

	$\implies$ $(\mathbf{A} - \lambda\mathbf{I})$ is invertible and $\mathbf{v} = \mathbf{0}$ so, for $(\mathbf{T} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$ and $(\mathbf{T} - \lambda\mathbf{I}) \neq \mathbf{0}$ $\implies$ $\mathbf{v}$ is not a charcteristic vector of $\mathbf{T}$ which is a contradiction. So, case 2 is not possible.
Conclusion	So, from the above 2 cases and from the theorem, it can be concluded that $\mathbf{A}$ and $\mathbf{T}$ have the same characteristic values

TABLE 0: **Solution summary**