

AIML

Artificial Intelligence And Machine Learning

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* Unit 3 Artificial Neural Network (ANN)

- Neural → Comes from Neurons
- Network → Neurons are connected with each other
- Artificial → Manmade.

* Artificial Neural Network

1. The visual information of an object is to be carried out to brain to recognize an object with the help of neurons.
2. We try to create a machine in such a way that a machine should have neurons (Artificial neurons) and they will be connected to each other in same manner as we do have neurons in our brain. Those artificial neurons will train & learn data to generate better output.

* Why Neuron Network

- Normal Computer

Input Data

→ Meaningful Output

Applies some algorithm

- Computer with ANN

Input
Data

→ Meaningful
Output

→ Learns, trains
& improves

Applies some algorithm

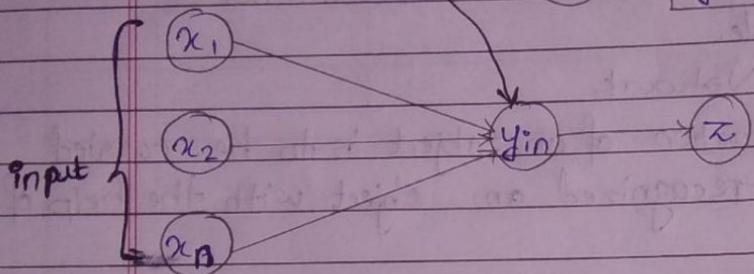
* Basic structure of Neural Network

Net input also written as

$$\sum_{i=1}^n w_i x_i$$

$$y_{in} = x_1 w_1 + x_2 w_2 + x_3 w_3$$

$$y_{in} = \sum_{i=1}^n x_i w_i$$

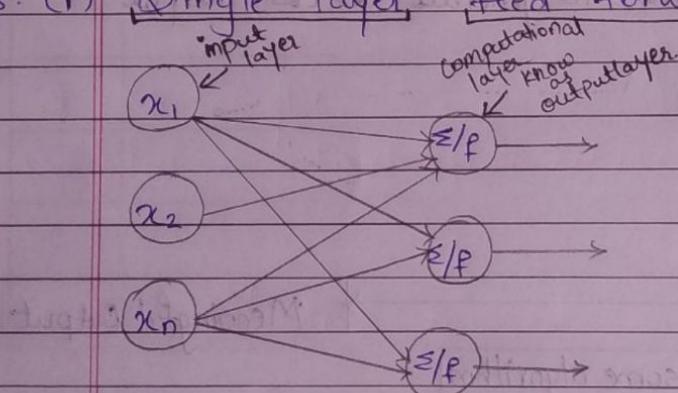


$$z = f(y_{in})$$

\nwarrow
activation
function.

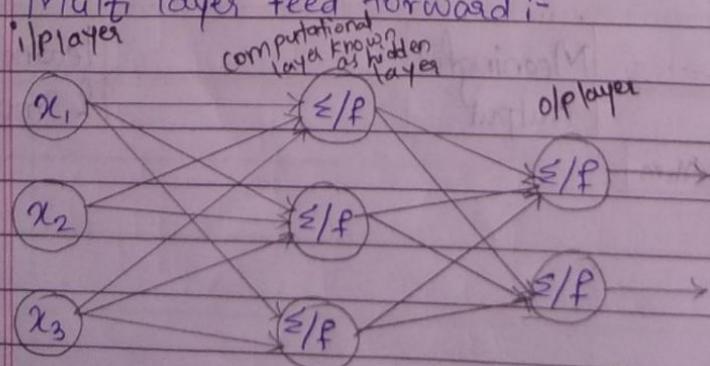
* What are the models of neural networks?

Ans. (1) Single layer, feed forward, Network



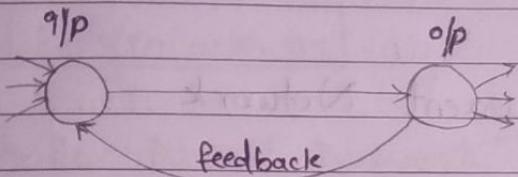
- There is only one computational layer that is output layer so it is a single layer network
- Input layer forward data to output layer only not viceversa

(2) Multi layer feed forward :-

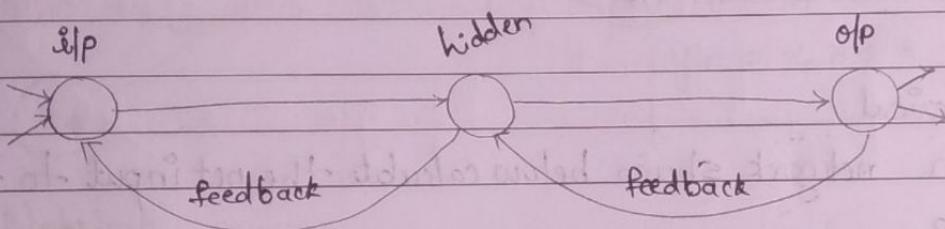


- Multi layered means multiple numbers of layers or more than one computational layer and there will be hidden layers which work as computational layer.

(3) Single node with its own feedback:-



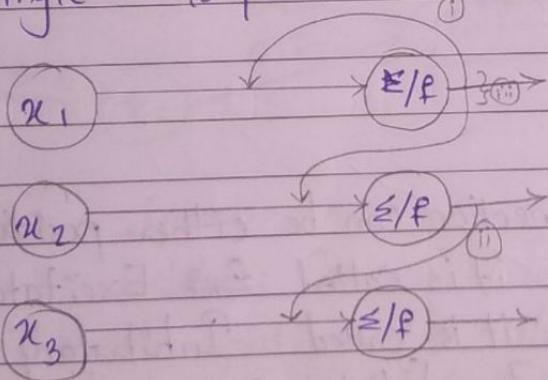
- Signals travels from Input node to Output node here signals can comeback to Input node that is direction of signals are allowed in both ways.



Note:-

Such type of networks are called as Recurrent/Recursive Network. They are complex used to generate

(4) Single layer Recurrent Network:-



- Single layer Recurrent Network is a network with feedback connection in which processing elements output can be directed back

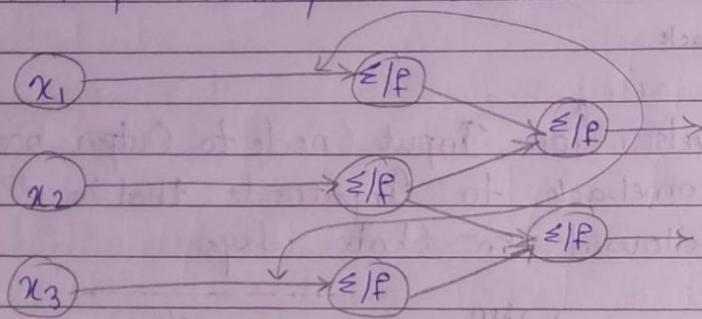
Conditions:-

(i) To processing element itself OR

(ii) To other processing element OR

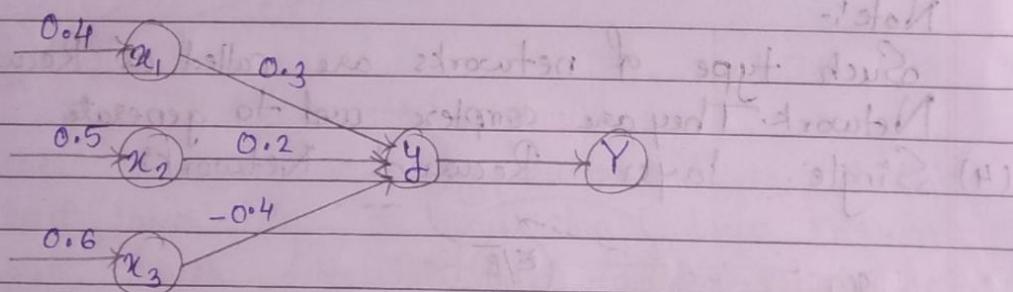
(iii) To both

(5) Multiple Multilayer Recurrent Network



* Numerical

5marks Q for a network shown below calculate the net input to the output neuron



Note:-

Weights of the connection can be either positive or negative
if the weight is positive it is called ~~Excitatory~~ Excitatory
if the weight is negative it is called ~~Inhibitory~~ Inhibitory

