Assignment 6

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***Abstract*—This is a simple document explaining how to determine the QR decomposition of a 2x2 matrix.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Find the QR decomposition of $\begin{pmatrix} 55 & -60 \\ -60 & 20 \end{pmatrix}$

2 Explanation

Let $\mathbf{a}$ and $\mathbf{b}$ are the column vectors of the given matrix. So,

$$\mathbf{a} = \begin{pmatrix} 55 \\ -60 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} = \begin{pmatrix} -60 \\ 20 \end{pmatrix} \tag{2.0.2}$$

$\mathbf{a}$ and $\mathbf{b}$ can be expressed as:

$$\mathbf{a} = K_1\mathbf{u_1} \tag{2.0.3}$$

$$\mathbf{b} = r_1\mathbf{u_1} + K_2\mathbf{u_2} \tag{2.0.4}$$

Where,

$$K_1 = \|\mathbf{a}\| \tag{2.0.5}$$

$$\implies \mathbf{u_1} = \frac{\mathbf{a}}{\|\mathbf{a}\|} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u_1}^T\mathbf{b}}{\|\mathbf{u_1}\|^2} \tag{2.0.7}$$

$$\mathbf{u_2} = \frac{\mathbf{b} - r_1\mathbf{u_1}}{\|\mathbf{b} - r_1\mathbf{u_1}\|} \tag{2.0.8}$$

$$K_2 = \mathbf{u_2}^T\mathbf{b} \tag{2.0.9}$$

Then we can express the given matrix as:

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix}\begin{pmatrix} K_1 & r_1 \\ 0 & K_2 \end{pmatrix} \tag{2.0.10}$$

$$\text{or,} \begin{pmatrix} a & b \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now,

$$K_1 = \|\mathbf{a}\| = \sqrt{55^2 + (-60)^2} = 5\sqrt{265} \tag{2.0.12}$$

$$\mathbf{u_1} = \frac{\mathbf{a}}{5\sqrt{265}} = \frac{1}{\sqrt{265}}\begin{pmatrix} 11 \\ -12 \end{pmatrix} \tag{2.0.13}$$

$$\|\mathbf{u_1}\| = 1 \tag{2.0.14}$$

$$r_1 = \mathbf{u_1}^T\mathbf{b} = \frac{1}{\sqrt{265}}\begin{pmatrix} 11 & -12 \end{pmatrix}\begin{pmatrix} -60 \\ 20 \end{pmatrix} \tag{2.0.15}$$

$$\implies r_1 = -\frac{900}{\sqrt{265}} \tag{2.0.16}$$

So,

$$r_1\mathbf{u_1} = -\frac{900}{\sqrt{265}}\frac{1}{\sqrt{265}}\begin{pmatrix} 11 \\ -12 \end{pmatrix} \tag{2.0.17}$$

$$\implies r_1\mathbf{u_1} = \begin{pmatrix} -\frac{1980}{53} \\ \frac{2160}{53} \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{b} - r_1\mathbf{u_1} = \begin{pmatrix} -60 \\ 20 \end{pmatrix} - \begin{pmatrix} -\frac{1980}{53} \\ \frac{2160}{53} \end{pmatrix} \tag{2.0.19}$$

$$\implies \mathbf{b} - r_1\mathbf{u_1} = -\frac{100}{53}\begin{pmatrix} 12 \\ 11 \end{pmatrix} \tag{2.0.20}$$

$$\|\mathbf{b} - r_1\mathbf{u_1}\| = \frac{100\sqrt{5}}{\sqrt{53}} \tag{2.0.21}$$

Similarly, we can obtain:

$$\mathbf{u_2} = -\frac{1}{\sqrt{265}}\begin{pmatrix} 12 \\ 11 \end{pmatrix} = \begin{pmatrix} -\frac{12}{\sqrt{265}} \\ -\frac{11}{\sqrt{265}} \end{pmatrix} \tag{2.0.22}$$

$$K_2 = \mathbf{u_2}^T\mathbf{b} = -\frac{1}{\sqrt{265}}\begin{pmatrix} 12 & 11 \end{pmatrix}\begin{pmatrix} -60 \\ 20 \end{pmatrix} \tag{2.0.23}$$

$$= \frac{500}{\sqrt{265}} \tag{2.0.24}$$

Now, after QR decomposition of the given matrix we can get:

$$\begin{pmatrix} 55 & -60 \\ -60 & 20 \end{pmatrix} = \begin{pmatrix} \frac{11}{\sqrt{265}} & \frac{-12}{\sqrt{265}} \\ \frac{-12}{\sqrt{265}} & \frac{-11}{\sqrt{265}} \end{pmatrix}\begin{pmatrix} 5\sqrt{265} & \frac{-900}{\sqrt{265}} \\ 0 & \frac{500}{\sqrt{265}} \end{pmatrix} \tag{2.0.25}$$