Math Document Template

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***Abstract*—This is a simple document explaining how to solve equations in a matrix form.**

Download all and latex-tikz codes from

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svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Find the values of a,b,c and d from the following equation:

$$\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix} \tag{1.0.1}$$

2 Construction

2.1. List the design parameters for construction
Solution: See Table. 2.1.

Parameters	Values
$2a+b$	4
$a-2b$	-3
$5c-d$	11
$4c+3d$	24

TABLE 2.1: Values of the Equations

2.2. Verification of the solution by using python code
Solution: The following Python code verifies the solutions of a, b, c and d

```
codes/eq_verify.py
```

3 Solution

From the problem statement, we got that:

$$2a+b=4 \tag{2.1}$$

$$a-2b=-3 \tag{2.2}$$

$$2c-d=11 \tag{2.3}$$

$$4c+3d=24 \tag{2.4}$$

These equations can be written as:

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 11 \\ 24 \end{pmatrix} \tag{2.5}$$

So the coefficient matrix A can be expressed as:

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \tag{2.6}$$

And the augmented matrix B can be expressed as:

$$B = \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{pmatrix} \tag{2.7}$$

Now, if we express the augmented matrix as Echelon form, then it will be:

$$
\begin{aligned}
B &= \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{pmatrix} \\
&\xleftrightarrow[R_4 \leftarrow R_4 - R_3]{R_2 \leftarrow 2R_2 - R_1} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & -5 & 0 & 0 & -10 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & -1 & 4 & 13 \end{pmatrix} \\
&\xleftrightarrow[R_4 \leftarrow 5R_4 + R_3]{R_2 \leftarrow \frac{R_2}{(-5)}} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & 19 & 76 \end{pmatrix} \\
&\xleftrightarrow{R_4 \leftarrow \frac{R_4}{(19)}} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix} \\
&\xleftrightarrow[R_1 \leftarrow R_1 - R_2]{R_3 \leftarrow R_3 + R_4} \begin{pmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & 0 & 15 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix} \\
&\xleftrightarrow[R_1 \leftarrow \frac{R_1}{2}]{R_3 \leftarrow \frac{R_3}{5}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}
\end{aligned} \tag{2.8}
$$

So, from here we can say that the the rank of A = 4 and the rank of B = 4. So the equations have unique solution.

$$
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \tag{2.9}
$$

$\implies$ a = 1, b= 2, c= 3 and d= 4.