Assignment 8

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Abstract—**This is a simple document explaining how to determine the nullspace solution from the Row Reduced Echelon Form of a coefficient matrix.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Find all solutions to the following system of equations by row-reducing the co-efficient matrix:

$$\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0 \tag{1.0.1}$$
$$-4x_1 \qquad + 5x_3 = 0 \tag{1.0.2}$$
$$-3x_1 + 6x_2 - 13x_3 = 0 \tag{1.0.3}$$
$$-\frac{7}{3}x_1 + 2x_2 - \frac{8}{3}x_3 = 0 \tag{1.0.4}$$

2 Explanation

The coefficient matrix is:

$$A = \begin{pmatrix} \frac{1}{3} & 2 & -6 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -\frac{7}{3} & 2 & -\frac{8}{3} \end{pmatrix} \tag{2.0.1}$$

The number of rows of this coefficient matrix is m = 4 and the number of columns is n= 3, So in this case, $n < m$. Now the row operations are:

$$\begin{pmatrix} \frac{1}{3} & 2 & -6 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -\frac{7}{3} & 2 & -\frac{8}{3} \end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1 \times 3]{R_4 \leftarrow R_4 \times 3} \begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -7 & 6 & -8 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_3 \leftarrow R_2 + R_3} \begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -7 & 6 & -8 \\ -7 & 6 & -8 \end{pmatrix} \xleftrightarrow{R_4 \leftarrow R_4 - R_3} \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -7 & 6 & -8 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 + 7R_1]{R_2 \leftarrow R_2 + 4R_1} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 48 & -138 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3 \leftarrow R_3/2} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 24 & -69 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 - R_2} \tag{2.0.5}$$

$$\begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow \frac{R_3}{(-2)}} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 24 & -67 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{4}} \begin{pmatrix} 1 & 6 & -18 \\ 0 & 6 & -\frac{67}{4} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \tag{2.0.7}$$

$$\begin{pmatrix} 1 & 0 & -\frac{5}{4} \\ 0 & 6 & -\frac{67}{4} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{6}} \begin{pmatrix} 1 & 0 & -\frac{5}{4} \\ 0 & 1 & -\frac{67}{24} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 + \frac{5R_3}{4}]{R_2 \leftarrow R_2 + \frac{67R_3}{24}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.9}$$

Now the coefficient matrix A can be written as row reduced matrix R and R can be expressed as:

$$R = \begin{pmatrix} I \\ 0 \end{pmatrix} \tag{2.0.10}$$

Now the rank of the matrix A is ρ = 3 =n, so the coefficient matrix has the full column rank. ρ =

3 =n,this implies that the solution may or may not exist. But as $\rho = n < m$, so if solution exists then it will be either 1 or 0.

Now,

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \tag{2.0.11}$$

$$\implies x_1 = 0 \tag{2.0.12}$$

$$x_2 = 0 \tag{2.0.13}$$

$$x_3 = 0 \tag{2.0.14}$$

$$\tag{2.0.15}$$

As x_1,x_2 and x_3 all are pivot variables, but $x_1 = x_2 = x_3$ =0