Assignment 9

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***Abstract*—This is a simple document explaining how to express a matrix by the linear combination of the rows of another matrix.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
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1 Problem

Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix}$ Is there any matrix C such that $CA = B$?

2 Explanation

The matrix B is obtained by multiplying the matrix A with matrix C. B is a 2×2 matrix and A is a 3×2 matrix. so matrix C must be a 2×3 matrix. Let the matrix C is:

$$C = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \tag{2.0.1}$$

$$\implies C^T = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} \tag{2.0.2}$$

So, after multiplying with A matrix we get,

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a_1 + 2b_1 + c_1 & -a_1 + 2b_1 \\ a_2 + 2b_2 + c_2 & -a_2 + 2b_2 \end{pmatrix} \tag{2.0.3}$$

Matrix A is a rectangular matrix. Now, Considering CA =B and by transposing both side,

$$(CA)^T = B^T \tag{2.0.4}$$

$$\implies A^T C^T = B^T \tag{2.0.5}$$

$$\implies \begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} | & | \\ C_1 & C_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & 4 \end{pmatrix} \tag{2.0.6}$$

Where C_1 and C_2 are the column vectors of matrix C. Now, row redution operation is applied to the matrix A^T:

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & \frac{1}{2} \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 2 & \frac{1}{2} \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \end{pmatrix} \tag{2.0.7}$$

Now,

$$\begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} | & | \\ C_1 & C_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & 4 \end{pmatrix} \tag{2.0.8}$$

$$\implies \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{2.0.10}$$

Here a_1, b_1, a_2, b_2 are pivot variables but c_1 and c_2 are free variables. So the pivot variables can be expressed in terms of the free variables. So,

$$a_1 = 3 - \frac{c_1}{2} \tag{2.0.11}$$

$$b_1 = -1 - \frac{c_1}{4} \tag{2.0.12}$$

$$a_2 = -4 - \frac{c_2}{2} \tag{2.0.13}$$

$$b_2 = 4 - \frac{c_2}{4} \tag{2.0.14}$$

Now,

$$C = \begin{pmatrix} (3 - \frac{c_1}{2}) & (-1 - \frac{c_1}{4}) & c_1 \\ (-4 - \frac{c_2}{2}) & (4 - \frac{c_2}{4}) & c_2 \end{pmatrix} \tag{2.0.15}$$