```
with(plots) :
```

#Calculations

```
Inveps := 1 - (es - 1)/(es + 1) · exp(-2·h·k);
```

$$Inveps := 1 - \frac{(es-1)\,e^{-2\,h\,k}}{es+1} \tag{1}$$

```
K := 2·Pi·n0/(v^2 - alpha);
```

$$K := \frac{2\,\pi\,n0}{v^2-\alpha} \tag{2}$$

```
L := 2·Pi·n0/alpha;
```

$$L := \frac{2\,\pi\,n0}{\alpha} \tag{3}$$

```
assume(KK > 0);
assume(hh > 0) :
assume(ees > 1);
solve(1 - (ees - 1)/(ees + 1) · exp(-2·hh·k) - k/KK, k);
```

$$\frac{2\,KK\sim\,hh\sim + \mathrm{LambertW}\left(-\frac{2\,KK\sim\,hh\sim\,(ees\sim-1)\,e^{-2\,KK\sim\,hh\sim}}{ees\sim+1}\right)}{2\,hh\sim} \tag{4}$$

```
kc := (2·K·h + LambertW(-2·K·h·(es - 1)·exp(-2·K·h)/(es + 1)))/(2·h);
```

$$kc := \frac{\frac{4\,\pi\,n0\,h}{v^2-\alpha} + \mathrm{LambertW}\left(-\frac{4\,\pi\,n0\,h\,(es-1)\,e^{-\frac{4\,\pi\,n0\,h}{v^2-\alpha}}}{(v^2-\alpha)\,(es+1)}\right)}{2\,h} \tag{5}$$

```
Fs := K·sqrt(1 - alpha/v^2)·Heaviside(v - sqrt(alpha))·Int(Inveps^2·exp(-2·k·z0)
      ·sqrt(k/(k - K·Inveps)), k = kc..infinity);
```

$$Fs := \frac{1}{v^2-\alpha}\left(2\,\pi\,n0\,\sqrt{1-\frac{\alpha}{v^2}}\ \text{Heaviside}\left(v-\sqrt{\alpha}\right)\left(\int_{\frac{4\,\pi\,n0\,h}{v^2-\alpha}+\text{LambertW}\left(-\frac{4\,\pi\,n0\,h\,(es-1)\,e^{-\frac{4\,\pi\,n0\,h}{v^2-\alpha}}}{(v^2-\alpha)\,(es+1)}\right)}^{\infty}\left(1-\frac{(es-1)\,e^{-2\,h\,k}}{es+1}\right)^2 e^{-2\,k\,z0}\sqrt{\frac{k}{-\frac{2\left(1-\frac{(es-1)\,e^{-2\,h\,k}}{es+1}\right)\pi\,n0}{v^2-\alpha}+k}}\,dk\right)\right) \tag{6}$$

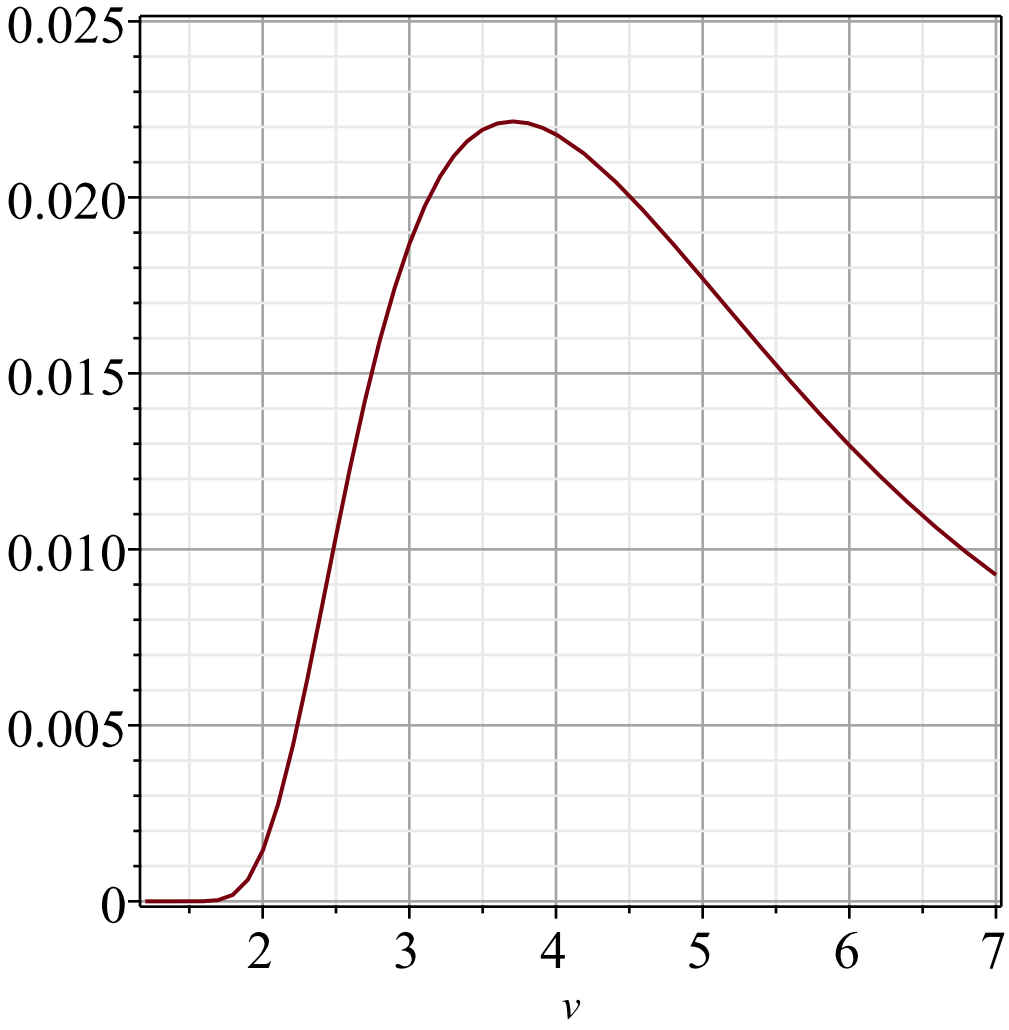
$evalf(subs(v=4, z0=3, es=3.9, h=0.1, \text{alpha}=\text{Pi}\cdot n0, n0=0.428, Fs));$

