Assignment 3

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Abstract—This is a simple document explaining how to calculate the modulus of complex numbers in vector form.

1 PROBLEM

Find the modulus of :

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \tag{1.0.1}$$

2 EXPLANATION

In general, any complex number can be expressed in matrix representation as follows:

$$\begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}a_1 & -a_2\\a_2 & a_1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.1}$$

In our case,

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.2}$$

&

$$\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.3}$$

Now,

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} = \begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.5}$$

Similarly,

$$\frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} = \begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.6}$$

$$= \begin{pmatrix}0\\-1\end{pmatrix} \tag{2.0.7}$$

So,

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}0\\-1\end{pmatrix} \tag{2.0.8}$$

$$= \begin{pmatrix}0\\2\end{pmatrix} \tag{2.0.9}$$

3 SOLUTION

Now, according to the problem statement:

$$\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}} \tag{3.0.1}$$

$$= \begin{pmatrix}0\\2\end{pmatrix} \tag{3.0.2}$$

$$\therefore$$

$$\left\|\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}} - \frac{\begin{pmatrix}1\\-1\end{pmatrix}}{\begin{pmatrix}1\\1\end{pmatrix}}\right\| \tag{3.0.3}$$

$$= \left\|\begin{pmatrix}0\\2\end{pmatrix}\right\| = \sqrt{0^2 + 2^2} = 2 \tag{3.0.4}$$

So, we can say that the modulus value of

$$\frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} - \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} \qquad (3.0.5)$$

is 2.

3.1. Verification of the above problem using python code.

codes/complex_verify.py