

Assignment 18

Jayati Dutta

Abstract

This is a simple document explaining about the properties of linear operator \mathbf{T} on the vector space \mathbf{V} especially the characteristic values and characteristic vectors.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

Let \mathbf{V} be the space of $n \times n$ matrices over F . Let \mathbf{A} be a fixed $n \times n$ matrix over F . Let \mathbf{T} be the linear operator 'left multiplication by \mathbf{A} ' on \mathbf{V} . Is it true that \mathbf{A} and \mathbf{T} have the same characteristic values?

2 SOLUTION

Given	\mathbf{V} is the space of $n \times n$ matrices over F \mathbf{A} is a fixed $n \times n$ matrix over F \mathbf{T} be the linear operator on \mathbf{V} such that $\mathbf{T}(\mathbf{B}) = \mathbf{AB}$
To prove	\mathbf{A} and \mathbf{T} have the same characteristic values
Theorem	Let λ be a characteristic value of \mathbf{T} and $\lambda \in F$ and \mathbf{v} is the corresponding characteristic vector, then if \mathbf{T} is a linear operator on a finite dimensional space \mathbf{V} , it must be $\det(\mathbf{T} - \lambda\mathbf{I}) = 0$ and for $(\mathbf{T} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$
Proof	As per the problem statement, $\mathbf{T}\mathbf{v} = \lambda\mathbf{v}$ or , $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$, as $\mathbf{T}\mathbf{v} = \mathbf{A}\mathbf{v}$ or , $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$ From here 2 cases can be arrived.
Case 1:	$\det((\mathbf{A} - \lambda\mathbf{I})) = 0$, where $\mathbf{v} \neq \mathbf{0}$ $\implies \lambda$ is characteristic value of \mathbf{A} .
Case 2:	$\det((\mathbf{A} - \lambda\mathbf{I})) \neq 0$ $\implies (\mathbf{A} - \lambda\mathbf{I})$ is invertible and $\mathbf{v} = \mathbf{0}$

	<p>so, for $(\mathbf{T} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$ and $(\mathbf{T} - \lambda \mathbf{I}) \neq \mathbf{0}$ $\Rightarrow \mathbf{v}$ is not a characteristic vector of \mathbf{T} which is a contradiction. So, case 2 is not possible.</p>
Conclusion	<p>So, from the above 2 cases and from the theorem, it can be concluded that \mathbf{A} and \mathbf{T} have the same characteristic values</p>

TABLE 0: **Solution summary**