$$0.005753723325 \tag{7}$$

$vthr := evalf(\text{sqrt}(\text{Pi}\cdot 0.428));$

$$vthr := 1.159569599 \tag{8}$$

$plot(subs(z0=3, es=3.9, h=10, \text{alpha}=\text{Pi}\cdot n0, n0=0.428, Fs), v=1.2..7, 0..0.025, axes=boxed, numpoints=30);$



$$Fup := \frac{1}{4\cdot(z0+h)^2}\cdot\frac{es-1}{es+1} + \text{Heaviside}(v-\text{sqrt}(\text{alpha}))\cdot Int\left(\frac{k\cdot Inveps^2\cdot\exp(-2\cdot k\cdot z0)}{\text{sqrt}\left(Inveps-\frac{k}{K}\right)\cdot\text{sqrt}\left(Inveps+\frac{k}{L}\right)}, k=0..kc\right);$$

$$Fup := \frac{es-1}{4\,(z0+h)^2\,(es+1)} + \text{Heaviside}\left(v-\sqrt{\alpha}\right)\left(\tag{9}$$

$$\frac{\frac{4\pi n0\,h}{v^2-\alpha}+\mathrm{LambertW}\left(-\frac{4\pi n0\,h\,(es-1)\,e^{-\frac{4\pi n0\,h}{v^2-\alpha}}}{(v^2-\alpha)\,(es+1)}\right)}{2h}$$

$$\int_0 \frac{4k\left(1-\frac{(es-1)\,e^{-2hk}}{es+1}\right)^2 e^{-2kz0}}{\sqrt{4-\frac{4\,(es-1)\,e^{-2hk}}{es+1}-\frac{2k\,(v^2-\alpha)}{\pi n0}}\sqrt{4-\frac{4\,(es-1)\,e^{-2hk}}{es+1}+\frac{2k\alpha}{\pi n0}}}\,dk$$

evalf(*subs*(*v* = 4, *z0* = 3, *es* = 3.9, *h* = 0.1, alpha = Pi·*n0*, *n0* = 0.428, *Fup*));

0.01759576092 **(10)**

pup := *plot*(*subs*(*z0* = 3, *es* = 3.9, *h* = 10, alpha = Pi·*n0*, *n0* = 0.428, *Fup*), *v* = 1.2 ..7, *axes* = *boxed*, *numpoints* = 20) :

$$Flow := \frac{1}{4\cdot(z0+h)^2}\cdot\frac{es-1}{es+1}+\mathrm{Heaviside}(-v+\mathrm{sqrt}(\mathrm{alpha}))\cdot Int\left(\frac{k\cdot Inveps^2\cdot\exp(-2\cdot k\cdot z0)}{\mathrm{sqrt}\left(Inveps-\frac{k}{K}\right)\cdot\mathrm{sqrt}\left(Inveps+\frac{k}{L}\right)},\ k=0\,..\,\mathrm{infinity}\right);$$

