

Math Document Template

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Abstract—This is a simple document explaining how to solve equations in a matrix form.

Download all and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

Find the values of a,b,c and d from the following equation:

$$\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix} \quad (1.0.1)$$

2 CONSTRUCTION

2.1. List the design parameters for construction

Solution: See Table. 2.1.

Parameters	Values
$2a + b$	4
$a - 2b$	-3
$5c - d$	11
$4c + 3d$	24

TABLE 2.1: To construct the Parallelogram OACB

2.2. Verification of the solution by using python code

Solution: The following Python code verifies the solutions of a, b, c and d

codes/eq_verify.py

3 SOLUTION

From the problem statement, we got that:

$$2a + b = 4 \quad (2.1)$$

$$a - 2b = -3 \quad (2.2)$$

These equations can be written as:

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad (2.3)$$

So the coefficient matrix A can be expressed as:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \quad (2.4)$$

And the augmented matrix B can be expressed as:

$$B = \begin{pmatrix} 2 & 1 & 4 \\ 1 & -2 & -3 \end{pmatrix} \quad (2.5)$$

Now, if we express the augmented matrix as Echelon form, then it will be:

$$B = \begin{pmatrix} 2 & 1 & 4 \\ 1 & -2 & -3 \end{pmatrix} \xrightarrow{R_2 \leftarrow 2R_2 - R_1} \begin{pmatrix} 2 & 1 & 4 \\ 0 & -5 & -10 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{R_2}{(-5)}} \begin{pmatrix} 2 & 1 & 4 \\ 0 & 1 & 2 \end{pmatrix} \quad (2.6)$$

So, from here we can say that the the rank of $A = 2$ and the rank of $B = 2$. So the equations have unique solution.

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (2.7)$$

This indicates that:
 $2a + b = 4$

and $b = 2$
 $\implies a = 1.$

Similarly, from the problem statement, we got that:

$$2c - d = 11 \quad (2.8)$$

$$4c + 3d = 24 \quad (2.9)$$

These equations can be written as:

$$\begin{pmatrix} 5 & -1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 24 \end{pmatrix} \quad (2.10)$$

So the coefficient matrix A can be expressed as:

$$A = \begin{pmatrix} 5 & -1 \\ 4 & 3 \end{pmatrix} \quad (2.11)$$

And the augmented matrix B can be expressed as:

$$B = \begin{pmatrix} 5 & -1 & 11 \\ 4 & 3 & 24 \end{pmatrix} \quad (2.12)$$

Now, if we express the augmented matrix as Echelon form, then it will be:

$$\begin{aligned} B = \begin{pmatrix} 5 & -1 & 11 \\ 4 & 3 & 24 \end{pmatrix} &\xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 5 & -1 & 11 \\ -1 & 4 & 13 \end{pmatrix} \xrightarrow{R_2 \leftarrow 5R_2 + R_1} \\ &\begin{pmatrix} 5 & -1 & 11 \\ 0 & 19 & 76 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{R_2}{19}} \\ &\begin{pmatrix} 5 & -1 & 11 \\ 0 & 1 & 4 \end{pmatrix} \quad (2.13) \end{aligned}$$

So, from here we can say that the the rank of $A = 2$ and the rank of $B = 2$. So the equations have unique solution.

$$\begin{pmatrix} 5 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \quad (2.14)$$

This indicates that:

$$5c - d = 11$$

$$\text{and } d = 4$$

$$\implies c = 3.$$

Hence, $a = 1$, $b = 2$, $c = 3$ and $d = 4$.