Solⁿ Inputs $\Rightarrow [x_1 \ x_2 \ x_3] = [0.4 \ 0.5 \ 0.6]$

Weights $\Rightarrow [w_{11} \ w_{22} \ w_{32}] = [0.3 \ 0.2 \ -0.4]$

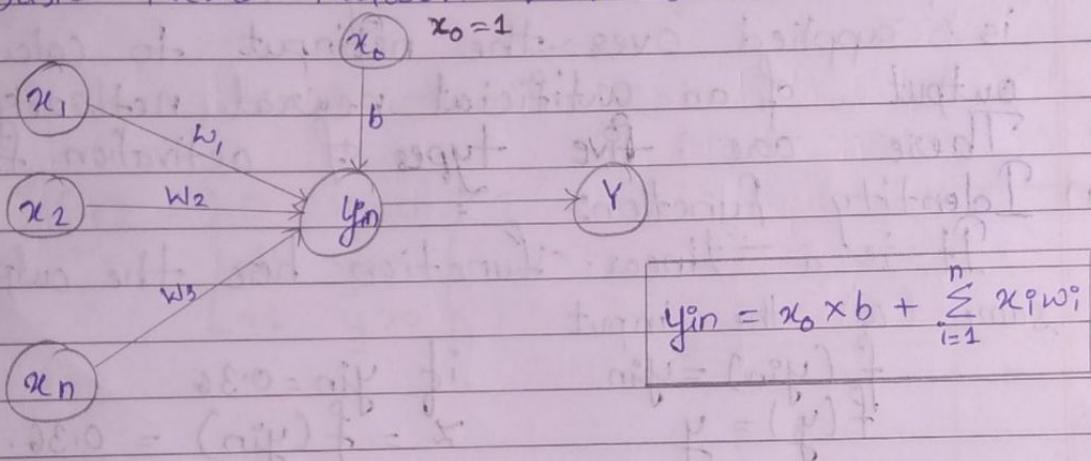
\therefore The net input can be calculated as:

$$y_{in} = \sum_{i=1}^n x_i w_{ip}$$

$$\begin{aligned}
 &= x_1 w_1 + x_2 w_2 + x_3 w_3 \\
 &= 0.4(0.3) + 0.5(0.2) + 0.6(-0.4) \\
 &= 0.12 + 0.10 - 0.24 \\
 &= 0.22 - 0.24 \\
 &= -0.02
 \end{aligned}$$

$\therefore y_{in} = -0.02$

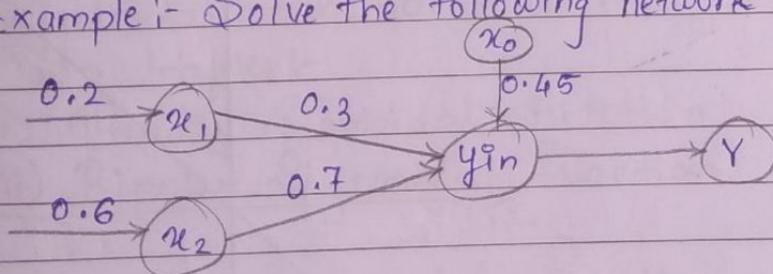
* Basic Neural Network with Biased connection.



Note:-

The Input value of biased connection will always be 1

* Example:- Solve the following network to calculate input



Solⁿ Given = inputs = $[x_1 \ x_2 \ x_0] = [0.2 \ 0.6 \ 1]$, $b = 0.45$
 weights = $[w_1 \ w_2] = [0.3 \ 0.7]$

\therefore The net input can be calculated as

$$\begin{aligned}
 y_{in} &= x_0 \times b + \sum_{i=1}^2 x_i w_i \\
 &= 1 \times 0.45 + 0.2 \times 0.3 + 0.6 \times 0.7 \\
 &= 0.45 + 0.6 + 0.42 \\
 &= 0.93
 \end{aligned}$$

$\therefore y_{in} = 0.93$

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8 to
10 marks *

What is activation function and what are different types of activation functions

Ans.

- Activation Function:-

Let us assume a person is performing some work to make the work more efficient and to get exact output some force or activation may be given this activation helps in achieving the exact output. In a similar way the activation function is applied over the net input to calculate the output of an artificial neural network.

- There are five types of activation function

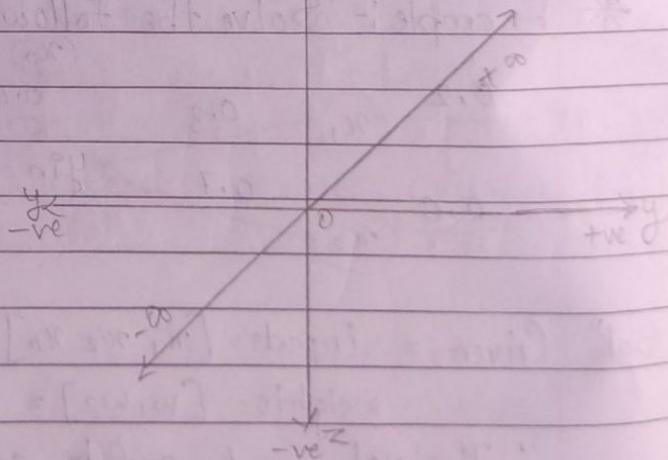
- (1) Identity Function:-

It is a Linear function here the output remains same as the input

$$f_p(y_{in}) = y_{in} \quad \text{if } y_{in} = 0.36$$

$$f(y) = y \quad z = f(y_{in}) = 0.36.$$

range = -∞ to +ve



(2) Binary Step Function:-

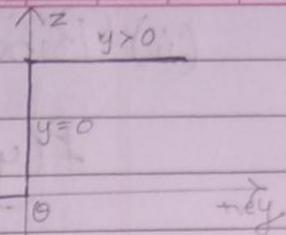
$$f(y) = \begin{cases} 1 & y \geq \theta \\ 0 & y < \theta \end{cases}$$

θ = threshold value

if $y=0$ then $z=1$

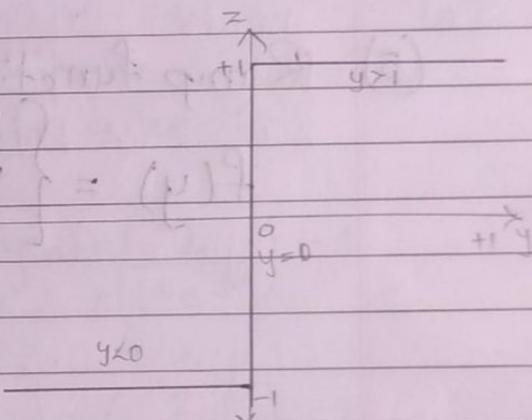
if $y<0$ then $z=-1$

if ~~y~~ $y>0$ ~~then~~ then $z=1$ & $y=1$



(3) Bipolar Step function:-

$$f(y) = \begin{cases} 0 & y=0 \\ -1 & y \leq 0 \\ +1 & y > 0 \end{cases}$$



(4) Sigmoidal function :- [Mostly used]

Two types:-

(i) Binary Sigmoidal function

(ii) Bipolar Sigmoidal function.

(i) Binary Sigmoidal Function

$$f(y) = \frac{1}{1+e^{-y}}$$

$x=1$ in basic neural network

$$f(y) = \frac{1}{1+e^{-y}}$$

(ii) Bipolar Sigmoidal function:-

$$f(y) = \left(\frac{2}{1+e^{-\lambda y}} \right) - 1$$

$\lambda = 1$ in basic neural network

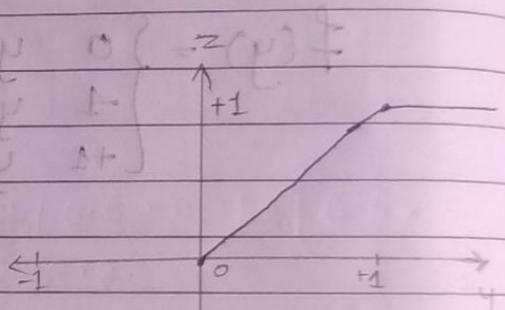
$$f(y) = \left(\frac{2}{1+e^{-y}} \right) - 1$$

where λ is steepness parameter

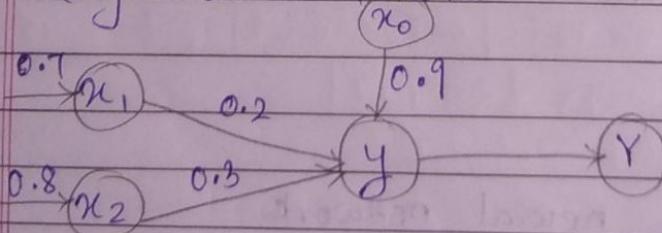
$\lambda = 1$ in basic neural network always

(5) Ramp function:-

$$f(y) = \begin{cases} 1 & y > 1 \\ y & 0 \leq y \leq 1 \\ 0 & y < 0 \end{cases}$$



* Calculate the output of the neuron y for the net given below use binary and bipolar Sigmoidal function.



* $x_0 = 1$ always.

Sol Inputs $\Rightarrow [x_1 \ x_2] = [0.7 \ 0.8]$

Weights $\Rightarrow [w_1 \ w_2] = [0.2 \ 0.3]$

bias $\Rightarrow [b] = [0.9]$

\therefore The net input can be calculated as.

$$y_{in} = x_0 \times b + \sum_{i=1}^n x_i w_i$$

$$= 1 \times 0.9 + (0.7 \times 0.2) + (0.8 \times 0.3)$$

$$= 0.9 + 0.14 + 0.24$$

$$\therefore \underline{y_{in} = 1.28}$$

(i) Binary Sigmoidal

$$\therefore y_{in} = 1.28$$

$$f(y_{in}) = \frac{1}{1 + e^{-y_{in}}}$$

$$= \frac{1}{1 + e^{-1.28}}$$

$$= \frac{1}{1 + 0.2780}$$

$$= \frac{1}{1.2780}$$

$$\underline{f(y_{in}) = 0.7824}$$

(ii) Bipolar Sigmoidal

$$\therefore y_{in} = 1.28$$

$$f(y_{in}) = \left(\frac{2}{1 + e^{-y_{in}}} \right) - 1$$

$$= \left(\frac{2}{1 + e^{-1.28}} \right) - 1$$

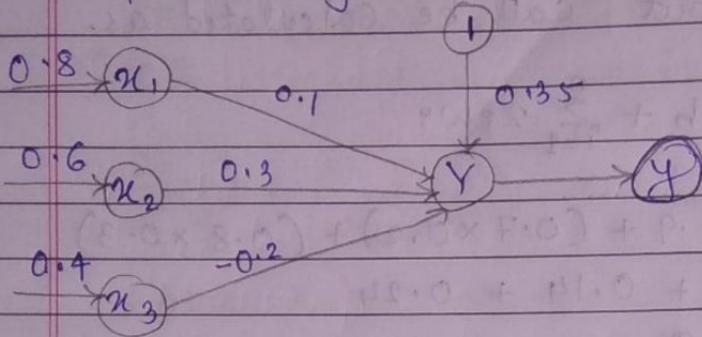
$$= \left(\frac{2}{1 + 0.2780} \right) - 1$$

$$= \left(\frac{2}{1.2780} \right) - 1$$

$$= 1.5649 - 1$$

$$\underline{f(y_{in}) = 0.5649}$$

Q. Obtain the output of neuron of the following net given below using Binary Sigmoidal & Bipolar Sigmoidal activation function.



