Math Document Template

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***Abstract*—This is a simple document explaining a question about the concept that the median of a triangle divides it into two triangles with equal areas.**

Download all python codes from

```
svn co https://github.com/JayatiD93/trunk/
    My_solution_design/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

Verify that the median of a triangle $\triangle ABC$ with vertices A = $(4\ -6)^T$, B = $(3\ -2)^T$, C = $(5\ 2)^T$ divides it into two triangles of equal areas.

2 Construction

2.1. The figure for the triangle obtained in the question looks like Fig. 2.1. with vertices A,B and C

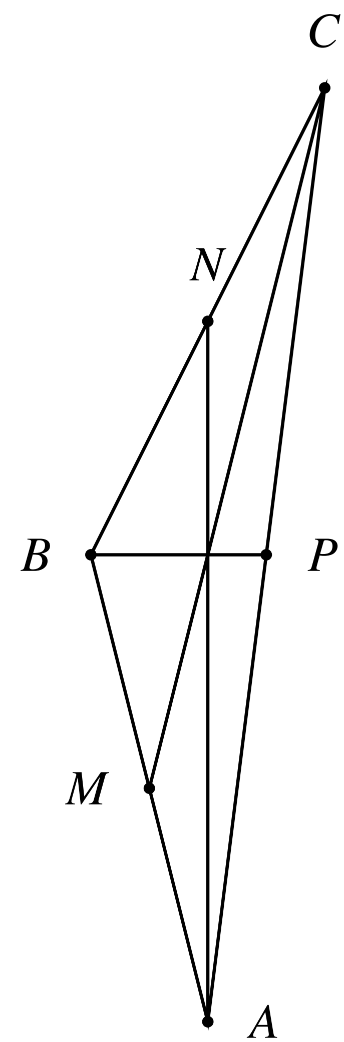

Fig. 2.1: Scalene Triangle by Latex-Tikz

2.2. List the design parameters for construction
Solution: See Table. 2.2.

Parameter	Value
AB (c)	4.12
BC (a)	4.47
CA (b)	8.06

TABLE 2.2: To construct $\triangle ABC$

2.3. Find the coordinates of the various points in Fig. 2.1
Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \tag{2.3.3}$$

$\because$ **M** is the midpoint of AB,

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} 7 \\ -8 \end{pmatrix} \tag{2.3.4}$$

$\because$ **N** is the midpoint of BC,

$$\mathbf{N} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{2.3.5}$$

$\because$ **P** is the midpoint of CA,

$$\mathbf{P} = \frac{\mathbf{C}+\mathbf{A}}{2} = \frac{1}{2}\begin{pmatrix} 9 \\ -4 \end{pmatrix} \tag{2.3.6}$$

The values are listed in Table. 2.3

Derived Values.	
M	$\begin{pmatrix} 3.5 \\ -4 \end{pmatrix}$
N	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$
P	$\begin{pmatrix} 4.5 \\ -2 \end{pmatrix}$

TABLE 2.3: To get Mid-points of $\triangle ABC$

2.4. Draw Fig. 2.1.
Solution: The following Python code generates Fig. 2.4

```
codes/triangle.py
```

The following Python code verifies the determinant values.

```
codes/determinant_check.py
```

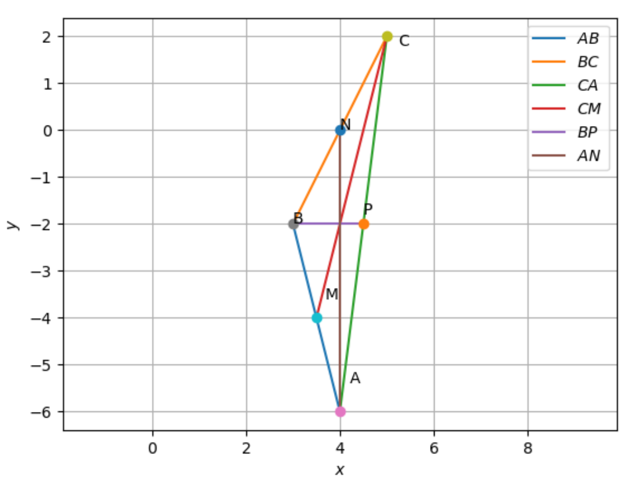

Fig. 2.4: Triangle generated using python

and the equivalent latex-tikz code generating Fig. 2.1 is

