

# Assignment 5

Jayati Dutta

**Abstract**—This is a simple document explaining how to prove the congruence of two triangles.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

## 1 PROBLEM

$\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base  $BC$  and the vertices  $A$  and  $D$  are on the same side of  $BC$ . If  $AD$  is extended to intersect  $BC$  at  $P$ , show that

- $\triangle ABD \cong \triangle ACD$
- $\triangle ABP \cong \triangle ACP$
- $AP$  bisects  $\angle A$  as well as  $\angle D$
- $AP$  is the perpendicular bisector of  $BC$

## 2 EXPLANATION

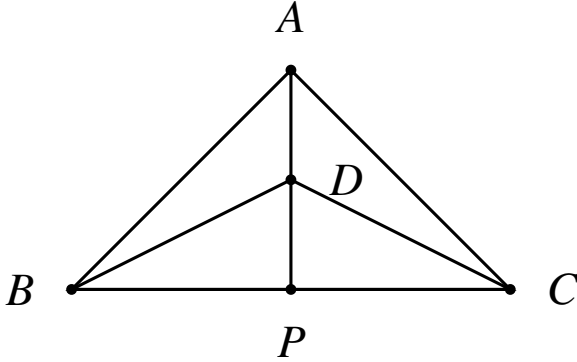


Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure 0 where the vertices are:  $A$ ,  $B$  and  $C$  for  $\triangle ABC$  and  $D$ ,  $B$  and  $C$  for  $\triangle DBC$ . For  $\triangle ABC$  the sides  $AB$ ,  $BC$  and  $CA$  are represented by the vectors  $\mathbf{A} - \mathbf{B}$ ,  $\mathbf{B} - \mathbf{C}$  and  $\mathbf{C} - \mathbf{A}$  and for  $\triangle DBC$  the sides  $DB$ ,  $BC$  and  $CD$  are represented by  $\mathbf{D} - \mathbf{B}$ ,  $\mathbf{B} - \mathbf{C}$  and  $\mathbf{C} - \mathbf{D}$ .

From the problem statement we get that:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.1)$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \quad (2.0.2)$$

$$\mathbf{P} - \mathbf{C} = K_2(\mathbf{B} - \mathbf{C}) \quad (2.0.3)$$

$$\mathbf{P} - \mathbf{B} = K_1(\mathbf{B} - \mathbf{C}) \quad (2.0.4)$$

$$\begin{aligned} \|\mathbf{A} - \mathbf{B}\| &= \|\mathbf{A} - \mathbf{C}\| \\ \Rightarrow \|\mathbf{A} - \mathbf{B}\|^2 &= \|\mathbf{A} - \mathbf{C}\|^2 \\ \Rightarrow \|(\mathbf{A} - \mathbf{P}) + (\mathbf{P} - \mathbf{B})\|^2 &= \|(\mathbf{A} - \mathbf{P}) + (\mathbf{P} - \mathbf{C})\|^2 \\ \Rightarrow \|\mathbf{A} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) &= \\ \|\mathbf{A} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) & \\ \Rightarrow \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) &= \\ \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) & \\ \Rightarrow \|\mathbf{P} - \mathbf{B}\|^2 - \|\mathbf{P} - \mathbf{C}\|^2 & \\ + 2(\mathbf{A} - \mathbf{P})^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) &= 0 \\ \Rightarrow ((\mathbf{P} - \mathbf{B}) + (\mathbf{P} - \mathbf{C}))^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) & \\ + 2(\mathbf{A} - \mathbf{P})^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) &= 0 \\ \Rightarrow (\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T(\mathbf{B} - \mathbf{C}) & \\ + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{P} - \mathbf{B} - \mathbf{P} + \mathbf{C}) &= 0 \\ \Rightarrow (\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T(\mathbf{B} - \mathbf{C}) + 2(\mathbf{A} - \mathbf{P})^T(\mathbf{B} - \mathbf{C}) &= 0 \\ \Rightarrow (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T - 2(\mathbf{A} - \mathbf{P})^T &= 0 \\ \Rightarrow (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{P}) - (2\mathbf{A} - 2\mathbf{P})^T &= 0 \\ \Rightarrow (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{A})^T &= 0 \\ \Rightarrow (\mathbf{B} + \mathbf{C} - 2\mathbf{A})(\mathbf{B} - \mathbf{C})^T &= 0 \end{aligned} \quad (2.0.5)$$

Similarly,

$$\begin{aligned}
& \|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \\
& \implies \|\mathbf{D} - \mathbf{B}\|^2 = \|\mathbf{D} - \mathbf{C}\|^2 \\
& \implies \|(\mathbf{D} - \mathbf{P}) + (\mathbf{P} - \mathbf{B})\|^2 = \|(\mathbf{D} - \mathbf{P}) + (\mathbf{P} - \mathbf{C})\|^2 \\
& \implies \|\mathbf{D} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) = \\
& \quad \|\mathbf{D} - \mathbf{P}\|^2 + \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) \\
& \implies \|\mathbf{P} - \mathbf{B}\|^2 + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{P} - \mathbf{B}) = \\
& \quad \|\mathbf{P} - \mathbf{C}\|^2 + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{P} - \mathbf{C}) \\
& \implies \|\mathbf{P} - \mathbf{B}\|^2 - \|\mathbf{P} - \mathbf{C}\|^2 \\
& \quad + 2(\mathbf{D} - \mathbf{P})^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) = 0 \\
& \implies ((\mathbf{P} - \mathbf{B}) + (\mathbf{P} - \mathbf{C}))^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) \\
& \quad + 2(\mathbf{D} - \mathbf{P})^T((\mathbf{P} - \mathbf{B}) - (\mathbf{P} - \mathbf{C})) = 0 \\
& \implies (\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T(\mathbf{B} - \mathbf{C}) \\
& \quad + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{P} - \mathbf{B} - \mathbf{P} + \mathbf{C}) = 0 \\
& \implies (\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T(\mathbf{B} - \mathbf{C}) + 2(\mathbf{D} - \mathbf{P})^T(\mathbf{B} - \mathbf{C}) = 0 \\
& \implies (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{P})^T - 2(\mathbf{D} - \mathbf{P})^T = 0 \\
& \implies (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{P}) - (2\mathbf{D} - 2\mathbf{P})^T = 0 \\
& \implies (\mathbf{B} - \mathbf{C})(\mathbf{B} + \mathbf{C} - 2\mathbf{D})^T = 0 \\
& \implies (\mathbf{B} + \mathbf{C} - 2\mathbf{D})(\mathbf{B} - \mathbf{C})^T = 0
\end{aligned}$$

(2.0.6)