Solⁿ Inputs $\rightarrow [0.8 \ 0.6 \ 0.4]$
 Weights $\rightarrow [0.1 \ 0.3 \ -0.2]$
 bias $\rightarrow [0.35]$

$$\begin{aligned} y_{in} &= 0.35 \times 1 + [0.8 \times 0.1] + [0.6 \times 0.3] + [0.4 \times (-0.2)] \\ &= 0.35 + 0.08 + 0.18 - 0.08 \\ \therefore y_{in} &= 0.53 \end{aligned}$$

(i) Binary Sigmoidal (ii) Bipolar Sigmoidal

$$f(y_{in}) = \frac{1}{1 + e^{-y_{in}}}$$

$$f(y_{in}) = \left(\frac{2}{1 + e^{-y_{in}}} \right) - 1$$

$$= \frac{1}{1 + e^{-0.53}}$$

$$\left(\frac{2}{1 + e^{-0.53}} \right) - 1$$

$$= \frac{1}{1 + 0.5886}$$

$$\left(\frac{2}{1 + 0.5886} \right) - 1$$

$$= \frac{1}{1.5886}$$

$$= \left(\frac{2}{1.5886} \right) - 1$$

$$f(y_{in}) = 0.6294$$

$$= 1.2589 - 1$$

$$f(y_{in}) = 0.2589$$

* Models. (i) Mc Culloc Pitt

(i) Mc Culloc Pitt
Implement AND Function using Mc Culloc Pitt neuron network.

Soln: The truth table for AND Function is :-

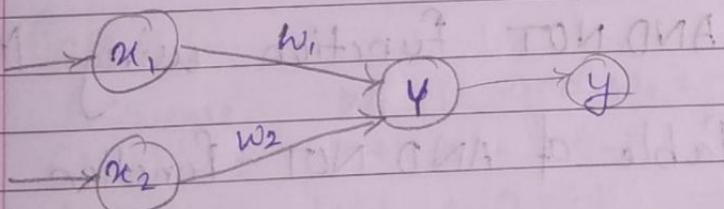
x_1	x_2	y
1	1	1
0	1	0
1	0	0
0	0	0

Let us assume the weights be $w_1 = 1$, $w_2 = 1$

Note:-

In Mc Culloc Pitt Neuron only analysis is being performed.

Hence we can draw the basic neural network according to the basic neural network



With the above assumed weights the net input can be calculated for 4 inputs as follows:-

$$(1, 1) = y_{in} = x_1 w_1 + x_2 w_2 = 1 \times 1 + 1 \times 1 = 2$$

$$(0, 1) = y_{in} = 0 \times 1 w_1 + x_2 w_2 = 0 \times 1 + 1 \times 1 = 1$$

$$(1, 0) = y_{in} = x_1 w_1 + 0 \times w_2 = 1 \times 1 + 0 \times 1 = 1$$

$$(0, 0) = y_{in} = x_1 w_1 + x_2 w_2 = 0 \times 1 + 0 \times 1 = 0$$

For AND Function output is high if both input is high

Thus, for this condition p.e (1,1) the net input is high calculated as $x_1 \cdot 2$ and on the basis of this ~~threshold~~^{input} value the threshold value is set.

If threshold value is greater than or equal to $x_1 \cdot 2$ ^{then only} neuron fires else it does not fire. Threshold value can be calculated by using the formula.

$$\theta \geq nw - p$$

where n = net input w = excitatory weight
 p = inhibitory weight

$$\theta \geq nw - p$$

$$\theta \geq 2 \times 1 - 0$$

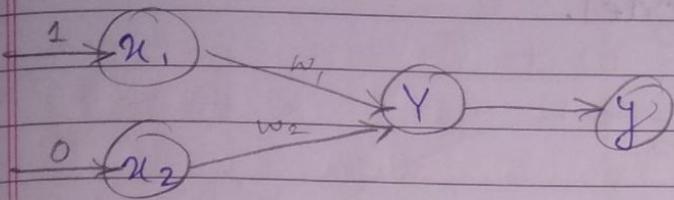
$$f(y_{in}) = \begin{cases} 1 & y_{in} \geq 2 \\ 0 & y_{in} < 2 \end{cases}$$

Q. Implement AND NOT function using McCulloch Pitt neuron

Sol' Truth Table of AND NOT Function

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

The Given function gives an output only when $x_1 = 1$ and $x_2 = 0$ the network can be represented as.



Case 1 :-

Assume both the weights w_1 and w_2 as 1

$$(1, 1) = y_{in} = 2$$

$$(1, 0) = y_{in} = 1$$

$$(0, 1) = y_{in} = 1$$

$$(0, 0) = y_{in} = 0$$

Comparing calculated output with truth table all 3 are going to fire means there is error. To overcome this problem we have to update weights but there is no training algorithm so they will assume weights by oneself.

for case 1 $\theta \geq 2$

Case 2 :-

Assume $w_1 = 1$ and $w_2 = -1$

$$(1, 1) = 1 \times 1 + 1 \times -1 = 0$$

$$(1, 0) = 1 \times 1 + 0 \times -1 = 1$$

$$(0, 1) = 0 \times 1 + 1 \times -1 = -1$$

$$(0, 0) = 0 \times 1 + 0 \times -1 = 0$$

$$\theta \geq nw - p$$

$$\theta \geq 1 \times 1 - 1$$

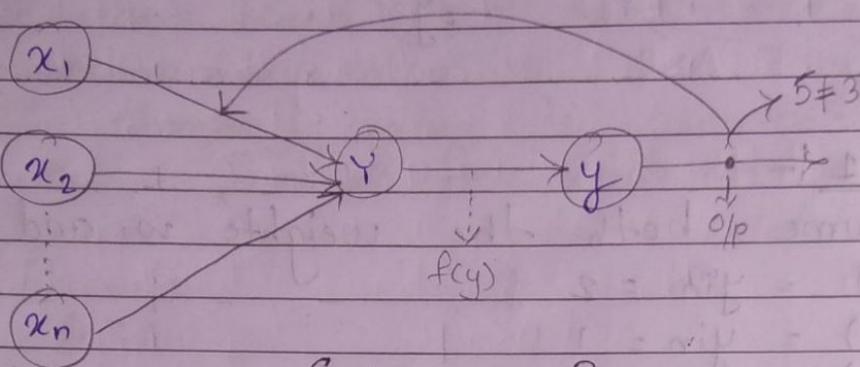
$$\theta \geq 0$$

$$f(y_{in}) = \begin{cases} 1 & y_{in} \geq 0 \\ 0 & y_{in} < 0 \end{cases}$$

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(2) Perception Model.



$$f(y_{in}) = \begin{cases} 1 & y_{in} > \theta \\ 0 & \theta \leq y_{in} \leq \theta \\ -1 & y_{in} < \theta \end{cases}$$

If there will be any error at the output layer then we will update weight and bias values.

* Weights.

$$w_1(\text{new}) = w_1(\text{old}) + d \cdot t \cdot x_1$$

$$w_2(\text{new}) = w_2(\text{old}) + d \cdot t \cdot x_2$$

where d = learning rate

t = desired output.

$$\text{bias}(\text{new}) = \text{bias}(\text{old}) + d \cdot t$$

* Q. Implement AND function using Perception Network

k for Bipolar inputs and target

Soln The Perceptron Network Architecture for 2 input are as follows:-

{ Output of f(x,y) }
{ Output of f(x,y) }

INPUT	α_1	α_2	α_3	α_4	Epoch 1	Target Net input	(t)	Y _m	Y	Weight changes			Weights		
										Δw_1	Δw_2	Δw_3	w_1	w_2	b
	1	1	1	1		0	0	0	0	1	1	1	1	1	1
	1	1	1	1		0	0	0	0	-1	-1	-1	0	0	0
	1	1	1	1		1	1	1	1	1	1	1	1	1	1
	1	1	1	1		2	2	2	2	2	2	2	2	2	2
	1	1	1	1		3	3	3	3	3	3	3	3	3	3

- The input patterns are presented to the network one by one, when all four input patterns are presented then 1 Epoch is said to be completed
- Initializing weights and bias:-
 $w_1 = 0 \quad w_2 = 0 \quad b = 0$

