Assignment 5

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***Abstract*—This is a simple document explaining how to prove the congruence of two triangles.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that
a) $\triangle ABD \cong \triangle ACD$
b) $\triangle ABP \cong \triangle ACP$
c) AP bisects $\angle A$ as well as $\angle D$
d) AP is the parpendicular bisector of BC

2 Explanation

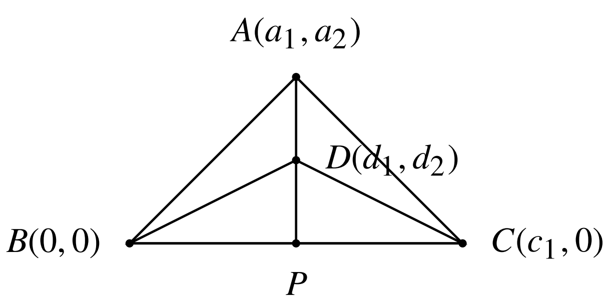

Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure 0 where the vertices are: $\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} c_1 \\ 0 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$

From the problem statement we get that:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.1}$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \tag{2.0.2}$$

$\angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$

Now, for $\triangle ABD$ and $\triangle ACD$,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.3}$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \tag{2.0.4}$$

and

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{D}\| \tag{2.0.5}$$

So, using SSS theorem it can be concluded that $\triangle ABD \cong \triangle ACD$.

As $\triangle ABD \cong \triangle ACD$ and

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \tag{2.0.6}$$

$$\angle BAD = \angle CAD \implies \angle BAP = \angle CAP \tag{2.0.7}$$

Now, for $\triangle ABP$ and $\triangle ACP$,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.8}$$

$$\|\mathbf{A} - \mathbf{P}\| = \|\mathbf{A} - \mathbf{P}\| \tag{2.0.9}$$

$$\angle BAP = \angle CAP \tag{2.0.10}$$

So, using SAS theorem it can be concluded that $\triangle ABP \cong \triangle ACP$.

As $\angle BAP = \angle PAC$ which implies that **AP** bisects $\angle A$.

It is already proved that $\triangle ABP \cong \triangle ACP$ and $\angle BAP = \angle PAC$, so

$$\|\mathbf{B} - \mathbf{P}\| = \|\mathbf{P} - \mathbf{C}\| \tag{2.0.11}$$

Now for $\triangle DBP$ and $\triangle DCP$,

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \tag{2.0.12}$$

$$\|\mathbf{D}-\mathbf{P}\| = \|\mathbf{D}-\mathbf{P}\| \tag{2.0.13}$$

$$\|\mathbf{B}-\mathbf{P}\| = \|\mathbf{P}-\mathbf{C}\| \tag{2.0.14}$$

So, according to SSS theorem $\triangle DBP \cong \triangle DCP$ and as

$$\|\mathbf{B}-\mathbf{P}\| = \|\mathbf{P}-\mathbf{C}\| \tag{2.0.15}$$

so, $\angle BDP = \angle PDC$ which implies that $\mathbf{AP}$ bisects $\angle D$.

From here we can say that $\mathbf{AP}$ bisects $\angle A$ as well as $\angle D$.

Now, as $\|\mathbf{B}-\mathbf{P}\| = \|\mathbf{P}-\mathbf{C}\|$, we can say that $\mathbf{AP}$ bisects $\mathbf{BC}$.

It is already proved that $\triangle ABP \cong \triangle ACP$, so

$$\angle BAP = \angle CAP \tag{2.0.16}$$

$$\angle ABP = \angle ACP \tag{2.0.17}$$

$$\angle APB = \angle APC \tag{2.0.18}$$

So, we can say that for $\triangle ABC$, $\angle A + \angle B + \angle C = 180\ ^\circ$.

$\implies \angle A + 2\angle B = 180\ ^\circ$.

For $\triangle ABP$,

$$\frac{\angle A}{2} + \angle B + \angle APB = 180^\circ \tag{2.0.19}$$
$$\implies \angle A + 2\angle B + 2\angle APB = 360^\circ \tag{2.0.20}$$
$$\implies 180^\circ + 2\angle APB = 360^\circ \tag{2.0.21}$$
$$\implies 2\angle APB = 180^\circ \tag{2.0.22}$$
$$\implies \angle APB = 90^\circ \tag{2.0.23}$$
$$\implies \mathbf{AP} \perp \mathbf{BC} \tag{2.0.24}$$