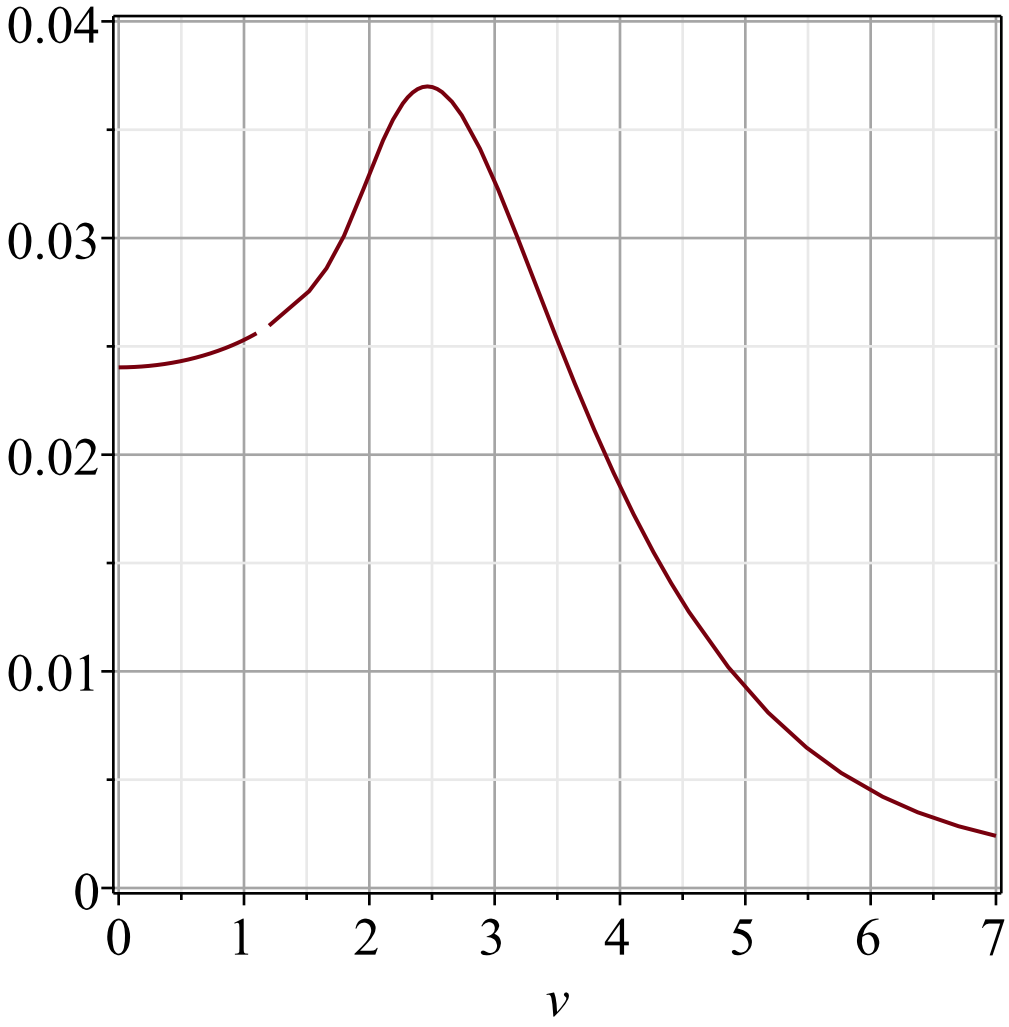
$$Flow := \frac{es-1}{4\,(z0+h)^2\,(es+1)}+\mathrm{Heaviside}\left(-v+\sqrt{\alpha}\right)\left(\int_0^{\infty}\frac{4k\left(1-\frac{(es-1)\,e^{-2hk}}{es+1}\right)^2 e^{-2kz0}}{\sqrt{4-\frac{4\,(es-1)\,e^{-2hk}}{es+1}-\frac{2k\,(v^2-\alpha)}{\pi n0}}\sqrt{4-\frac{4\,(es-1)\,e^{-2hk}}{es+1}+\frac{2k\alpha}{\pi n0}}}\,dk\right) \quad \textbf{(11)}$$

evalf(*subs*(*v* = 0, *z0* = 3, *es* = 3.9, *h* = 0.1, alpha = Pi·*n0*, *n0* = 0.428, *Flow*));

0.02465049890 **(12)**

plo := *plot*(*subs*(*z0* = 3, *es* = 3.9, *h* = 10, alpha = Pi·*n0*, *n0* = 0.428, *Flow*), *v* = 0 ..1.1, *axes* = *boxed*, *numpoints* = 10) :

