

Assignment 9

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Abstract—This is a simple document explaining how to express a matrix by the linear combination of the rows of another matrix.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix}$ Is there any matrix C such that $CA = B$?

2 EXPLANATION

The matrix B is obtained by multiplying the matrix A with matrix C . B is a 2×2 matrix and A is a 3×2 matrix. so matrix C must be a 2×3 matrix. Let the matrix C is:

$$C = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow C^T = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{pmatrix} \quad (2.0.2)$$

So, after multiplying with A matrix we get,

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a_1 + 2b_1 + c_1 & -a_1 + 2b_1 \\ a_2 + 2b_2 + c_2 & -a_2 + 2b_2 \end{pmatrix} \quad (2.0.3)$$

Matrix A is a rectangular matrix. Now, Considering $CA = B$ and by transposing both side,

$$(CA)^T = B^T \quad (2.0.4)$$

$$\Rightarrow A^T C^T = B^T \quad (2.0.5)$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} (\mathbf{c}_1 \quad \mathbf{c}_2) = \begin{pmatrix} 3 & -4 \\ 1 & 4 \end{pmatrix} \quad (2.0.6)$$

We can represent it like this:

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} \mathbf{c}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (2.0.7)$$

$$(2.0.8)$$

Now the augmented matrix is:

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ -1 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 1 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & \frac{1}{2} & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 2 & \frac{1}{2} & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{4} & 1 \end{pmatrix} \quad (2.0.9)$$

Similarly,

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 0 \end{pmatrix} \mathbf{c}_2 = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \quad (2.0.10)$$

$$(2.0.11)$$

Now the augmented matrix is:

$$\begin{pmatrix} 1 & 2 & 1 & -4 \\ -1 & 2 & 0 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 2 & 1 & -4 \\ 0 & 4 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 2 & 1 & -4 \\ 0 & 2 & \frac{1}{2} & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -4 \\ 0 & 2 & \frac{1}{2} & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -4 \\ 0 & 1 & \frac{1}{4} & 0 \end{pmatrix} \quad (2.0.12)$$

From equations 2.0.9 and 2.0.12, it can be observed that solutions exist and there is a matrix C such that

$CA = B$. Now,

$$\mathbf{c}_1 = \begin{pmatrix} 1 - \frac{c_1}{2} \\ 1 - \frac{c_1}{4} \\ c_1 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{c}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{4} \\ 1 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{c}_2 = \begin{pmatrix} -4 - \frac{c_2}{2} \\ -\frac{c_2}{4} \\ c_2 \end{pmatrix} \quad (2.0.15)$$

$$\Rightarrow \mathbf{c}_2 = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{4} \\ 1 \end{pmatrix} \quad (2.0.16)$$

Now,

$$\begin{aligned} C^T &= \begin{pmatrix} 1 & -4 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} + c_1 \begin{pmatrix} -\frac{1}{2} & 0 \\ -\frac{1}{4} & 0 \\ 1 & 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & -\frac{1}{4} \\ 0 & 1 \end{pmatrix} \\ \Rightarrow C &= \begin{pmatrix} 1 & 1 & 0 \\ -4 & 0 & 0 \end{pmatrix} + c_1 \begin{pmatrix} -\frac{1}{2} & -\frac{1}{4} & 1 \\ 0 & 0 & 0 \end{pmatrix} \\ &\quad + c_2 \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{4} & 1 \end{pmatrix} \quad (2.0.17) \end{aligned}$$

Now,

$$\begin{aligned} CA &= \begin{pmatrix} 1 & 1 & 0 \\ -4 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \\ &\quad + c_1 \begin{pmatrix} -\frac{1}{2} & -\frac{1}{4} & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \\ &\quad + c_2 \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \\ \Rightarrow CA &= \begin{pmatrix} 3 & 1 \\ -4 & 4 \end{pmatrix} + c_1 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &\Rightarrow CA = B \quad (2.0.18) \end{aligned}$$

Hence, it is proved that there exist a matrix C such that $CA = B$.

2.1. Verification of the above problem using python code.

Solution: The following Python code verifies the above solution.

codes/multiplication_test.py