#

Epoch 1

(1) for the first input pattern i.e. $w_1=0$, $w_2=0$ and $b=0$ i.e. $x_1=1$, $x_2=1$ & $t=1$
Net input will be as follows:-

$$y_{in} = b + x_1 w_1 + x_2 w_2 \\ = 0 + 1 \times 0 + 1 \times 0$$

$$y_{in} = 0$$

Threshold condition given:-

$$y = f(y_{in}) = \begin{cases} 1 & y_{in} > 0 \\ 0 & y_{in} = 0 \\ -1 & y_{in} < 0 \end{cases}$$

Since net input i.e. $y_{in}=0$ ~~is 0~~
therefore, $\boxed{y=0}$

but, $t \neq y \therefore$ weight^{bias} update required

$$w_1(\text{new}) = w_1(\text{old}) + \alpha \times t \times x_1 \\ = 0 + 1 \times 1 \times 1$$

$$\therefore w_1(\text{new}) = 1$$

$$w_2(\text{new}) = w_2(\text{old}) + \alpha \times t \times x_2 \\ = 0 + 1 \times 1 \times 1$$

$$\therefore w_2(\text{new}) = 1$$

$$b(\text{new}) = b(\text{old}) + \alpha \times t \times 1 \\ = 0 + 1 \times 1 \times 1$$

$$\therefore b(\text{new}) = 1$$

(2) for the second input pattern i.e. $x_1 = 1$
 $x_2 = -1$ and $t = -1$ we will use updated weight
 and bias. $\therefore w_1 = 1$ $w_2 = 1$ $b = 1$

Net input will be as follows:-

$$\begin{aligned} y_{in} &= b + x_1 w_1 + x_2 w_2 \\ &= 1 + (1)(1) + (-1)(1) \\ &= 1 + 1 - 1 \end{aligned}$$

$$\therefore y_{in} = 1$$

Since net input is greater than 0 i.e.

$$\begin{aligned} y_{in} &= 1 \\ \text{therefore } y &= 1 \end{aligned}$$

but, $t \neq y$ but $t = -1 \therefore t \neq y$

\therefore weight and bias changes are required

$$\begin{aligned} w_1(\text{new}) &= w_1(\text{old}) + \alpha \times t \times x_1 \\ &= 1 + 1 \times -1 \times 1 \end{aligned}$$

$$\begin{aligned} w_2(\text{new}) &= w_2(\text{old}) + \alpha \times t \times x_2 \\ &= 1 + 1 \times (-1) \times (-1) \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$b(\text{new}) = b(\text{old}) + \alpha \times t \times 1$$

$$\begin{aligned} &= 1 + 1 \times (-1) \\ &= 0 \end{aligned}$$

(3) for the third input pattern i.e. $x_1 = -1$, $x_2 = 1$, $t = 1$ we will use updated weight and bias value i.e. $w_1 = 0$, $w_2 = 2$, $b = 0$.
Net input will be as follows

$$y_{in} = b + x_1 w_1 + x_2 w_2 \\ = 0 + (-1)(0) + (1)(2)$$

$$\therefore y_{in} = \underline{\underline{2}}$$

Since net input i.e. $y_{in} = 2$ therefore $y = 1$
but $t = -1 \therefore t \neq y$

$$w_1(\text{new}) = w_1(\text{old}) + \alpha \times t \times x_1 \\ = 0 + 1 \times 1 \times (-1)$$

$$\therefore w_1(\text{new}) = 1 + \underline{\underline{1}} + 1$$

$$\therefore w_2(\text{new}) = w_2(\text{old}) + \alpha \times t \times x_2$$

$$= 2 + 2(1 + 1 \times (-1) \times 1) \\ = 2 + 2 - 2 = 2$$

$$\therefore w_2(\text{new}) = \underline{\underline{1}}$$

$$b(\text{new}) = b(\text{old}) + \alpha \times (-t)$$

$$= 0 + 1(1)(-1)$$

$$\therefore b(\text{new}) = \underline{\underline{-1}}$$

(4) For the fourth input pattern i.e. $x_1 = -1$ $x_2 = -1$
 $t = -1$ we will use update weight & bias
 i.e. $w_1 = 1$ $w_2 = 1$ $b = -1$

Net input will be as follows:-

$$\begin{aligned}y_{in} &= b + x_1 w_1 + x_2 w_2 \\&= -1 + (-1)(1) + (-1)(1)\end{aligned}$$

$$= -1 - 1 - 1$$

$$= -3$$

Since net input i.e. $y_{in} = -3$ therefore $y = -1$
 $\therefore t = y$ So no updation required.

Epoch 2

Input	target	Net (t)	Calculated Input	Output (y)	Weight changes $\Delta w_1, \Delta w_2, \Delta b$	Weights w_1, w_2, b
x_1	x_2	1	y_{in}	$\text{Output } (y)$	$\Delta w_1, \Delta w_2, \Delta b$	w_1, w_2, b
-1	-1	-1	-1	-1	+1	+1 -1
-1	-1	-1	-1	-1	+1	+1 -1
-1	-1	-1	-1	-1	+1	+1 -1
-1	-1	-1	-1	-1	+1	+1 -1

* Initial weights and bias for Epoch 2 is
 $w_1 = 1$, $w_2 = 1$ and $b = -1$

(1) for the first input pattern in Epoch 2 i.e $x_1 = 1$ $x_2 = 1$
 $t_1 = 1$ we will use the updated weights & bias i.e
 $w_1 = 1$ $w_2 = 1$ $b = 1$

Net input will be as follows:-

$$y_{in} = b + x_1 w_1 + x_2 w_2$$

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$$y_{in} = -1 + (1)(1) + (1)(1)$$

$$\therefore \text{output} = -1 + 1 + 1$$

$$\therefore y_{in} = +1 \quad \text{and} \quad d = 0$$

Since net input i.e. $y_{in} = +1 \therefore y = +1$
 $t = y$ so ~~there is no update required~~

(2) $x_1 = 1 \quad x_2 = 1 \quad t = 1$

Weight & bias are $w_1 = 1 \quad w_2 = 1 \quad b = -1$

$$\begin{aligned}y_{in} &= b + x_1 w_1 + x_2 w_2 \\&= -1 + (1)(1) + (-1)(1) \\&= -1 + 1 - 1 \\&= -1\end{aligned}$$

$$y_{in} = -1$$

$$y = 1 \quad \therefore t = -1$$

$y = t$ So no updation in weight and bias are required.

(3) For third input pattern i.e. $x_1 = -1 \quad x_2 = 1 \quad t = -1$

Weight & bias are $w_1 = 1 \quad w_2 = 1 \quad b = -1$

Net input will be as follows:-

$$\begin{aligned}y_{in} &= b + x_1 w_1 + x_2 w_2 \\&= -1 + (-1)(1) + (1)(1)\end{aligned}$$

$$y_{in} = -1 + (-1) + 1 = 0$$

$$y_{in} = 0 \neq t \quad (0 \neq -1)$$

$\therefore y_{in} = -1 \quad \boxed{t = y}$ So there no updation in weight & bias required.

(4) For fourth input pattern i.e. $x_1 = -1 \quad x_2 = -1 \quad t = -1$

Weight and bias are $w_1 = 1 \quad w_2 = 1 \quad b = -1$

Net input will be as follows:-

$$\begin{aligned}y_{in} &= b + x_1 w_1 + x_2 w_2 \\&= -1 + (-1)(1) + (-1)(1) \\&= -1 - 1 - 1\end{aligned}$$

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$t = -3$ (given by input) for y_{out}
 $\therefore y_{in} = -3x$ & $y = -1$ ($y \propto x$) for output
 $\therefore t = -1$ $\because t = y_{out}$ & $y \propto x$
So no updation in weight & bias required.

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* Practical

- ' ' completes the fact mean period.
- What we write in a parenthesis we call it arguments and those arguments are called object
- Outside the parenthesis are relations
- Save the file in D drive with extension .pli All files.
- in GNU console select file → change dir → select d drive.
- command [prog]. (it compiles the program)
- command cat(X). (output X = tom)
- Note :- Donot use capital letter in code
- Facts can be defined as explicit between object and properties of object
- Rules they are conditionally true. It defines implicit relationship between object
- ';' is 'IF' condition.
- ',' for 'and' and ';' for 'or' or
- In Rules we use more than one fact

- Rules have two parts

(i) Conclusion part

(ii) Conditional part

Example:-

Code:- rules.pl [file]

likes (pooja, geeta).

likes (geeta, pooja).

friendship (X, Y) :- likes (X, Y), likes (Y, X).

- GNU console

[rules].

friendship (X, Y).

To run second fact ';' then enter.

* Relationship in Prolog.

- Relationship can also be a Rule

- We can define Relationship in form of Rules.

Code:- Relationship using rules.

parent (smith, seema).

parent (smith, seema).

female (seema)

female (seema)

sister (X, Y) :- parent (Z, X), parent (Z, Y), female (X), female (Y)

GNU console:-

[sister].

? sister (X, Y).

