Assignment 5

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***Abstract*—This is a simple document explaining how to prove the congruence of two triangles.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that
a) $\triangle ABD \cong \triangle ACD$
b) $\triangle ABP \cong \triangle ACP$
c) AP bisects $\angle A$ as well as $\angle D$
d) AP is the parpendicular bisector of BC

2 Explanation

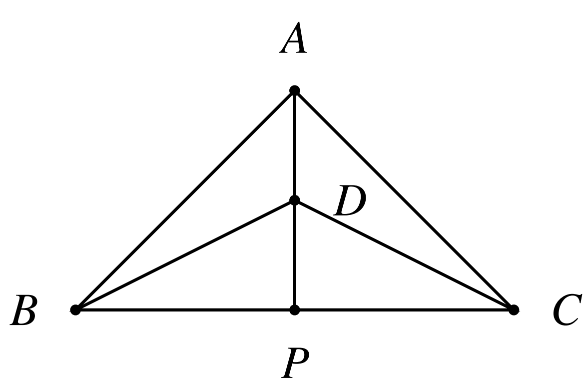

Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure 0 where the vertices are: A, B and C for $\triangle ABC$ and D, B and C for $\triangle DBC$. For $\triangle ABC$ the sides AB, BC and CA are represented by the vectors $\mathbf{A}-\mathbf{B}$, $\mathbf{B}-\mathbf{C}$ and $\mathbf{C}-\mathbf{A}$ and for $\triangle DBC$ the sides DB, BC and CD are represented by $\mathbf{D}-\mathbf{B}$, $\mathbf{B}-\mathbf{C}$ and $\mathbf{C}-\mathbf{D}$.

From the problem statement we get that:

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.1}$$

$$\|\mathbf{D}-\mathbf{B}\| = \|\mathbf{D}-\mathbf{C}\| \tag{2.0.2}$$

As the $\triangle ABC$ is an iso-scelen triangle,

$$\angle ABC = \angle ACB = \gamma \tag{2.0.3}$$

Similarly, as the $\triangle DBC$ is an iso-scelen triangle,

$$\angle DBC = \angle DCB \tag{2.0.4}$$

Let consider $\angle APB = \phi$,$\angle BAP = \theta_1$, $\angle CAP = \theta_2$,$\angle BDP = \alpha$ and $\angle CDP = \beta$. From the triangular law of vector addition, we can also get that:

$$\mathbf{A}-\mathbf{P} = (\mathbf{A}-\mathbf{B}) + (\mathbf{B}-\mathbf{P}) \tag{2.0.5}$$
$$\mathbf{A}-\mathbf{P} = (\mathbf{A}-\mathbf{C}) + (\mathbf{C}-\mathbf{P}) \tag{2.0.6}$$
$$\mathbf{D}-\mathbf{P} = (\mathbf{D}-\mathbf{B}) + (\mathbf{B}-\mathbf{P}) \tag{2.0.7}$$
$$\mathbf{D}-\mathbf{P} = (\mathbf{D}-\mathbf{C}) + (\mathbf{C}-\mathbf{P}) \tag{2.0.8}$$
$$\mathbf{B}-\mathbf{C} = (\mathbf{B}-\mathbf{A}) + (\mathbf{A}-\mathbf{C}) \tag{2.0.9}$$
$$\implies \mathbf{B}-\mathbf{C} = -(\mathbf{A}-\mathbf{B}) + (\mathbf{A}-\mathbf{C}) \tag{2.0.10}$$

According to the Parallelogram law of vector addition,

$$\mathbf{A}-\mathbf{P} = (\mathbf{A}-\mathbf{B}) + (\mathbf{A}-\mathbf{C}) \tag{2.0.11}$$

Now,

$$\begin{aligned}(\mathbf{B}-\mathbf{C}).(\mathbf{A}-\mathbf{P}) &= \|\mathbf{A}-\mathbf{C}\|^2 - \|\mathbf{A}-\mathbf{B}\|^2\\ &\implies (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{P}) = 0\\ &\implies \cos\phi = 0\\ &\implies \phi = 90^\circ\end{aligned} \tag{2.0.12}$$

Now, as the angle between $\mathbf{A}-\mathbf{P}$ and $\mathbf{B}-\mathbf{C}$ is 90 °. So we can say that,

$$\begin{aligned}\|\mathbf{A}-\mathbf{B}\|^2 &= \|\mathbf{A}-\mathbf{P}\|^2 + \|\mathbf{B}-\mathbf{P}\|^2\\ \|\mathbf{A}-\mathbf{C}\|^2 &= \|\mathbf{A}-\mathbf{P}\|^2 + \|\mathbf{P}-\mathbf{C}\|^2\end{aligned} \tag{2.0.13}$$

But,

$$\|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2$$
$$\implies \|\mathbf{A}-\mathbf{P}\|^2 + \|\mathbf{B}-\mathbf{P}\|^2 = \|\mathbf{A}-\mathbf{P}\|^2 + \|\mathbf{P}-\mathbf{C}\|^2$$
$$\implies \|\mathbf{B}-\mathbf{P}\|^2 = \|\mathbf{P}-\mathbf{C}\|^2$$
$$\implies \|\mathbf{B}-\mathbf{P}\| = \|\mathbf{P}-\mathbf{C}\| \quad (2.0.14)$$

$$\cos\theta_1 = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{P})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{P}\|}$$
$$\implies \cos\theta_1 = \frac{(\mathbf{A}-\mathbf{B})^T((\mathbf{A}-\mathbf{B})+(\mathbf{B}-\mathbf{P}))}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{P}\|}$$
$$\implies \cos\theta_1 = \frac{\|\mathbf{A}-\mathbf{B}\|}{\|\mathbf{A}-\mathbf{P}\|} - \cos\gamma \quad (2.0.15)$$

Similarly,

$$\cos\theta_2 = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{P})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{A}-\mathbf{P}\|}$$
$$\implies \cos\theta_2 = \frac{(\mathbf{A}-\mathbf{C})^T((\mathbf{A}-\mathbf{C})+(\mathbf{C}-\mathbf{P}))}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{P}\|}$$
$$\implies \cos\theta_2 = \frac{\|\mathbf{A}-\mathbf{B}\|}{\|\mathbf{A}-\mathbf{P}\|} - \cos\gamma \quad (2.0.16)$$

This proves that $\theta_1 = \theta_2$ Similarly, for $\triangle DBC$ we can prove that $\alpha = \beta$. Now,

Now we can conclude that AP bisects $\angle A$ as well as $\angle D$ and AP bisects BC perpendicularly.