Assignment 3

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***Abstract*—This is a simple document explaining how to calculate the modulus of complex numbers in vector form.**

Download all and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Find the modulus of :

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \tag{1.0.1}$$

2 Explanation

In general if $\mathbf{z_1}$ and $\mathbf{z_2}$ are two complex numbers, then they can be expressed as:

$$\mathbf{z_1} = r_1\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{z_2} = r_2\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix} \tag{2.0.2}$$

&

$$\mathbf{z_1}\mathbf{z_2} = r_1 r_2\begin{pmatrix}\cos(\theta_1+\theta_2)\\ \sin(\theta_1+\theta_2)\end{pmatrix} \tag{2.0.3}$$

Now,

$$\mathbf{z_1} = \begin{pmatrix}1\\1\end{pmatrix} = \sqrt{2}\begin{pmatrix}\cos 45^\circ\\ \sin 45^\circ\end{pmatrix} \tag{2.0.4}$$

&

$$\mathbf{z_2} = \begin{pmatrix}1\\-1\end{pmatrix}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos 45^\circ\\ \sin 45^\circ\end{pmatrix} \tag{2.0.5}$$

So,

$$\mathbf{z_1}\mathbf{z_2} = \frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} = \begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}^{-1} = \sqrt{2}\frac{1}{\sqrt{2}}\begin{pmatrix}\cos(45^\circ+45^\circ)\\ \sin(45^\circ+45^\circ)\end{pmatrix} = \begin{pmatrix}\cos 90^\circ\\ \sin 90^\circ\end{pmatrix} \tag{2.0.6}$$

Similarly,

$$\mathbf{z_3} = \begin{pmatrix}1\\-1\end{pmatrix} = \sqrt{2}\begin{pmatrix}\cos 45^\circ\\ -\sin 45^\circ\end{pmatrix} \tag{2.0.7}$$

&

$$\mathbf{z_4} = \begin{pmatrix}1\\1\end{pmatrix}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos 45^\circ\\ -\sin 45^\circ\end{pmatrix} \tag{2.0.8}$$

So,

$$\mathbf{z_3}\mathbf{z_4} = \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} = \begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}^{-1} = \sqrt{2}\frac{1}{\sqrt{2}}\begin{pmatrix}\cos(45^\circ+45^\circ)\\ \sin(-45^\circ-45^\circ)\end{pmatrix} = \begin{pmatrix}\cos 90^\circ\\ -\sin 90^\circ\end{pmatrix} \tag{2.0.9}$$

3 Solution

Now, according to the problem statement:

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \tag{3.0.1}$$

$$= \mathbf{z_1}\mathbf{z_2} - \mathbf{z_3}\mathbf{z_4} \tag{3.0.2}$$

$$= \begin{pmatrix}\cos 90^\circ\\ \sin 90^\circ\end{pmatrix} - \begin{pmatrix}\cos 90^\circ\\ -\sin 90^\circ\end{pmatrix} \tag{3.0.3}$$

$$= \begin{pmatrix}0\\2\end{pmatrix} \tag{3.0.4}$$

$\therefore$

$$\left\| \frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \right\| \tag{3.0.5}$$

$$= \|\mathbf{z_1 z_2} - \mathbf{z_3 z_4}\| \tag{3.0.6}$$

$$= \left\| \begin{pmatrix}0\\2\end{pmatrix} \right\| = \sqrt{0^2 + 2^2} = 2 \tag{3.0.7}$$

So, we can say that the modulus value of

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \tag{3.0.8}$$

is 2.

3.1. Verification of the above problem using python code.

```
codes/complex_verify.py
```