Introduction to crisp set

- Crisp set is a collection of **unordered distinct** elements, which are derived from a Universal set.
- A Universal set consists of all possible elements which take part in any experiment.
- A set is a quite useful and important way of representing data.

  Let X represents a set of natural numbers, so
  X = {1, 2, 3, 4, …}
  Sets are always defined with respect to some universal set. Let us derive two sets A and B from this universal set X.
  A = Set of even numbers = {2, 4, 6, …}
  B = Set of odd number = {1, 3, 5, …}

- Elements in the set are **unique**, i.e. A = {1, 1, 2, 2, 3, 3}, B = {1, 2, 3}, C = {1, 2, 2, 3, 3, 3} all are the same.
- **The order of elements** in the set is not important, i.e. A = {1, 2, 3}, B = {2, 1, 3}, C = {3, 1, 2}, all correspond to identical set.
- The element of the set is called a **member of the set. If any element is present in the set then it is considered a member of the set otherwise it is not a** member.
- In a crisp set, there is no concept of partial membership. Element is either fully present in the set or it is fully outside the set.
- A crisp set is very important to model or represents many real-world entities, such as a set of boys, a set of books, a set of elements, a set of employees, a set of colours etc.

The membership function can be used to define a set A given by

$$\chi A(x)=\{1,0, if x \in A if x \notin A$$

The function χ (read as '*chi*') is known as the crisp **membership function**, which assigns membership value to the element of the universal set based on certain properties.

The best example of crisp set representation is the number system in mathematics, where,

- N: Set of natural numbers
- R: Set of real numbers
- Z: Set of integers
- Q: Set of rational numbers

Notations used in Crisp Set

We will discuss the various set notations with respect to the following sets:

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8, 10}
B = {1, 3, 5, 7, 9}
C = {4, 6, 8}
D = {x | x is perfect square and x > 10} = Φ
Various notations used in set theory are defined below:

- Φ: **Empty set** is represented by the symbol, Φ is a set which does not have any element in it. For given data, D = Φ
- x ∈ A represents element x is a **member** of set A. For given data, 2 ∈ A
- x ∉ A represents an element x that is **not a member** of set A. For given data, 3 ∉ A
- A ⊆ B represents every element of set A that is present in set B as well. In other words, A is a **subset** of B. For given data, A ⊆ X
- A ⊇ B represents every element of B is a member of set A as well. In other words, A is a **superset** of B. For given sets, A ⊇ C

- A ⊂ B represents every element of A in B as well as B has some additional element which is not in A. This notation says that A is a **proper subset** of B.
- A ⊃ B represents all the elements of B in set A as well as A has some additional element which is not in B. This notation says that A is a **proper superset** of B.
- if set A and B are identical then we can say A is a subset of B or B is a subset of A, but we cannot say that A is a proper superset of B or A is a proper subset of B
- A = B represents **Equal sets**, i.e. sets A and B have identical elements
- A ≠ B represents **Not equal sets**, i.e. sets A and B have different elements. For given sets, A ≠ B
- |A| represents the **Cardinality** of set A (i.e. a number of elements in set A). For given sets, |A|= 5
- p(A) represents the **Power set** of set A. For the given sets, p(c) = { Φ, {4}, {6}, {8}, {4, 6}, {4, 8}, {6, 8}, {4, 6, 8} }
- |p(A)|: $2^{|A|}$, i.e. power set of any set contains 2^n elements

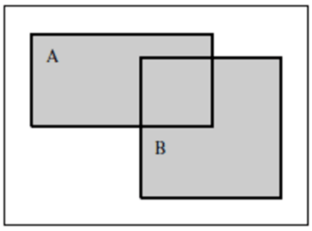
Operations on CRISP Set

We can perform a wide range of operations on a crisp set. Before you explore the operations of the crisp set, it is recommended to understand what a crisp set is.
Let us understand various operations on set with the help of examples. We will consider the following data to execute various operations:
X = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {1, 2, 3, 4, 5}
B = {3, 4, 5, 6}
C = {6, 7, 8, 9}

1: Union of sets is the collection of all the elements which are either in A **or** in B. Common elements from both sets are considered only once. Mathematically, we can represent union operation as follow:

A ∪ B = { x | x ∈ A **or** x ∈ B }

For the given data
A ∪ B = {1, 2, 3, 4, 5, 6}
Graphically, we can describe union operation as shown below. The grey region represents the output of the operation.