Output:-

X = seema

Y = seema ? ;

X = seema

Y = seema ? ;

X = seema

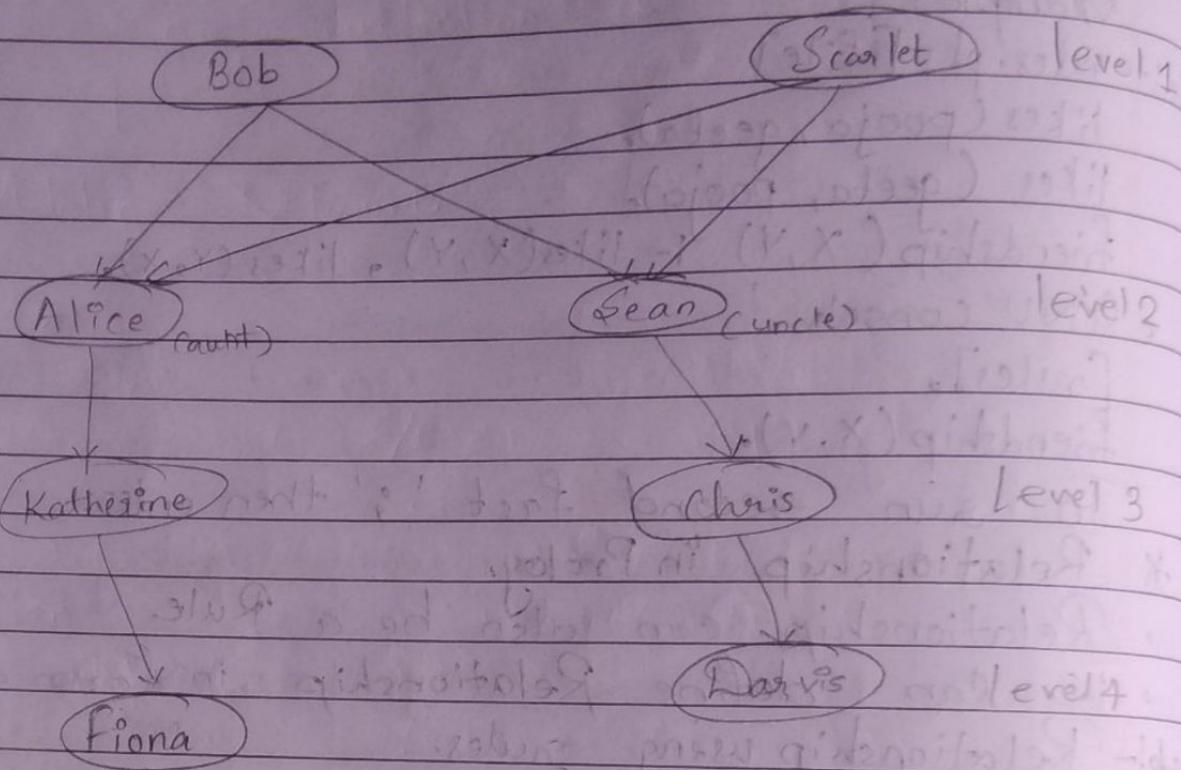
Y = seema ? ;

X = seema

Y = seema.

Yes.

* Creating knowledge based on the basis of facts and rules.



Code:-

- female (alice).
- female (scarlet).
- female (katherine).
- female (fiona).
- male (bob) male (sean).
- male (bob).
- male (chris).
- male (davis).
- parent (bob, alice).
- parent (bob, sean).
- parent (scarlet, alice).
- parent (scarlet, sean).
- parent (alice, katherine).
- parent (sean, chris).
- parent (katherine, fiona).

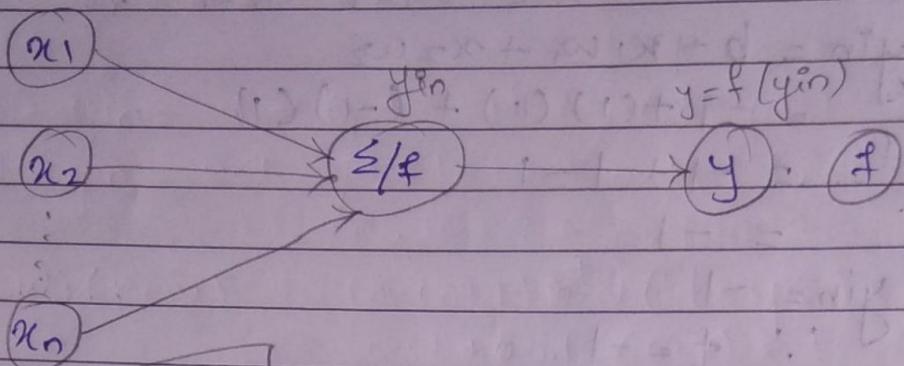
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parent(chris, davis).

grandparent(X, Y) :- parent(X, Z), parent(Z, Y).

sister(X, Y) :- parent(Z, X), parent(Z, Y), female(X), male(Y).

(3) Adaline Network



- $(t - y_{in})^2 \rightarrow$ least mean square error
- According to Adaline Network we perform weight & bias changes as follows:-

$$w_1(\text{new}) = w_1(\text{old}) + \eta(t - y_{in})x_1$$

$$w_2(\text{new}) = w_2(\text{old}) + \eta(t - y_{in})x_2$$

$$b(\text{new}) = b(\text{old}) + \eta(t - y_{in})$$

* Implement OR function with Bipolar inputs & targets using Adaline Network

Solⁿ Step 1

Truth table for OR function with Bipolar inputs & targets can be drawn as follows:

x_1	x_2	t
1	1	1
1	-1	1
-1	1	1
-1	-1	-1

For $w_1 = 0.1$, $w_2 = 0.1$, $b = 0.1$. \rightarrow initial value of weights and bias.

$$\alpha = 0.1$$

Epoch 1

Input	target	net	Calculated	Error	Weight	weights
x_1, x_2	1	(t)	Input o/p	$t - y_{in}$	changes w_1, w_2, b	w_1, w_2, b
			(y_{in})	(y)	$\Delta w_1, \Delta w_2, \Delta b$	$0.1, 0.1, 0.1$

Epoch 1

Inputs	target	net	Calculated o/p	Error	Weight changes	weights	$(t - y_{in})^2$
x_1, x_2	(t)	Input	y_{in}	y	$\Delta w_1, \Delta w_2, \Delta b$	w_1, w_2, b	
1 1	1	0.3		0.7	0.07 0.07 0.07	0.17 0.17 0.17	0.49
1 -1	1	0.17		0.83	0.083 -0.083	0.253 0.087	0.253 0.087
-1 1	1	0.087		0.913		0.1617 0.17 0.34	53 43 100
-1 -1	-1						0.688

(1) For 1st input pattern i.e. $x_1 = 1$, $x_2 = 1$, $t = 1$
 we use initial weights w_{10} , $w_1 = 0.1$, $w_2 = 0.1$

$$b = 0.1$$

$$\begin{aligned} y_{in} &= b + (x_1)(w_1) + (x_2)(w_2) \\ &= 0.1 + (1)(0.1) + (1)(0.1) \\ &= 0.1 + 0.1 + 0.1 \\ &= 0.3 \end{aligned}$$

$$y_{in} = 0.3$$

$$t - y_{in} = 1 - 0.3$$

$$t - y_{in} = 0.7$$

$$\begin{aligned} w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in})x_1 \\ &= 0.1 + 0.1(0.7) \times 1 \\ &= 0.17 \end{aligned}$$

$$\begin{aligned} w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in})x_2 \\ &= 0.1 + 0.1(0.7) \times 1 \\ &= 0.17 \end{aligned}$$

$$\begin{aligned} b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\ &= 0.1 + 0.1(0.7) \\ &= 0.17 \end{aligned}$$

(2) For second input pattern $x_1 = 1$, $x_2 = 1$, $t = 1$
 we use updated weight & bias
 i.e. $w_1 = 0.17$, $w_2 = 0.17$, $b = 0.17$

$$\begin{aligned} y_{in} &= b + x_1w_1 + x_2w_2 \\ &= 0.17 + 1(0.17) + 1(0.17) \\ &= 0.17 + 0.17 - 0.17 \\ &= 0.17 \end{aligned}$$

$$\begin{aligned} t - y_{in} &= 1 - 0.17 \\ &= 0.83 \end{aligned}$$

$$\begin{aligned}
 w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_1 \\
 &= 0.17 + 0.1(0.83) \times 1 \\
 &= 0.17 + 0.083 \\
 &= 0.253
 \end{aligned}$$

$$\begin{aligned}
 w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in}) \times x_2 \\
 &= 0.17 + 0.1(0.83) \times -1 \\
 &= 0.17 - 0.083 \\
 &= 0.087
 \end{aligned}$$

$$\begin{aligned}
 b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\
 &= 0.17 + 0.1(0.83) \\
 &= 0.17 + 0.083 \\
 &= 0.253
 \end{aligned}$$

(3) For third input pattern i.e. $x_1 = -1$ $x_2 = 1$ $t = 1$
 we use updated weights $w_1 = 0.253$ $w_2 = 0.087$
 $b = 0.253$.

