Assignment 5

Jayati Dutta

***Abstract*—This is a simple document explaining how to prove the congruence of two triangles.**

Download all and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and the vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that
a) $\triangle ABD \cong \triangle ACD$
b) $\triangle ABP \cong \triangle ACP$
c) AP bisects $\angle A$ as well as $\angle D$
d) AP is the parpendicular bisector of BC

2 Explanation

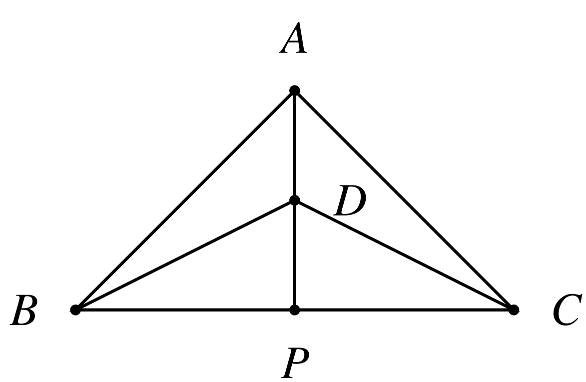

Fig. 0: Iso-sceles Triangles by Latex-Tikz

The above problem statement is depicted in the figure 0 where the vertices are: A, B and C for $\triangle ABC$ and D, B and C for $\triangle DBC$. For $\triangle ABC$ the sides AB, BC and CA are represented by the vectors $\mathbf{A}-\mathbf{B}$, $\mathbf{B}-\mathbf{C}$ and $\mathbf{C}-\mathbf{A}$ and for $\triangle DBC$ the sides DB, BC and CD are represented by $\mathbf{D}-\mathbf{B}$, $\mathbf{B}-\mathbf{C}$ and $\mathbf{C}-\mathbf{D}$.

From the problem statement we get that:

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.1}$$

$$\|\mathbf{D}-\mathbf{B}\| = \|\mathbf{D}-\mathbf{C}\| \tag{2.0.2}$$

$$\mathbf{P}-\mathbf{C} = K_2(\mathbf{B}-\mathbf{C}) \tag{2.0.3}$$

$$\mathbf{P}-\mathbf{B} = K_1(\mathbf{B}-\mathbf{C}) \tag{2.0.4}$$

As the $\triangle ABC$ is an iso-scelen triangle,

$$\angle ABC = \angle ACB = \gamma \tag{2.0.5}$$

Similarly, as the $\triangle DBC$ is an iso-scelen triangle,

$$\angle DBC = \angle DCB \tag{2.0.6}$$

Let consider $\angle APB = \phi_1$, $\angle APC = \phi_2$ and $\phi_1 + \phi_2 = 180\ ^\circ$. From the triangular law of vector addition, we can also get that:

$$\mathbf{A}-\mathbf{B} = (\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{B}) \tag{2.0.7}$$

$$\mathbf{A}-\mathbf{C} = (\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{C}) \tag{2.0.8}$$

$$\mathbf{D}-\mathbf{B} = (\mathbf{D}-\mathbf{P}) + (\mathbf{P}-\mathbf{B}) \tag{2.0.9}$$

$$\mathbf{D}-\mathbf{C} = (\mathbf{D}-\mathbf{P}) + (\mathbf{P}-\mathbf{C}) \tag{2.0.10}$$

Now squaring both side of equation 2,

$$\begin{gathered}
\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \\
\text{or, } \|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \\
\text{or, } \|(\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{B})\|^2 = \|(\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{C})\|^2 \\
\text{or,}((\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{B}))^T((\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{B})) = \\
((\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{C}))^T((\mathbf{A}-\mathbf{P}) + (\mathbf{P}-\mathbf{C})) \\
\text{or,}((\mathbf{A}-\mathbf{P})^T + K_1(\mathbf{B}-\mathbf{C})^T)((\mathbf{A}-\mathbf{P}) + K_1(\mathbf{B}-\mathbf{C})) = \\
((\mathbf{A}-\mathbf{P})^T + K_2(\mathbf{B}-\mathbf{C})^T)((\mathbf{A}-\mathbf{P}) + K_2(\mathbf{B}-\mathbf{C})) \\
\text{or,}(K_1 - K_2)((\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{P}) \\
+ (\mathbf{A}-\mathbf{P})^T(\mathbf{B}-\mathbf{C}) + (K_1 + K_2)\,\|\mathbf{B}-\mathbf{C}\|^2) = 0
\end{gathered} \tag{2.0.11}$$