

# Assignment 9

Jayati Dutta

**Abstract**—This is a simple document explaining how to express a matrix by the linear combination of the rows of another matrix.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

## 1 PROBLEM

Let  $A = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix}$  Is there any matrix  $C$  such that  $CA = B$ ?

## 2 EXPLANATION

The matrix  $B$  is obtained by multiplying the matrix  $A$  with matrix  $C$ .  $B$  is a  $2 \times 2$  matrix and  $A$  is a  $3 \times 2$  matrix. so matrix  $C$  must be a  $2 \times 3$  matrix. Let the matrix  $C$  is:

$$C = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow C^T = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} \quad (2.0.2)$$

So, after multiplying with  $A$  matrix we get,

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a_1 + 2b_1 + c_1 & -a_1 + 2b_1 \\ a_2 + 2b_2 + c_2 & -a_2 + 2b_2 \end{pmatrix} \quad (2.0.3)$$

Matrix  $A$  is a rectangular matrix. Now, Considering  $CA = B$  and by transposing both side,

$$(CA)^T = B^T \quad (2.0.4)$$

$$\Rightarrow A^T C^T = B^T \quad (2.0.5)$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} (C_1 \ C_2) = \begin{pmatrix} 3 & -4 \\ 1 & 4 \end{pmatrix} \quad (2.0.6)$$

We can represent it like this:

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} C_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (2.0.7)$$

$$(2.0.8)$$

Now the augmented matrix is:

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ -1 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 1 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & \frac{1}{2} & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 2 & \frac{1}{2} & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{4} & 1 \end{pmatrix} \quad (2.0.9)$$

Similarly,

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} C_2 = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \quad (2.0.10)$$

$$(2.0.11)$$

Now the augmented matrix is:

$$\begin{pmatrix} 1 & 2 & 1 & -4 \\ -1 & 2 & 0 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & 1 & -4 \\ 0 & 4 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & 1 & -4 \\ 0 & 2 & \frac{1}{2} & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -4 \\ 0 & 2 & \frac{1}{2} & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -4 \\ 0 & 1 & \frac{1}{4} & 0 \end{pmatrix} \quad (2.0.12)$$

From equations ?? and ??, it can be observed that solutions exist and there is a matrix  $C$  such that  $CA = B$ .