```
figs/triangle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle_fig.tex
```

3 Solution

From the values of the parameters of table, it is clear that the triangle $\triangle ABC$ is scalens.

For $\triangle ABC$, the vertices are **A** , **B** and **C**. So the area of the triangle $\triangle ABC$ by using determinant will be :

$$\begin{aligned} Area = \frac{1}{2}\begin{vmatrix} 4 & -6 & 1 \\ 3 & -2 & 1 \\ 5 & 2 & 1 \end{vmatrix} &\xleftrightarrow{C_2\leftarrow\frac{C_2}{2}} \frac{2}{2}\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix} \\ &\xleftrightarrow[R_3\leftarrow R_3-R_1]{R_2\leftarrow R_2-R_1} \begin{vmatrix} 4 & -3 & 1 \\ -1 & 2 & 0 \\ 1 & 4 & 0 \end{vmatrix} \\ &\xleftrightarrow{R_3\leftarrow R_3+R_2} \begin{vmatrix} 4 & -3 & 1 \\ -1 & 2 & 0 \\ 0 & 6 & 0 \end{vmatrix} \\ &\xleftrightarrow{R_3\leftarrow\frac{R_3}{6}} 6\begin{vmatrix} 4 & -3 & 1 \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} \\ &= -6 \end{aligned} \tag{4.1}$$

Now, we will consider the absolute value of area only. So, $Area = |-6| = 6$.
To verify the problem statement we have to check 3 cases:
Case 1: When **BP** is median, we will consider $\triangle ABP$ triangle. In that case, the vertices will be **A**, **B** and **P**.
Now, the area of $\triangle ABP$ will be :

$$\begin{aligned} A1 = \frac{1}{2}\begin{vmatrix} 4 & -6 & 1 \\ 3 & -2 & 1 \\ 4.5 & -2 & 1 \end{vmatrix} &\xleftrightarrow{C_2\leftarrow\frac{C_2}{(-2)}} \frac{(-2)}{2}\begin{vmatrix} 4 & 3 & 1 \\ 3 & 1 & 1 \\ 4.5 & 1 & 1 \end{vmatrix} \\ &\xleftrightarrow[R_3\leftarrow R_3-R_1]{R_2\leftarrow R_2-R_1} (-1)\begin{vmatrix} 4 & 3 & 1 \\ -1 & -2 & 0 \\ 0.5 & -2 & 0 \end{vmatrix} \\ &\xleftrightarrow{R_3\leftarrow R_3-R_2} (-1)\begin{vmatrix} 4 & 3 & 1 \\ -1 & -2 & 0 \\ 1.5 & 0 & 0 \end{vmatrix} \\ &= -3 \end{aligned} \tag{4.2}$$

But, we will consider the absolute value of area only. So, $\mathbf{A1} = |-3| = 3$.

or, **A1** = $\frac{1}{2}$(Area of $\triangle ABC$)
Case 2: When **AN** is median,we will consider $\triangle ABN$ triangle. In that case, the vertices will be **A**, **B** and **N**.
Now, the area of $\triangle ABN$ will be :

$$\begin{aligned} A2 = \frac{1}{2}\begin{vmatrix}4 & -6 & 1\\ 3 & -2 & 1\\ 4 & 0 & 1\end{vmatrix} &\xleftrightarrow{C_2\leftarrow\frac{C_2}{(-2)}} \frac{(-2)}{2}\begin{vmatrix}4 & 3 & 1\\ 3 & 1 & 1\\ 4 & 0 & 1\end{vmatrix} \\ &\xleftrightarrow[R_3\leftarrow R_3-R_1]{R_2\leftarrow R_2-R_1} (-1)\begin{vmatrix}4 & 3 & 1\\ -1 & -2 & 0\\ 0 & -3 & 0\end{vmatrix} \\ &\xleftrightarrow{R_3\leftarrow\frac{R_3}{(-3)}} 3\begin{vmatrix}4 & 3 & 1\\ -1 & -2 & 0\\ 0 & 1 & 0\end{vmatrix} \\ &= -3 \end{aligned} \tag{4.3}$$

But, we will consider the absolute value of area only. So, **A2** = $|-3|$ = 3.
or, **A2** = $\frac{1}{2}$(Area of $\triangle ABC$)
Case 3: When **CM** is median,we will consider $\triangle CAM$ triangle. In that case, the vertices will be **A**, **C** and **M**.
Now, the area of $\triangle CAM$ will be :

$$\begin{aligned} A3 = \frac{1}{2}\begin{vmatrix}5 & 2 & 1\\ 4 & -6 & 1\\ 3.5 & -4 & 1\end{vmatrix} &\xleftrightarrow{C_2\leftarrow\frac{C_2}{2}} \frac{2}{2}\begin{vmatrix}5 & 1 & 1\\ 4 & -3 & 1\\ 3.5 & -2 & 1\end{vmatrix} \\ &\xleftrightarrow[R_3\leftarrow R_3-R_1]{R_2\leftarrow R_2-R_1} \begin{vmatrix}5 & 1 & 1\\ -1 & -4 & 0\\ -1.5 & -3 & 0\end{vmatrix} \\ &\xleftrightarrow[R_3\leftarrow\frac{R_3}{(-1.5)}]{R_2\leftarrow\frac{R_2}{(-1)}} 1.5\begin{vmatrix}5 & 1 & 1\\ 1 & 4 & 0\\ 1 & 2 & 0\end{vmatrix} \\ &\xleftrightarrow{R_3\leftarrow R_3-R_2} 1.5\begin{vmatrix}5 & 1 & 1\\ 1 & 4 & 0\\ 0 & -2 & 0\end{vmatrix} \\ &= -3 \end{aligned} \tag{4.4}$$

But, we will consider the absolute value of area only. So, **A3** = $|-3|$ = 3.
or, **A3** = $\frac{1}{2}$(Area of $\triangle ABC$)

Hence, the above problem statement is verified.