Math Document Template

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***Abstract*—This is a simple document explaining how to get the area of a parallelogram with the given adjacent sides that are vectors.**

Download all and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Find the area of a parallelogram whose adjacent sides are given by the vectors $(3\ 1\ 4)^T$ and $(1\ \text{-}1\ 1)^T$

2 Construction

2.1. The figure for the parallelogram obtained in the question looks like Fig. 2.1. with with vectors $\mathbf{a}$ and $\mathbf{b}$.

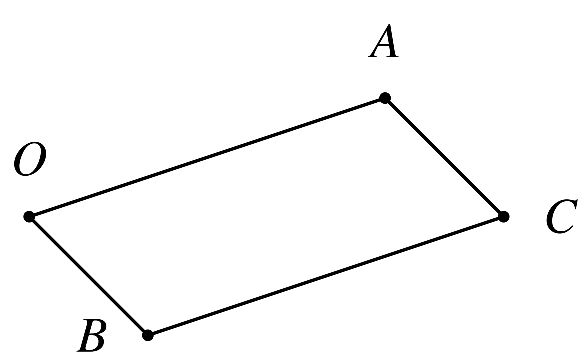

Fig. 2.1: Parallelogram by Latex-Tikz

2.2. List the design parameters for construction
Solution: See Table. 2.2.

2.3. Find the various values in Fig. 2.1
Solution: From the given information,

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \tag{2.3.2}$$

Parameters	Values
OA ($\mathbf{a}$)	$\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$
OB ($\mathbf{b}$)	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

TABLE 2.2: To construct the Parallelogram OACB

The vector $\mathbf{a}$ can also be written as:
$\mathbf{a} = 3\hat{i} + \hat{j} + 4\hat{k}$,
And the vector $\mathbf{b}$ can also be written as:
$\mathbf{b} = \hat{i} - \hat{j} + \hat{k}$
The values are listed in Table. 2.4

2.4. List the derived values.
Solution: See Table. 2.4

2.5. Draw Fig. 2.1.
Solution: The following Python code generates Fig. 2.5

```
codes/parallelogram.py
```

The following Python code verifies the cross-product value.

```
codes/cross_product_check.py
```

and the equivalent latex-tikz code generating Fig. 2.1 is

```
figs/parallelo.tex
```

3 Solution

The area of a parallelogram can be defined as:
$Area = |\mathbf{a} \times \mathbf{b}|$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} \tag{5.1}$$

Derived Values.	
O	$\begin{pmatrix}0\\0\\0\end{pmatrix}$
A	$\begin{pmatrix}3\\1\\4\end{pmatrix}$
B	$\begin{pmatrix}1\\-1\\1\end{pmatrix}$
C	$\begin{pmatrix}4\\0\\5\end{pmatrix}$

TABLE 2.4: To get the vertices of the Parallelogram *OACB*

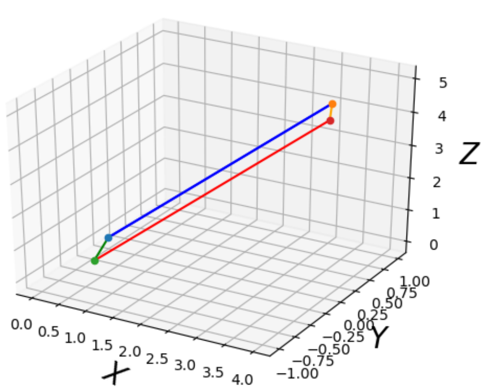

Fig. 2.5: Parallelogram generated using python 3D-plot

So, $\mathbf{a} \times \mathbf{b} = 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$.

Now, $|\mathbf{a} \times \mathbf{b}| = \sqrt{5^2 + 1^2 + (-1)^2}$
or, $|\mathbf{a} \times \mathbf{b}| = \sqrt{27}$
or, $Area = \sqrt{27}$
Hence, the area of the parallelogram in the above problem statement is $\sqrt{27}$.
or,
The cross-product can also be calculated as:

$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_x\mathbf{b}$ where $[\mathbf{a}]_x = \mathbf{a} \times \hat{e}$ and $\hat{e}$ is the unit vector.
If $\mathbf{a}$ can be expressed as:

$$\mathbf{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} = \begin{pmatrix}3\\1\\4\end{pmatrix} \tag{5.2}$$

and

$$\mathbf{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{5.3}$$

Then $[\mathbf{a}]_x$ can be expressed as:
$[\mathbf{a}]_x = [\mathbf{a} \times \hat{\imath} \quad \mathbf{a} \times \hat{\jmath} \quad \mathbf{a} \times \hat{k}]$

$$\text{or, } [\mathbf{a}]_x = \begin{pmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{pmatrix} = \begin{pmatrix}0 & -4 & 1\\ 4 & 0 & -3\\ -1 & 3 & 0\end{pmatrix} \tag{5.4}$$

So, the $[\mathbf{a}]_x\mathbf{b}$ can be calculated as:

$$[\mathbf{a}]_x\mathbf{b} = \begin{pmatrix}0 & -4 & 1\\ 4 & 0 & -3\\ -1 & 3 & 0\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}5\\1\\-4\end{pmatrix} \tag{5.5}$$

or, $[\mathbf{a}]_x\mathbf{b} = 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$.
Now, $|[\mathbf{a}]_x\mathbf{b}| = \sqrt{5^2 + 1^2 + (-1)^2}$
or, $|[\mathbf{a}]_x\mathbf{b}| = \sqrt{27}$
or, $Area = |\mathbf{a} \times \mathbf{b}| = |[\mathbf{a}]_x\mathbf{b}| = \sqrt{27}$
Hence, the area of the parallelogram in the above problem statement is $\sqrt{27}$.