

Assignment 5

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Abstract—This is a simple document explaining how to prove the congruence of two triangles.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC . If AD is extended to intersect BC at P , show that

- $\triangle ABD \cong \triangle ACD$
- $\triangle ABP \cong \triangle ACP$
- AP bisects $\angle A$ as well as $\angle D$
- AP is the perpendicular bisector of BC

2 EXPLANATION

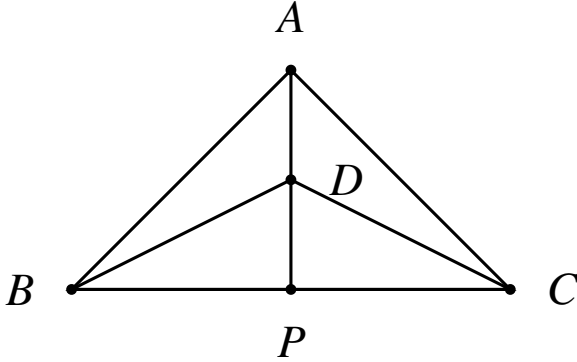


Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure 0 where the vertices are: A , B and C for $\triangle ABC$ and D , B and C for $\triangle DBC$. For $\triangle ABC$ the sides AB , BC and CA are represented by the vectors $\mathbf{A} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$ and $\mathbf{C} - \mathbf{A}$ and for $\triangle DBC$ the sides DB , BC and CD are represented by $\mathbf{D} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$ and $\mathbf{C} - \mathbf{D}$.

From the problem statement we get that:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.1)$$

$$\|\mathbf{D} - \mathbf{B}\| = \|\mathbf{D} - \mathbf{C}\| \quad (2.0.2)$$

As the $\triangle ABC$ is an iso-sceles triangle,

$$\angle ABC = \angle ACB = \gamma \quad (2.0.3)$$

Similarly, as the $\triangle DBC$ is an iso-sceles triangle,

$$\angle DBC = \angle DCB \quad (2.0.4)$$

Let consider $\angle APB = \phi$, $\angle BAP = \theta_1$, $\angle CAP = \theta_2$, $\angle BDP = \alpha$ and $\angle CDP = \beta$. From the triangular law of vector addition, we can also get that:

$$\mathbf{A} - \mathbf{P} = (\mathbf{A} - \mathbf{B}) + (\mathbf{B} - \mathbf{P}) \quad (2.0.5)$$

$$\mathbf{A} - \mathbf{P} = (\mathbf{A} - \mathbf{C}) + (\mathbf{C} - \mathbf{P}) \quad (2.0.6)$$

$$\mathbf{D} - \mathbf{P} = (\mathbf{D} - \mathbf{B}) + (\mathbf{B} - \mathbf{P}) \quad (2.0.7)$$

$$\mathbf{D} - \mathbf{P} = (\mathbf{D} - \mathbf{C}) + (\mathbf{C} - \mathbf{P}) \quad (2.0.8)$$

$$\mathbf{B} - \mathbf{C} = (\mathbf{B} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}) \quad (2.0.9)$$

$$\Rightarrow \mathbf{B} - \mathbf{C} = -(\mathbf{A} - \mathbf{B}) + (\mathbf{A} - \mathbf{C}) \quad (2.0.10)$$

According to the Parallelogram law of vector addition,

$$\mathbf{A} - \mathbf{P} = (\mathbf{A} - \mathbf{B}) + (\mathbf{A} - \mathbf{C}) \quad (2.0.11)$$

Now,

$$(\mathbf{B} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{P}) = \|\mathbf{A} - \mathbf{C}\|^2 - \|\mathbf{A} - \mathbf{B}\|^2$$

$$\Rightarrow (\mathbf{B} - \mathbf{C})^T (\mathbf{A} - \mathbf{P}) = 0$$

$$\Rightarrow \cos \phi = 0$$

$$\Rightarrow \phi = 90^\circ \quad (2.0.12)$$

$$\cos \theta_1 = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{P})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{P}\|}$$

$$\Rightarrow \cos \theta_1 = \frac{(\mathbf{A} - \mathbf{B})^T ((\mathbf{A} - \mathbf{B}) + (\mathbf{B} - \mathbf{P}))}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{P}\|}$$

$$\Rightarrow \cos \theta_1 = \frac{\|\mathbf{A} - \mathbf{B}\|}{\|\mathbf{A} - \mathbf{P}\|} - \cos \gamma$$

$$(2.0.13)$$

Similarly,

$$\begin{aligned}
 \cos \theta_2 &= \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{P})}{\|\mathbf{A} - \mathbf{C}\| \|\mathbf{A} - \mathbf{P}\|} \\
 \implies \cos \theta_2 &= \frac{(\mathbf{A} - \mathbf{C})^T ((\mathbf{A} - \mathbf{C}) + (\mathbf{C} - \mathbf{P}))}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{P}\|} \\
 \implies \cos \theta_2 &= \frac{\|\mathbf{A} - \mathbf{B}\|}{\|\mathbf{A} - \mathbf{P}\|} - \cos \gamma
 \end{aligned}
 \tag{2.0.14}$$

This proves that $\theta_1 = \theta_2$. Similarly, for $\triangle DBC$ we can prove that $\alpha = \beta$.

Now we can conclude that AP bisects $\angle A$ as well as $\angle D$ and AP bisects BC perpendicularly.