

Math Document Template

Jayati Dutta

Abstract—This is a simple document explaining how to solve equations in a matrix form.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

Find the values of a,b,c and d from the following equation:

$$\begin{pmatrix} 2a + b & a - 2b \\ 5c - d & 4c + 3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix} \quad (1.0.1)$$

2 CONSTRUCTION

2.1. List the design parameters for construction

Solution: See Table. 2.1.

Parameters	Values
$2a + b$	4
$a - 2b$	-3
$5c - d$	11
$4c + 3d$	24

TABLE 2.1: Values of the Equations

2.2. Verification of the solution by using python code

Solution: The following Python code verifies the solutions of a, b, c and d

codes/eq_verify.py

3 SOLUTION

From the problem statement, we got that:

$$2a + b = 4 \quad (2.1)$$

$$a - 2b = -3 \quad (2.2)$$

$$2c - d = 11 \quad (2.3)$$

$$4c + 3d = 24 \quad (2.4)$$

These equations can be written as:

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 11 \\ 24 \end{pmatrix} \quad (2.5)$$

So the coefficient matrix A can be expressed as:

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 4 & 3 \end{pmatrix} \quad (2.6)$$

And the augmented matrix B can be expressed as:

$$B = \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{pmatrix} \quad (2.7)$$

Now, if we express the augmented matrix as Echelon form, then it will be:

$$\begin{aligned}
B &= \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 4 & 3 & 24 \end{pmatrix} \\
&\xleftrightarrow[R_4 \leftarrow R_4 - R_3]{R_2 \leftarrow 2R_2 - R_1} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & -5 & 0 & 0 & -10 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & -1 & 4 & 13 \end{pmatrix} \\
&\xleftrightarrow[R_4 \leftarrow 5R_4 + R_3]{R_2 \leftarrow \frac{R_2}{(-5)}} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & 19 & 76 \end{pmatrix} \\
&\xleftrightarrow[R_4 \leftarrow \frac{R_4}{(19)}]{} \begin{pmatrix} 2 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & -1 & 11 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}
\end{aligned} \tag{2.8}$$

So, from here we can say that the rank of $A = 4$ and the rank of $B = 4$. So the equations have unique solution.

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 11 \\ 4 \end{pmatrix} \tag{2.9}$$

This indicates that:

$$2a + b = 4$$

$$\text{and } b = 2$$

$$\implies a = 1.$$

And,

$$5c - d = 11$$

$$\text{and } d = 4$$

$$\implies c = 3.$$

Hence, $a = 1$, $b = 2$, $c = 3$ and $d = 4$.