Net input will be as follows:-

$$\begin{aligned}
 y_{in} &= b + x_1 w_1 + x_2 w_2 \\
 &= 0.253 + 0.253(-1) + 1(0.087)
 \end{aligned}$$

$$\begin{aligned}
 y_{in} &= 0.087 \\
 t - y_{in} &= 0.913
 \end{aligned}$$

$$\begin{aligned}
 w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_1 \\
 &= 0.253 + 0.1(0.913) \times (-1) \\
 &= 0.1617
 \end{aligned}$$

$$\begin{aligned}
 w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in}) \times x_2 \\
 &= 0.087 + 0.1(0.913) \times 1 \\
 &= 0.1753
 \end{aligned}$$

$$\begin{aligned}
 b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\
 &= 0.253 + 0.1(0.913) \\
 &= 0.3443
 \end{aligned}$$

(4) for fourth input pattern i.e $x_1 = -1$ $x_2 = -1$
 $t = -1$ we use updated weights & bias.

$$w_1 = 0.1617 \quad w_2 = 0.1753 \quad b = 0.3443$$

Net input will be as follows

$$\begin{aligned} y_{in} &= b + x_1 w_1 + x_2 w_2 \\ &= 0.3443 + (-1)(0.1617) + (-1)(0.1753) \\ &= 0.3443 - 0.1617 - 0.1753 \\ &= 0.0073 \end{aligned}$$

$$\begin{aligned} w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in})x_1 \\ &= 0.1617 + 0.1(-1.0073) \times -1 \\ &= 0.1617 + 0.10073 \\ &= 0.26243 \end{aligned}$$

$$\begin{aligned} w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in})x_2 \\ &= 0.1753 + 0.10073 \\ &= 0.27609 \end{aligned}$$

$$\begin{aligned} b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\ &= 0.3443 - 0.10073 \\ &= 0.24357 \end{aligned}$$

$$(8 \times (y_{in} - t)) b + (b \text{ old}) = (w_{\text{new}})_1$$

$$(1) \times (8(1.0) - 1.0) + 0.10073 = 0.10073$$

$$(8 \times (y_{in} - t)) b + (b \text{ old}) = (w_{\text{new}})_2$$

$$(1) \times (8(1.0) - 1.0) + 0.27609 = 0.27609$$

$$(y_{in} - t) b + (b \text{ old}) = (b \text{ new})$$

$$(0.0073 - 1.0) + 0.24357 = 0.24357$$

$(t - y_{in})^2$

Weight

weight changes

Target net Error
input ($t - y_{in}$)

Input

 α_1 α_2 Δw_1 Δw_2

1

1

0.7847

1

1

0.2484

1

1

0.2089

1

1

0.1945

1

1

0.3599

 $(t - y_{in})^2$ w_1 w_2

0.836

0.2003

0.3589

0.2251

0.2791

0.2044

0.3849

0.3092

Epoch 2.

(1) first input pattern

$$x_1 = 1 \quad x_2 = 1 \quad t = 1$$

$$w_1 = 0.2621 \quad w_2 = 0.2787 \quad b = 0.2438$$

$$\begin{aligned} y_{in} &= b + w_1 x_1 + w_2 x_2 \\ &= 0.7847 \end{aligned}$$

$$\begin{aligned} w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_1 \\ &= 0.2621 + 0.1(1 - 0.7847) \times 1 \\ &\approx 0.2621 + 0.02153 \\ &\approx 0.28363 \end{aligned}$$

$$\begin{aligned} w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in}) \times x_2 \\ &= 0.2787 + 0.1(1 - 0.7847) \times 1 \\ &= 0.2787 + 0.02153 \\ &= 0.30023 \end{aligned}$$

$$\begin{aligned} b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\ &= 0.2438 + 0.1(1 - 0.7847) \\ &= 0.2438 + 0.02153 \\ &= 0.26533 \end{aligned}$$

(2) Second input pattern

$$x_1 = 1 \quad x_2 = -1 \quad t = 1$$

$$\begin{aligned} y_{in} &= b + w_1 x_1 + w_2 x_2 \\ &= 0.26533 + (1)(0.28363) + (-1) \\ &\quad (0.30023) \\ &= 0.24874 \end{aligned}$$

$$\begin{aligned} w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_1 \\ &= 0.28363 + 0.1(0.75126) \times 1 \\ &= 0.28363 + 0.075126 \\ &= 0.3587 \end{aligned}$$

$$\begin{aligned}
 w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in}) \times (-1) \\
 &= 0.3003 + 0.1(0.75126) \times (-1) \\
 &= 0.3003 + 0.075126
 \end{aligned}$$

$$\begin{aligned}
 b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\
 &= 0.2654 + 0.075126 \\
 &= 0.3405
 \end{aligned}$$

(3) for third input pattern

$$x_1 = -1 \quad x_2 = 1 \quad t = 1$$

$$\begin{aligned}
 y_{in} &= 0.3405 + (-1)(0.3587) + (1) \cancel{(-0.2251)} \\
 &= 0.3405 + -0.3587 + 0.2251 \\
 &= 0.2069
 \end{aligned}$$

$$\begin{aligned}
 w(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_1 \\
 &= 0.3587 + \cancel{0.1}(0.7931) \times -1 \\
 &= \cancel{0.4380} + 0.3587 - 0.07931 = 0.2794
 \end{aligned}$$

$$\begin{aligned}
 w_2(\text{new}) &= w_2(\text{old}) + \alpha(t - y_{in}) \times x_2 \\
 &= 0.2251 + \cancel{0.07931} \\
 &= 0.3044
 \end{aligned}$$

$$\begin{aligned}
 b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \cancel{\otimes} \\
 &= 0.3405 + \cancel{0.07931} \\
 &= 0.3898
 \end{aligned}$$

(4) for fourth input pattern

$$x_1 = -1 \quad x_2 = -1 \quad t = -1$$

$$y_{in} = b + x_1 w_1 + x_2 w_2$$

$$\begin{aligned}
 &= 0.3898 + (-1)(0.2794) + (-1)(0.3044) \\
 &= \\
 &= 0.3898 - 0.2794 - 0.3044
 \end{aligned}$$

$$= -0.1945$$

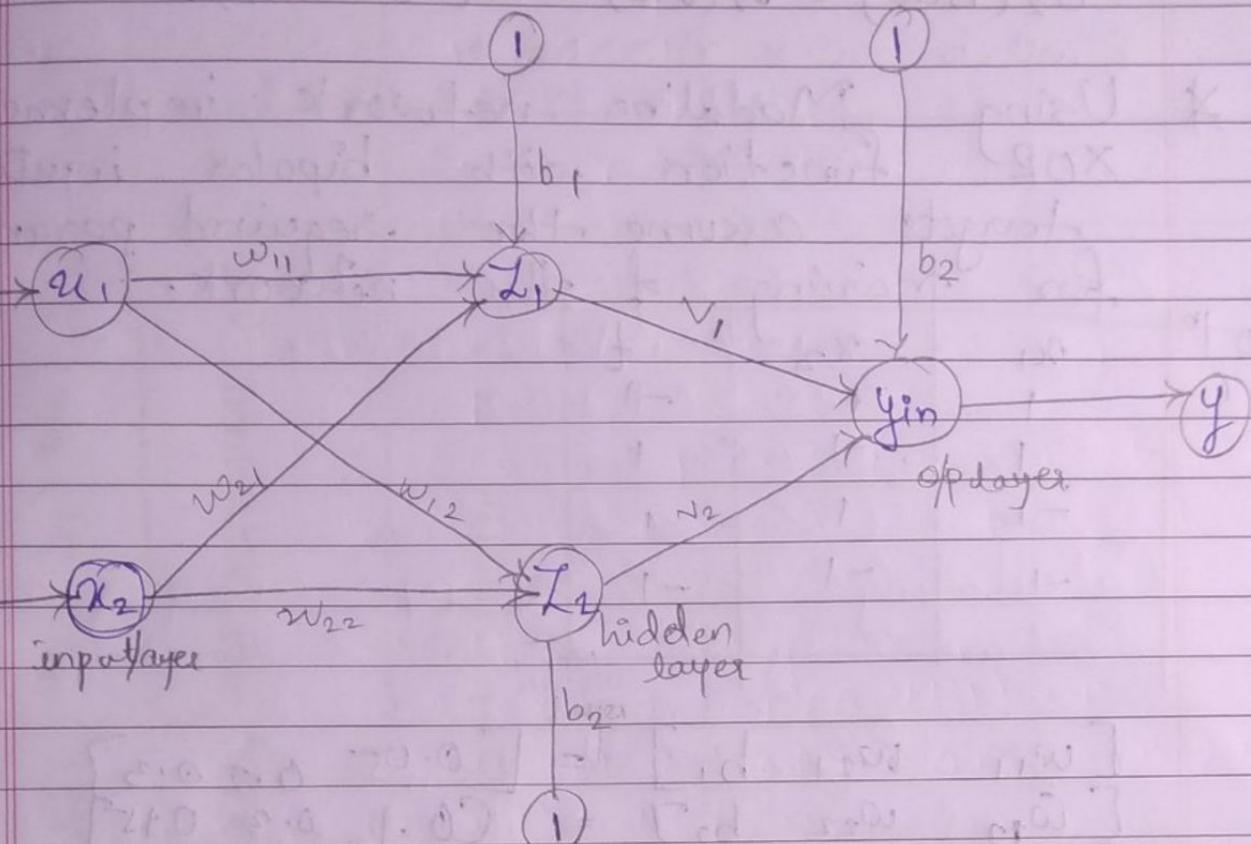
$$\begin{aligned}
 w_1(\text{new}) &= w_1(\text{old}) + \alpha(t - y_{in}) \times x_i \\
 &= 0.2194 + 0.1(-1)(-0.1945) \times 1 \\
 &= 0.2794 + 0 \\
 &= 0.3599
 \end{aligned}$$

$$\begin{aligned}
 w_2(\text{new}) &= 0.3044 + 0.1(-1)(-0.1945) \times 1 \\
 &= 0.3849
 \end{aligned}$$

$$\begin{aligned}
 b(\text{new}) &= b(\text{old}) + \alpha(t - y_{in}) \\
 &= 0.3092
 \end{aligned}$$

$$\begin{aligned}
 \alpha x(t_{in} - t) + (b_{10})d &= (w_{10})d \\
 18P10.0 + 18P88.0 &= 18P10.0 \\
 18P88.0 &= 18P88.0 \\
 \alpha x(t_{in} - t) + (b_{10})d &= (w_{10})d \\
 18P10.0 + 18P88.0 &= 18P88.0
 \end{aligned}$$