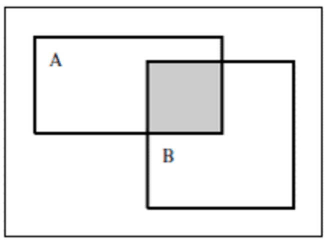
Union of crisp sets
If there are *n* sets, called $A_1, A_2, A_3, \ldots, A_n$, we can find the union of all by taking **unique elements** from each set, i.e. $A = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$
In shortened notation,

$$A=\bigcup_{i=1}^{n} A_i$$

2: Intersection of sets is the collection of all the common elements from sets A **and** B. Mathematically, we can represent intersection operation as follow:

A ∩ B = { x | x ∈ A **and** x ∈ B }

For the given data
A ∩ B = {3, 4, 5}
Graphically, we can describe intersection operation as shown below. The grey region represents the output of the operation.



The intersection of crisp sets
If there are *n* sets, called $A_1, A_2, A_3, \ldots, A_n$, we can find the intersection of all by taking **common elements** from each set, i.e. $A = A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$
In shortened notation,

$$A=\bigcap_{i=1}^{n} A_i$$

3: Complement operation is always represented with respect to some set. If complement is performed with respect to a universal set, then it is called **absolute complement**.

The complement of set A is a collection of all the elements which are **not in** A but are in the universal set. Mathematically,

$$A' = A^c = X - A = \{ x \mid x \in X \textbf{ and } x \notin A \}$$

The complement of set A is often represented as A' or A^c or

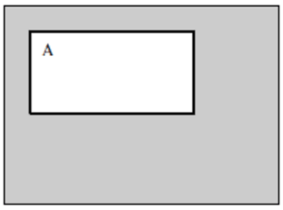
$$\bar{A}$$

. We will be using any notation interchangeably in our discussion.

For the given data

A' = {6, 7, 8, 9}

Graphically, we can describe the complement operation as shown below. The grey region represents the output of the operation.



4: Difference between set A with respect to set B is the collection of all the elements in A but not in B. It is also known as a **relative complement**.

Here the reference set is not the universal set, rather it is some set derived from the universe of discourse

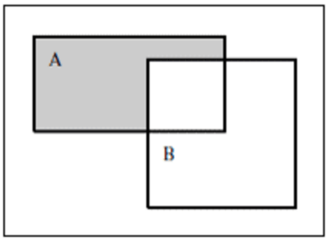
Mathematically,

$$A - B = \{ x \mid x \in A \textbf{ and } x \notin B \}$$

For the given data

A – B = {1, 2}

Graphically, difference operation can be represented as,



Difference of crisp sets

5: De Morgan's law is very popular in set operations and it is quite useful in simplifying many complex computations. It is also useful in reducing the process of some proof techniques. De Morgan's law enjoys a special place in crisp set operations. There are two laws of De Morgan.

Law 1: $(A \cup B)' = A' \cap B'$

For the given data:

$A \cup B$ = {1, 2, 3, 4, 5, 6}

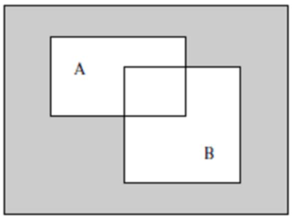
$(A \cup B)'$ = {7, 8, 9} → LHS

A' = {6, 7, 8, 9}

B' = {1, 2, 7, 8, 9}

$A' \cap B'$ = {7, 8, 9} → RHS

Graphically,



De Morgan's Law – 1

Law 2: $(A \cap B)' = A' \cup B'$

For the given data:

$A \cap B$={3, 4, 5}

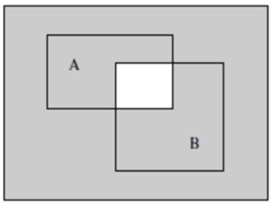
$(A \cap B)'$ = {1, 2, 6, 7, 8, 9} → LHS

A' = {6, 7, 8, 9}

B' = {1, 2, 7, 8, 9}

$A' \cup B'$ = {1, 2, 6, 7, 8, 9} → RHS

Graphically,



De Morgan's Law – 2

Properties of CRISP Set

Crisp set possesses the following properties. We will demonstrate each of them with a suitable example. We will be using the following sets for further discussion:

X = {1, 2, 3, 4, 5, 6}

A = {1, 2, 3},

B = {2, 3, 4},

C = {5, 6}

Involution:

Involution states that the complement of complement of set A would be set A itself.
For the given data,
$A' = X - A = \{4, 5, 6\}$
$(A')' = X - A' = \{1, 2, 3\} = A$

Commutativity:

The commutativity property states that the operation can be performed irrespective of the order of the operand. For example, addition is a commutative operator, so 2 + 3 or 3 + 2 yields the same result. But, subtraction is not commutative, so $3 - 2 \neq 2 - 3$.
Proving union is commutative:
$A \cup B = \{1, 2, 3, 4\} \rightarrow$ LHS
$B \cup A = \{1, 2, 3, 4\} \rightarrow$ RHS
Proving intersection is commutative:
$A \cap B = \{2, 3\} \rightarrow$ LHS
$B \cap A = \{2, 3\} \rightarrow$ RHS

Associativity:

The associativity property allows us to perform the operations by grouping the operands and keeping them in similar order.
$(A \cup B) \cup C = A \cup (B \cup C)$
For given data:
$A \cup B = \{1, 2, 3, 4\}$
$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \rightarrow$ LHS
$B \cup C = \{2, 3, 4, 5, 6\}$
$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6\} \rightarrow$ RHS
$(A \cap B) \cap C = A \cap (B \cap C)$
For given data:
$A \cap B = \{2, 3\}$
$(A \cap B) \cap C = \varphi \rightarrow$ LHS
$B \cap C = \varphi$
$A \cap (B \cap C) = \varphi \rightarrow$ RHS

Distributivity:

Mathematically it is defined as,
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$B \cap C = \varphi$
$A \cup (B \cap C) = \{1, 2, 3\} \rightarrow$ LHS
$A \cup B = \{1, 2, 3, 4\}$
$A \cup C = \{1, 2, 3, 5, 6\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3\} \rightarrow$ RHS
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$B \cup C = \{2, 3, 4, 5, 6\}$
$A \cap (B \cup C) = \{2, 3\} \rightarrow$ LHS
$A \cap B = \{2, 3\}$
$A \cap C = \varphi$
$(A \cap B) \cup (A \cap C) = \{2, 3\} \rightarrow$ RHS

Absorption:

Mathematically absorption is defined as,
$A \cup (A \cap B) = A$
For the given data:
$A \cap B = \{2, 3\}$

$A \cup (A \cap B) = \{1, 2, 3\} = A$

$A \cap (A \cup B) = A$

For the given data:

$A \cup B = \{1, 2, 3, 4\}$

$A \cap (A \cup B) = \{1, 2, 3\} = A$

Idempotency/Tautology:

Idempotency is defined as,

$A \cup A = A$

$A \cap A = A$

For the given data,

$A \cup A = \{1, 2, 3\} = A$

$A \cap A = \{1, 2, 3\} = A$

Identity:

Mathematically, we can define this property as,

$A \cup X = X$

$A \cap X = A$

$A \cup \varphi = A$

$A \cap \varphi = \varphi$

For the given data,

$A \cup X = \{1, 2, 3, 4, 5, 6\} = X$

$A \cap X = \{1, 2, 3\} = A$

$A \cup \varphi = \{1, 2, 3\} = A$

$A \cap \varphi = \{\ \} = \varphi$

De Morgan's Laws:

Mathematically, De Morgan's laws are defined as,

Law 1: $(A \cup B)' = A' \cap B'$

For the given data:

$A \cup B = \{1, 2, 3, 4\}$

$(A \cup B)' = \{5, 6\} \rightarrow$ LHS

$A' = \{4, 5, 6\}$

$B' = \{1, 5, 6\}$

$A' \cap B' = \{5, 6\} = (A \cup B)' \rightarrow$ RHS

$(A \cup B)' = A' \cap B'$

Law 2: $(A \cap B)' = A' \cup B'$

For the given data:

$A \cap B = \{2, 3\}$

$(A \cap B)' = \{1, 4, 5, 6\} \rightarrow$ LHS

$A' = \{4, 5, 6\}$

$B' = \{1, 5, 6\}$

$A' \cup B' = \{1, 4, 5, 6\} = (A \cap B)' \rightarrow$ RHS

$(A \cap B)' = A' \cup B$

Law of Contradiction:

Mathematically it is defined as,

$A \cap A' = \varphi$

For the given data:

$A' = \{4, 5, 6\}$

$A \cap A' = \{\ \} = \varphi$