Fuzzy Logic

What is Fuzzy Logic?

- We, as humans, don't always think in **binary terms** like computers do (i.e., 0 or 1). Sometimes, we think in **gray areas**.

- For example, when you say "It's warm today," you're not necessarily saying it's **hot** or **cold**. It's somewhere in between.

- Fuzzy Logic is a concept that deals with **uncertainty and approximation**—the kind of reasoning we do every day.

- It allows us to represent **vague** or **imprecise** information mathematically, which is useful when you need to make decisions with less-than-perfect data.

Crisp Logic v/s Fuzzy Logic

- In **Crisp Logic**, everything is **binary**. Something is either **true** or **false**. Think of this as a light switch—either the light is **on** or **off**.

- In **Fuzzy Logic**, things are not just true or false. They're on a **scale**. So, something could be **partially true** or **partially false**. For example, the temperature might be **60% hot**, or you might be **70% sure** about something.

- Example:-
  - ➢**Crisp Logic**: Temperature is either **hot** (above 30°C) or **cold** (below 30°C).
  - ➢**Fuzzy Logic**: Temperature could be **partly hot**, **partly cold**, or even **partly warm**.

Why Fuzzy Logic?

- Fuzzy Logic is **useful** because life is not always black-and-white, and often, you need a more nuanced approach to deal with complexity and uncertainty. Here's why we use it:

  ➤**Human-Like Thinking**: Just like how humans use approximate reasoning (e.g., "it's kinda hot"), fuzzy logic mimics this in machines.

  ➤**Handling Uncertainty**: In many real-world scenarios (like weather or human behavior), exact values aren't always available. Fuzzy logic works when you have **imperfect** or **ambiguous** data.

  ➤**Flexibility**: It can be applied to areas where traditional binary logic struggles, like **control systems**, **AI**, and **decision-making**.

Components of a Fuzzy Logic System

- A fuzzy logic system typically has **four main parts**:
    - ➢Fuzzification
    - ➢Rule Base
    - ➢Inference Engine
    - ➢Defuzzification

Fuzzification

- Fuzzification is the process of converting **crisp** inputs (like a temperature value) into **fuzzy values**. You take a precise, measurable value (like 25°C) and map it into fuzzy sets (like **"cold," "warm," and "hot"**).

- **Example:**
  ➢Temperature = 25°C
  ➢**Fuzzification**: The system decides that 25°C is **partially cold**, **mostly warm**, and **slightly hot**.

Contd.

- In fuzzy logic, instead of saying "temperature is 25°C," we say the **degree of membership** of that temperature in the fuzzy sets is:
    - ➤Cold: 0.2 (20% cold)
    - ➤Warm: 0.7 (70% warm)
    - ➤Hot: 0.1 (10% hot)
- This is how fuzzification works—**converting crisp inputs to fuzzy sets**.

Rule Base

- The rule base is essentially a set of **if-then** rules that define the behavior of the fuzzy system.

- **Example:**
  - ➤**Rule 1**: IF temperature is **cold**, THEN fan speed is **low**.
  - ➤**Rule 2**: IF temperature is **warm**, THEN fan speed is **medium**.
  - ➤**Rule 3**: IF temperature is **hot**, THEN fan speed is **high**.

- These rules help the system decide what to do based on the fuzzy inputs.

- So, we take the fuzzified input (say, temperature = 25°C) and apply the corresponding rules. The system looks at how **cold, warm, or hot** the temperature is and uses the rules to determine the right action (like fan speed).

Inference Engine

- The inference engine takes the **fuzzified inputs** and applies the **rule base** to **generate fuzzy outputs**.

- The inference engine uses the fuzzy rules to draw conclusions about the system's behavior.

- **Example:**
  - ➢Given that 25°C is **70% warm**, the inference engine will look at the rule base:
  - ➢Rule 2 (IF temperature is warm THEN fan speed is medium) applies strongly.
  - ➢So, the inference engine will likely suggest that the **fan speed should be medium**, based on the **degree of membership** of the input.

Defuzzification

- Defuzzification is the process of **converting fuzzy output** back into a crisp value (i.e., something we can use in the real world, like setting the fan speed to a specific number).

- The fuzzy output from the inference engine (like "medium fan speed") needs to be **converted** into a clear, precise value (like a speed setting of 3).

- **Example:**
  - ➢If the inference engine suggests that the fan speed should be **partially medium** and **partially high**, defuzzification will combine those fuzzy outputs and convert them into a single crisp value (e.g., fan speed = 5 on a scale of 1 to 10).
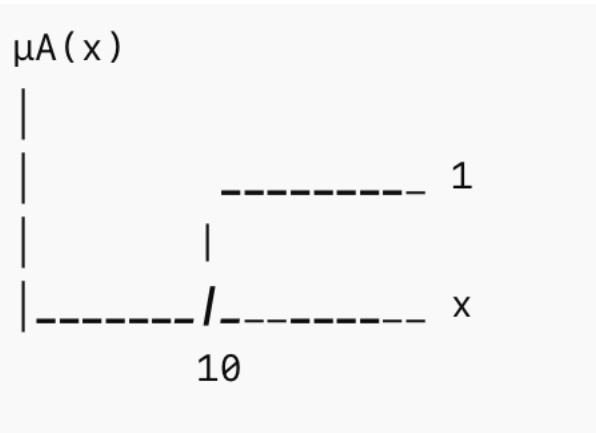
Practical Example:

- Imagine you have a **smart air conditioning system** that uses fuzzy logic to control the temperature of a room:

- **Step 1 (Fuzzification)**: The temperature sensor reads **25°C**. It maps this temperature into fuzzy sets: **20% cold, 70% warm, 10% hot**.