* Madaline (Multiple Adaline Network).
(More than one computation layer)



① Calculate net input at hidden layer

$$Z_{in1} = b_1 + x_1 \times w_{11} + x_2 \times w_{21}$$

$$Z_{in2} = b_2 + x_1 \times w_{12} + x_2 \times w_{22}$$

$$z_1 = f(Z_{in1})$$

$$z_2 = f(Z_{in2}).$$

② Calculating net input at output layer.

$$y_{in} = b_{in} + z_1 v_1 + z_2 v_2$$

$$y = f(y_{in}).$$

③ Updating & changing bias value.

$$w_{11}(\text{new}) = \underline{w_{11}(\text{old})} + \alpha(t - Z_{in1}) \times x_1 \quad Z_{in1}$$

$$w_{21}(\text{new}) = \underline{w_{21}(\text{old})} + \alpha(t - Z_{in1}) \times x_2 \quad Z_{in1}$$

$$b_1(\text{new}) = \underline{b_1(\text{old})} + \alpha(t - Z_{in1})$$

$$\left. \begin{aligned} w_{11}(\text{new}) &= w_{11}(\text{old}) + \alpha(t - z_{in2}) \times x_1 \\ w_{22}(\text{new}) &= w_{22}(\text{old}) + \alpha(t - z_{in2}) \times x_2 \\ b_2(\text{new}) &= b_2(\text{old}) + \alpha(t - z_{in2}). \end{aligned} \right\} z_{in2}$$

* Using Madaline network implement XOR function with bipolar inputs & targets assume the required parameters for training of the network.

So In

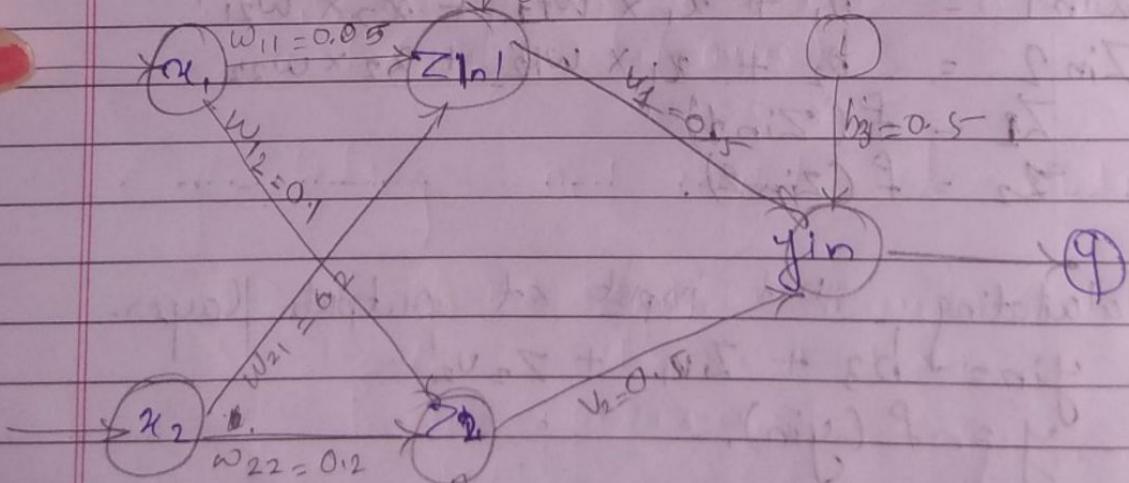
x_1	x_2	t
1	1	-1
1	-1	1
-1	1	1
-1	-1	-1

$$\begin{bmatrix} w_{11} & w_{21} & b_1 \end{bmatrix} = \begin{bmatrix} 0.05 & 0.2 & 0.3 \end{bmatrix}$$

$$\begin{bmatrix} \bar{w}_{12} & w_{22} & b_2 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 & 0.15 \end{bmatrix}$$

$$\begin{bmatrix} v_1 & v_2 & b_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}$$

learning rate = 0.5



$$\alpha \times (1.05 - t(1)) + (b10) \cdot v1 = (b10) \cdot v1$$

$$\alpha \times (1.05 - t(1)) + (b10) \cdot v1 = (b10) \cdot v1$$

$$(1.05 - t(1)) \cdot v1 + (b10) \cdot v1 = (b10) \cdot v1$$

χ_1	χ_2	t	Z_{in1}	Z_{in2}	Z_1	Z_2	y_{in}	y	w_{11}	w_{21}	b_1	w_{22}	w_{12}	b_2	v_1	v_2	b_3
1	1	-1	0.55	0.45	1	1	1.05	1	-0.725	-0.475	-0.625	-0.525	-0.515				
1	1	-1	-1	-1	-1	-1	-1	-1	-0.4875	-1.3875	0.3375						
1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Z_{in} = Epoch 1

(1) 1st input pattern

$$x_1 = 1 \quad x_2 = 1 \quad t = -1$$

$$\begin{aligned} Z_{in1} &= b_1 + x_1 \times w_{11} + x_2 \times w_{21} \\ &= 0.3 + 1 \times 0.5 + \cancel{1} \times 0.2 \\ &= 0.55 \end{aligned}$$

$$\begin{aligned} Z_{in2} &= b_2 + x_1 \times w_{12} + x_2 \times w_{22} \\ &= 0.15 \times 1 \times 0.1 + 1 \times 0.2 \\ &= 0.15 + 0.1 + 0.2 \end{aligned}$$

$$Z_{in2} = 0.45$$

Calculating Z_1 & Z_2 by applying activation function given as below.

$$Z = f(Z_{in}) = \begin{cases} 1 & \text{if } Z_{in} \geq 0 \\ -1 & \text{if } Z_{in} \leq 0 \end{cases}$$

$$Z_1 = f(Z_{in}) = f(0.55) = 1$$

$$\therefore Z_1 = 1$$

$$Z_2 = f(Z_{in2}) = f(0.45) = 1$$

Calculating net input at output layer

$$\begin{aligned} Y_{in} &= b_3 + Z_1 v_1 + Z_2 v_2 \\ &= 0.5 + 1 \times 0.5 + 1 \times 0.5 \\ &= 0.5 + 0.5 + 0.5 \end{aligned}$$

$$Y_{in} = 1.5$$

$$\begin{aligned} Y &= f(Y_{in}) \\ \therefore Y &= 1 \end{aligned}$$

$\therefore t = -1 \quad \therefore t \neq y$ So weight & bias changes required.

$$\begin{aligned} w_{11}(\text{new}) &= w_{11}(\text{old}) + \alpha(t - z_{in1}) \times x_1 \\ &= 0.05 + 0.5(-1.55) \times 1 \\ &= -0.725 \end{aligned}$$

$$\begin{aligned} w_{21}(\text{new}) &= w_{21}(\text{old}) + \alpha(t - z_{in1}) \times x_2 \\ &= 0.2 + 0.5(-1.55) \times 1 \\ &= 0.2 + (-0.775) \\ &= -0.575 \end{aligned}$$

$$\begin{aligned} b_1(\text{new}) &= b_1(\text{old}) + \alpha(t - z_{in}) \\ &= 0.3 - 0.775 \\ &= -0.475 \end{aligned}$$

$$\begin{aligned} w_{12}(\text{new}) &= w_{12}(\text{old}) + \alpha(t - z_{in2}) \times x_1 \\ &= 0.1 + 0.5(0.7 + 0.5(-1.45)) \times 1 \\ &= 0.1 + 0.5(-1.45) \times 1 \\ &= 0.1 - 0.725 \\ &= -0.625 \end{aligned}$$

$$\begin{aligned} w_{22}(\text{new}) &= \cancel{w_{22}(\text{old})} + \alpha(t - z_{in2}) \times x_2 \\ &= 0.2 - 0.725 \\ &= -0.525 \end{aligned}$$

$$\begin{aligned} b_2(\text{new}) &= b_2(\text{old}) + \alpha(t - z_{in2}) \\ &= 0.15 - 0.725 \\ &= -0.575 \end{aligned}$$

(2) 2nd input pattern.

$$x_1 = 1 \quad x_2 = -1 \quad t = 1.$$

$$\begin{aligned} Z_{in1} &= -0.475 + 1(-0.725) + (-1)(-0.575) \\ &= -0.475 - 0.725 + 0.575 \\ &= -0.625 \end{aligned}$$

$$\begin{aligned} Z_{in2} &= -0.475 + -0.575 + 1(0.625) + (-1)(0.525) \\ &= -0.575 - 0.625 + 0.525 \\ &= -0.675 \end{aligned}$$

$$\begin{aligned} Z_1 &= f(Z_{in1}) \\ &= f(-0.625) \end{aligned}$$

$$\begin{aligned} Z_2 &= f(Z_{in2}) \\ &= f(-0.675) \end{aligned}$$

$$\begin{aligned} y_{in} &= 0.5 + (-1)(0.5) + (-1)(0.5) \\ &= 0.5 - 0.5 - 0.5 \\ &= -0.5 \end{aligned}$$

$$y = -1$$

$\therefore t = 1 \quad \therefore t \neq y$ So weights & bias
change required.

