

Assignment 5

Jayati Dutta

Abstract—This is a simple document explaining how to prove the congruence of two triangles.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC . If AD is extended to intersect BC at P , show that

- $\triangle ABD \cong \triangle ACD$
- $\triangle ABP \cong \triangle ACP$
- AP bisects $\angle A$ as well as $\angle D$
- AP is the perpendicular bisector of BC

2 EXPLANATION

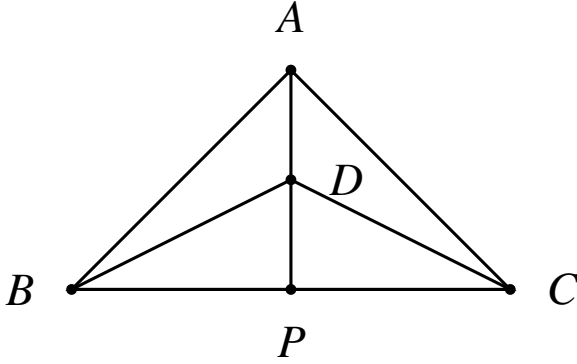


Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure ?? where the vertices are: A , B and C for $\triangle ABC$ and D , B and C for $\triangle DBC$. For $\triangle ABC$ the sides AB , BC and CA are represented by the vectors $\mathbf{A} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$ and $\mathbf{C} - \mathbf{A}$ and for $\triangle DBC$ the sides DB , BC and CD are represented by $\mathbf{D} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$ and $\mathbf{C} - \mathbf{D}$.

From the problem statement we get that:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.1)$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \quad (2.0.2)$$

$$\mathbf{P} - \mathbf{C} = K_2(\mathbf{B} - \mathbf{C}) \quad (2.0.3)$$

$$\mathbf{P} - \mathbf{B} = K_1(\mathbf{B} - \mathbf{C}) \quad (2.0.4)$$

$$\begin{aligned} \|\mathbf{A} - \mathbf{B}\| &= \|\mathbf{A} - \mathbf{C}\| \\ \Rightarrow \|\mathbf{A} - \mathbf{B}\|^2 &= \|\mathbf{A} - \mathbf{C}\|^2 \\ \Rightarrow \|(\mathbf{A} - \mathbf{P}) + (\mathbf{P} - \mathbf{B})\|^2 &= \|(\mathbf{A} - \mathbf{P}) + (\mathbf{P} - \mathbf{C})\|^2 \\ \Rightarrow \|\mathbf{A} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) &= \\ \|\mathbf{A} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) & \\ \Rightarrow \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) &= \\ \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) & \\ \Rightarrow \|\mathbf{P} - \mathbf{B}\|^2 - \|\mathbf{P} - \mathbf{C}\|^2 & \\ + 2(\mathbf{A} - \mathbf{P})^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) &= 0 \quad (2.0.5) \end{aligned}$$