Assignment 9

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***Abstract*—This is a simple document explaining how to express a matrix by the linear combination of the rows of another matrix.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix}$ Is there any matrix C such that $CA = B$?

2 Explanation

The matrix B is obtained by multiplying the matrix A with matrix C. B is a 2×2 matrix and A is a 3×2 matrix. so matrix C must be a 2×3 matrix. Let the matrix C is:

$$C = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \tag{2.0.1}$$

So, after multiplying with A matrix we get,

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} = \tag{2.0.2}$$

$$\begin{pmatrix} a_1 + 2b_1 + c_1 & -a_1 + 2b_1 \\ a_2 + 2b_2 + c_2 & -a_2 + 2b_2 \end{pmatrix} \tag{2.0.3}$$

Now equating this result with matrix B:

$$\begin{pmatrix} a_1 + 2b_1 + c_1 & -a_1 + 2b_1 \\ a_2 + 2b_2 + c_2 & -a_2 + 2b_2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix} \tag{2.0.4}$$

$$\implies a_1 + 2b_1 + c_1 = 3 \tag{2.0.5}$$

$$\implies a_1 + 2b_1 = 3 - c_1 \tag{2.0.6}$$

$$-a_1 + 2b_1 = 1 \tag{2.0.7}$$

$$\implies \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} 3 - c_1 \\ 1 \end{pmatrix} \tag{2.0.8}$$

Now, considering these two equations the augmented matrix is formed as:

$$\begin{pmatrix} 1 & 2 & (3 - c_1) \\ -1 & 2 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & (3 - c_1) \\ 0 & 4 & (4 - c_1) \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & (3 - c_1) \\ 0 & 2 & \frac{4-c_1}{2} \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2/2]{R_1 \leftarrow R_1 - R_2} \tag{2.0.10}$$

$$\begin{pmatrix} 1 & 0 & \frac{2-c_1}{2} \\ 0 & 1 & \frac{4-c_1}{4} \end{pmatrix} \tag{2.0.11}$$

$$\implies a_1 = \frac{2 - c_1}{2} \tag{2.0.12}$$

$$b_1 = \frac{4 - c_1}{4} \tag{2.0.13}$$

Now depending on the value of c_1, the values of a_1 and b_1 will be calculated. Let $c_1 = 4$, then a_1 = -1 and b_1=0. Similarly,

$$a_2 + 2b_2 + c_2 = -4 \tag{2.0.14}$$

$$\implies a_2 + 2b_2 = -4 - c_2 \tag{2.0.15}$$

$$-a_2 + 2b_2 = 4 \implies \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} -4 - c_2 \\ 4 \end{pmatrix} \tag{2.0.16}$$

Now, considering these two equations the augmented matrix is formed as:

$$\begin{pmatrix} 1 & 2 & (-4-c_2) \\ -1 & 2 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & (-4-c_2) \\ 0 & 4 & -c_2 \end{pmatrix} \tag{2.0.17}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & (-4-c_2) \\ 0 & 2 & -\frac{-c_2}{2} \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2/2]{R_1 \leftarrow R_1 - R_2} \tag{2.0.18}$$

$$\begin{pmatrix} 1 & 0 & -4-\frac{c_2}{2} \\ 0 & 1 & -\frac{c_2}{4} \end{pmatrix} \tag{2.0.19}$$

$$\implies a_2 = -4 - \frac{c_2}{2} \tag{2.0.20}$$

$$b_2 = -\frac{c_2}{4} \tag{2.0.21}$$

Now depending on the value of c_2, the values of a_2 and b_2 will be calculated. Let c_2 = 4, then a_2 = -6 and b_2=-1.

3 Solution

So, it can be oserved that matrix C exists and depending on the c_1 and c_2 values different C matrix can be generated. One of the C matrix is $= \begin{pmatrix} -1 & 0 & 4 \\ -6 & -1 & 4 \end{pmatrix}$ such that CA=B.

3.1. Verification of the above problem using python code.

Solution: The following Python code verifies the above solution.

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codes/multiplication_test.py
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