Assignment 8

Jayati Dutta

***Abstract*—This is a simple document explaining how to determine the nullspace solution from the Row Reduced Echelon Form of a coefficient matrix.**

Download all and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Find all solutions to the following system of equations by row-reducing the co-efficient matrix:

$$\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0 \tag{1.0.1}$$

$$-4x_1 \qquad + 5x_3 = 0 \tag{1.0.2}$$

$$-3x_1 + 6x_2 - 13x_3 = 0 \tag{1.0.3}$$

$$-\frac{7}{3}x_1 + 2x_2 - \frac{8}{3}x_3 = 0 \tag{1.0.4}$$

2 Explanation

The coefficient matrix is:

$$A = \begin{pmatrix} \frac{1}{3} & 2 & -6 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -\frac{7}{3} & 2 & -\frac{8}{3} \end{pmatrix} \tag{2.0.1}$$

The number of rows of this coefficient matrix is m = 4 and the number of columns is n= 3, So in this case, $n < m$. Now the row operations are:

$$\begin{pmatrix} \frac{1}{3} & 2 & -6 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -\frac{7}{3} & 2 & -\frac{8}{3} \end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1 \times 3]{R_4 \leftarrow R_4 \times 3} \begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -7 & 6 & -8 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_3 \leftarrow R_2 + R_3} \begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -7 & 6 & -8 \\ -7 & 6 & -8 \end{pmatrix} \xleftrightarrow{R_4 \leftarrow R_4 - R_3} \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -7 & 6 & -8 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 + 7R_1]{R_2 \leftarrow R_2 + 4R_1} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 48 & -138 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3 \leftarrow R_3/2} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 24 & -69 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - R_2} \tag{2.0.5}$$

$$\begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow \frac{R_3}{(-2)}} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{4}} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 6 & -\frac{67}{4} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \tag{2.0.7}$$

$$\begin{pmatrix} 1 & 0 & -\frac{5}{4} \\ 0 & 6 & -\frac{67}{4} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{6}} \begin{pmatrix} 1 & 0 & -\frac{5}{4} \\ 0 & 1 & -\frac{67}{24} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 + \frac{5R_3}{4}]{R_2 \leftarrow R_2 + \frac{67R_3}{24}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.9}$$

Now,

$$A\mathbf{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.0.10}$$

So,

$$\mathbf{I_3}\mathbf{x} = 0 \tag{2.0.11}$$

$$\implies \mathbf{x} = 0 \tag{2.0.12}$$

As $\mathbf{I_3}$ is invertible.