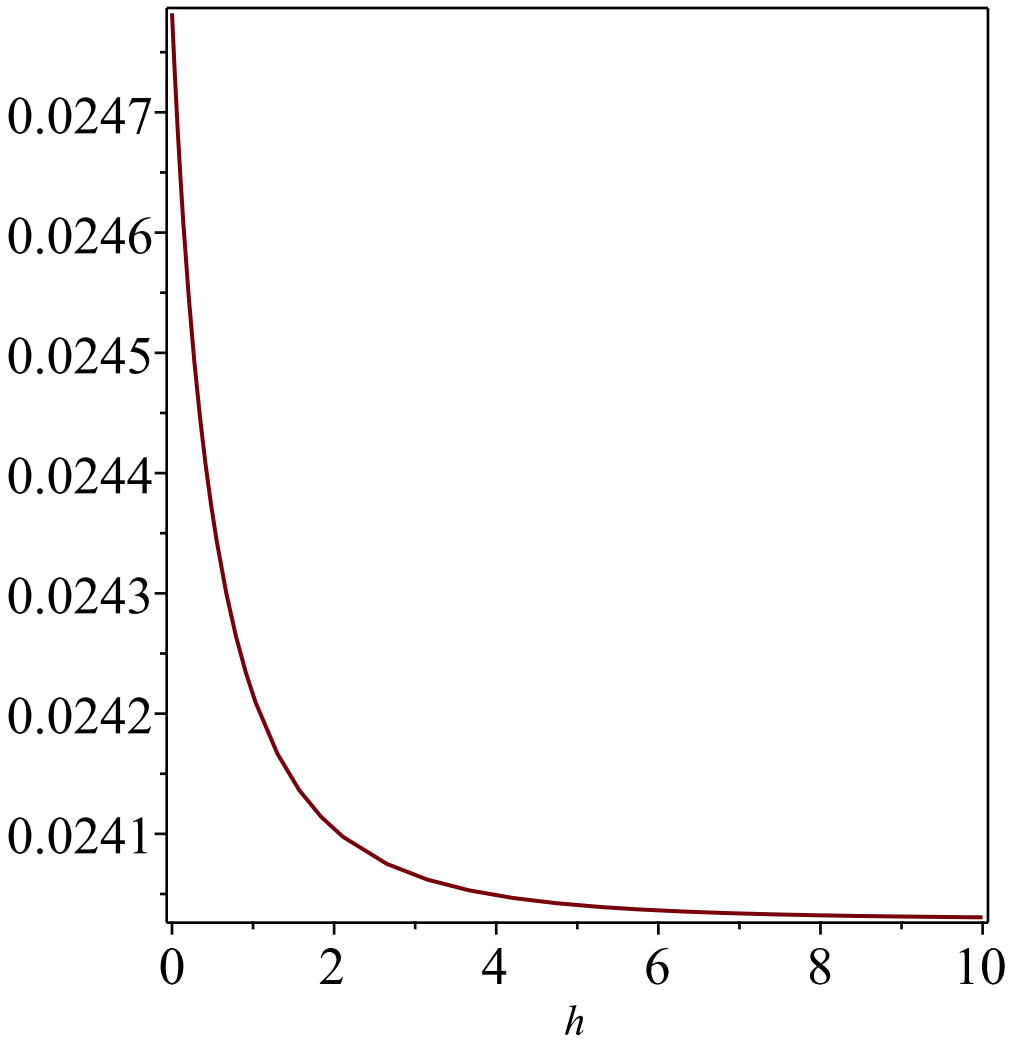
plots[*display*]({*pup*, *plo*}, *axes* = *boxed*, *view* = [0 ..7, 0 ..0.04]);



$$Fimstat := Int\left(k\cdot\exp(-2\cdot k\cdot z0)\cdot\left(\frac{1}{\frac{1}{1-\frac{es-1}{es+1}\cdot\exp(-2\cdot h\cdot k)}+\frac{L}{k}}-1\right), k=0\,..\,\text{infinity}\right);$$

$$Fimstat := \int_0^{\infty} k\,e^{-2\,k\,z0}\left(\frac{1}{\frac{1}{1-\frac{(es-1)\,e^{-2\,h\,k}}{es+1}}+\frac{2\,\pi\,n0}{\alpha\,k}}-1\right)dk \qquad \textbf{(13)}$$

$$plot(subs(z0=3, es=3.9, \text{alpha}=\text{Pi}\cdot n0, n0=0.428, -Fimstat), h=0\,..10, axes=boxed, numpoints=20);$$



$$evalf\left(subs\left(z0=3,\ es=3.9,\ h=0.1,\ \text{alpha}=\text{Pi}\cdot n0,\ n0=0.428,\ \frac{es-1}{4\,(z0+h)^2\,(es+1)}\right)\right);$$

0.01539637707 **(14)**