- **Step 2 (Rule Base)**: The system applies the rules based on fuzzified inputs:
  - ➢IF temperature is cold → THEN fan speed is low.
  - ➢IF temperature is warm → THEN fan speed is medium.
  - ➢IF temperature is hot → THEN fan speed is high.

- **Step 3 (Inference Engine)**: The system combines the rules and suggests that the fan speed should be medium (since the temperature is 70% warm).

- **Step 4 (Defuzzification)**: The system converts this fuzzy result into a crisp value, say **fan speed = 5**.

Summary

- So, to recap:

- **Fuzzy Logic** deals with uncertainty and approximation, just like how we think in everyday life.

- We contrast **crisp logic** (binary) with **fuzzy logic** (ranges, degrees).

- The **fuzzy logic system** works through:
  - ➢**Fuzzification**: Converting crisp inputs into fuzzy values.
  - ➢**Rule Base**: Using rules to define system behavior.
  - ➢**Inference Engine**: Processing the rules and inputs to infer outputs.
  - ➢**Defuzzification**: Converting fuzzy outputs back into crisp values.

Crisp Set

- A Crisp Set (also called a Classical Set) is a collection of objects having clearly defined boundaries- an element either belongs to the set or does not belong to it.

- Each element has membership value = 1 (if it belongs) or membership value = 0 (if it doesn't).

- Example: Let $A = \{x \mid x \geq 10\}$

- For $x = 12 \to$ member of A $\to \mu A(x) = 1$ & For $x = 8 \to$ not a member $\to \mu A(x) = 0$

- Hence, in a crisp set: $\mu A(x) \in \{0, 1\}$

```
μA(x)
|
|
|            --------- 1
|           |
|_____/_____ x
        10
```

Up to x = 9 → 0 (not in set)

From x = 10 onwards → 1 (in set)

Operations on Crisp Sets

- Let U = Universal Set
  Let A and B be subsets of U.

- **Union (A $\cup$ B)**-The set of all elements that belong to A or B or both.
  Formula:
  A $\cup$ B = {x | x $\in$ A or x $\in$ B}

- Example: A = {1,2,3}, B = {3,4,5}   then A $\cup$ B = {1,2,3,4,5}

Contd...

- **Intersection (A ∩ B)-** The set of all elements common to both A and B.
- Formula:
  A ∩ B = {x | x ∈ A and x ∈ B}

Example:

A = {1,2,3}, B = {3,4,5}

→ A ∩ B = {3}

Contd...

- **Complement (A')-**The set of all elements in the universal set U that are not in A.
  Formula:
  A' = {x | x $\in$ U and x $\notin$ A}

Example:

U = {1,2,3,4,5}, A = {2,4}

→ A' = {1,3,5}

Contd…

- **Difference (A − B)**-The set of elements that are in A but not in B.
  Formula:
  A − B = {x | x $\in$ A and x $\notin$ B}

- Example:
  A = {1,2,3,4}, B = {3,4,5} then A − B = {1,2}

- **Cartesian Product (A × B)-**The set of ordered pairs (a,b) where a $\in$ A and b $\in$ B.
  Example: A = {1,2}, B = {x,y} then A × B = {(1,x), (1,y), (2,x), (2,y)}

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Properties of Crisp Sets

Property	Formula / Rule
Commutative Law	$A \cup B = B \cup A$, $A \cap B = B \cap A$
Associative Law	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive Law	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Law	$A \cup \varnothing = A$, $A \cap U = A$
Complement Law	$A \cup A' = U$, $A \cap A' = \varnothing$
Idempotent Law	$A \cup A = A$, $A \cap A = A$
Domination Law	$A \cup U = U$, $A \cap \varnothing = \varnothing$
De Morgan's Law	$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

Practice

- U = {1,2,3,4,5,6}, A = {2,4,6}, B = {1,2,3}
  Find:

- A ∪ B

- A ∩ B

- A'

- (A – B)

- Verify De Morgan's Law for the above sets.

Practice

- Let
  U = {1, 2, 3, 4, 5, 6, 7, 8}.   A = {2, 4, 6, 8}.   B = {1, 2, 3, 4, 5}

- Find: $A \cup B$, $A \cap B$, A', B', $A - B$ and $B - A$

- Given
  U = {a, b, c, d, e, f, g}   A = {a, c, e, g}.   B = {b, c, d, e}

- Find:

- $(A \cup B)'$, $(A' \cap B')$ and Verify De Morgan's Law: $(A \cup B)' = A' \cap B'$