Note if the value of Z_{in1} and Z_{in2} are in -ve then we will apply weight changes only to the nearest to zero value of Z_{in} .

~~$$\begin{aligned} w_{11}(\text{new}) &= w_{11}(\text{old}) + \alpha(t - z_{in1}) \times x_1 \\ &= -0.725 + 0.5(0.375) \times 1 \\ &= -0.725 + 0.1875 \\ &= \underline{\underline{-0.9125}} \end{aligned}$$~~

~~$$\begin{aligned}
 w_{21}(\text{new}) &= w_{21}(\text{old}) + \alpha(t - z_{in1}) \times x_2 \\
 &= -0.575 + 0.1875 \times (-1) \\
 &= -0.575 - 0.1875 \\
 &= -0.7625
 \end{aligned}$$~~

~~$$\begin{aligned}
 b_1(\text{new}) &= b_1(\text{old}) + \alpha(t - z_{in1}) \\
 &= -0.475 + 0.1875 \\
 &= -0.2875
 \end{aligned}$$~~

~~$$\begin{aligned}
 w_{11}(\text{new}) &= w_{11}(\text{old}) + \alpha(t - z_{in1}) \times x_1 \\
 &= -0.725 + 0.5(1.625) \times 1 \\
 &= -0.725 + 0.8125 \\
 &= 0.0875
 \end{aligned}$$~~

~~$$\begin{aligned}
 w_{21}(\text{new}) &= w_{21}(\text{old}) + \alpha(t - z_{in1}) \times x_2 \\
 &= -0.575 + 0.8125 \times (-1) \\
 &= -0.575 - 0.8125 \\
 &= -1.3875
 \end{aligned}$$~~

~~$$\begin{aligned}
 b_1(\text{new}) &= b_1(\text{old}) + \alpha(t - z_{in1}) \\
 &= -0.475 + 0.8125 \\
 &= 0.3375
 \end{aligned}$$~~

(3) for 3rd input pattern

~~$$x_1 = -1, x_2 = +1, t = 1$$~~

~~$$\begin{aligned}
 z_{in1} &= 0.3375 + (0.0875)(-1) + (-1.3875)(1) \\
 &= 0.3375 - 0.0875 - 1.3875 \\
 &= -1.1375
 \end{aligned}$$~~

~~$$\begin{aligned}
 z_{in2} &= -0.575 + (-1)(0.625) + (1)(0.525) \\
 &= -0.575 + 0.625 - 0.525 \\
 &= -0.475
 \end{aligned}$$~~

$$\begin{aligned}
 y_{in} &= z_1 v_1 + z_2 v_2 + b_3 & y &= -1 \\
 &= 0.5 - 1(0.5) + (-1)(0.5) \\
 &= 0.5 - 0.5 - 0.5 \\
 &= -0.5
 \end{aligned}$$

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$$z_1 = -1 \quad z_2 = -1$$

$$\therefore y = -1 \because t = 1 \neq y$$

$$\begin{aligned}
 w_{12}(\text{new}) &= w_{12}(\text{old}) + \alpha(t - z_{in2}) \times x_2 \\
 &= -0.625 - 0.7375 \\
 &= \cancel{0.1125} - 1.3625
 \end{aligned}$$

$$\begin{aligned}
 w_{22}(\text{new}) &= w_{22}(\text{old}) + \alpha(t - z_{in2}) \times x_2 \\
 &= -0.525 + 0.7375 \times 1 \\
 &= 0.2125
 \end{aligned}$$

$$\begin{aligned}
 b_2(\text{new}) &= b_2(\text{old}) + \alpha(t - z_{in2}) \\
 &= -0.575 + 0.7375 \\
 &= 0.1625
 \end{aligned}$$

(4) For 4th input pattern

$$x_1 = -1 \quad x_2 = -1 \quad t = -1$$

$$\begin{aligned}
 z_{in1} &= 0.3375 + (0.0875)(-1) + (-1)(1.3875) \\
 &= 0.3375 - 0.0875 + 1.3875 \\
 &= 1.6375
 \end{aligned}$$

$$\begin{aligned}
 z_{in2} &= 0.1625 + (-1)(-1.3625) + (-1)(0.2125) \\
 &= 0.1625 + 1.3625 - 0.2125 \\
 &= 1.3125
 \end{aligned}$$

$$z_1 = f(z_{in1}) = 1$$

$$z_2 = f(z_{in2}) = 1$$

$$\begin{aligned}
 y_{in} &= b_3 + z_1 v_1 + z_2 v_2 \\
 &= (0.5) + (1)(0.5) + (1)(0.5) \\
 &= 1.5
 \end{aligned}$$

$$\begin{aligned}
 y &= f(y_{in}) = 1 \\
 \therefore t &= -1 \quad \because y = 1 \therefore t \neq y.
 \end{aligned}$$

$$\begin{aligned}
 w_{11}(\text{new}) &= w_{11}(\text{old}) + \alpha(t - z_{in1}) \times x_1 \\
 &= 0.0875 + 0.5(-1 - 1.6375) \times -1 \\
 &= 0.0875 + -2.6375(-1) \\
 &= 0.0875 + 2.6375 = 1.31875 \\
 &= 1.40625
 \end{aligned}$$

$$\begin{aligned}
 w_{21}(\text{new}) &= w_{21}(\text{old}) + \alpha(t - z_{in1}) \times x_2 \\
 &= -1.3875 + 1.31875 \\
 &= -6.06875
 \end{aligned}$$

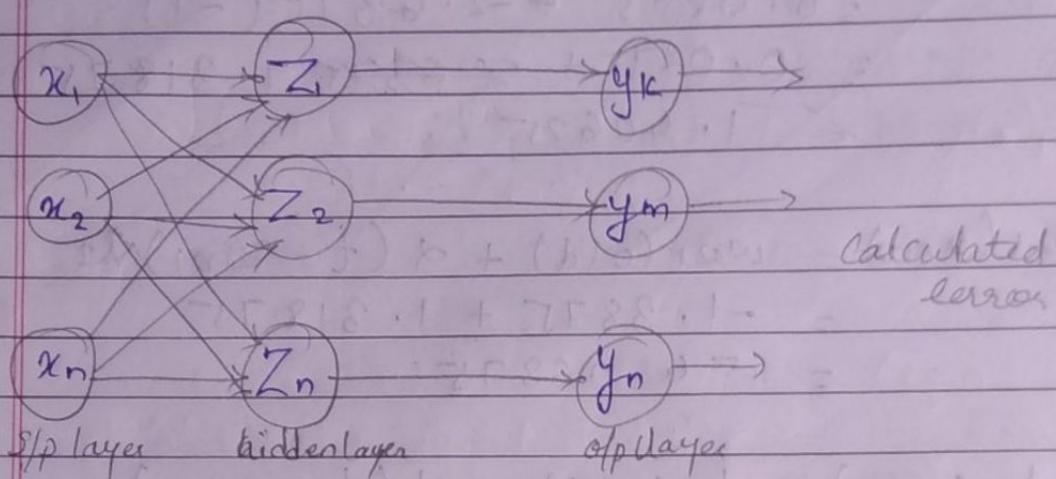
$$\begin{aligned}
 b_1(\text{new}) &= b_1(\text{old}) + \alpha(t - z_{in1}) \\
 &= 0.3375 - 1.31875 \\
 &= -0.98125
 \end{aligned}$$

$$\begin{aligned}
 w_{12}(\text{new}) &= w_{12}(\text{old}) + \alpha(t - z_{in2}) \times x_1 \\
 &\Rightarrow -1.3625 + 0.5(-1 - 1.3125) \times (-1) \\
 &= -1.3625 + 0.5(-3.29375) \times (-1) \\
 &= -1.3625 + 1.6468(-1) = 1.15625(-1) \\
 &= -1.3625 + 1.6468 = 1.15625 \\
 &= -0.0093 - 0.2843 = 2. - 0.2068
 \end{aligned}$$

$$\begin{aligned}
 w_{22}(\text{new}) &= w_{22}(\text{old}) + \alpha(t - z_{in2}) \times (x_2) \\
 &= 0.2125 + 1.6468 = 1.15625 \\
 &= 1.8593 = 1.36875
 \end{aligned}$$

$$\begin{aligned}
 b_2(\text{new}) &= b_2(\text{old}) + \alpha(t - z_{in2}) \\
 &= 0.1625 + 1.6468 = 1.15625 \\
 &= 1.4843 = 0.99375
 \end{aligned}$$

* Back propagation Algorithm



$\Delta \text{tta} = S$
↑
error

calculated
error

• Back propagation algorithm is one of the most important development in Neural Network.

This learning algorithm is applied to multilayer feed forward network. In this method error is propagated back to the hidden unit.

• To update weight we require (δ) delta which is calculated error.

• When targeted output is not equal to calculated output then there is some error so in back propagation algorithm we calculate error at the output layer. which is the difference between targeted output (t) & calculated output (y).

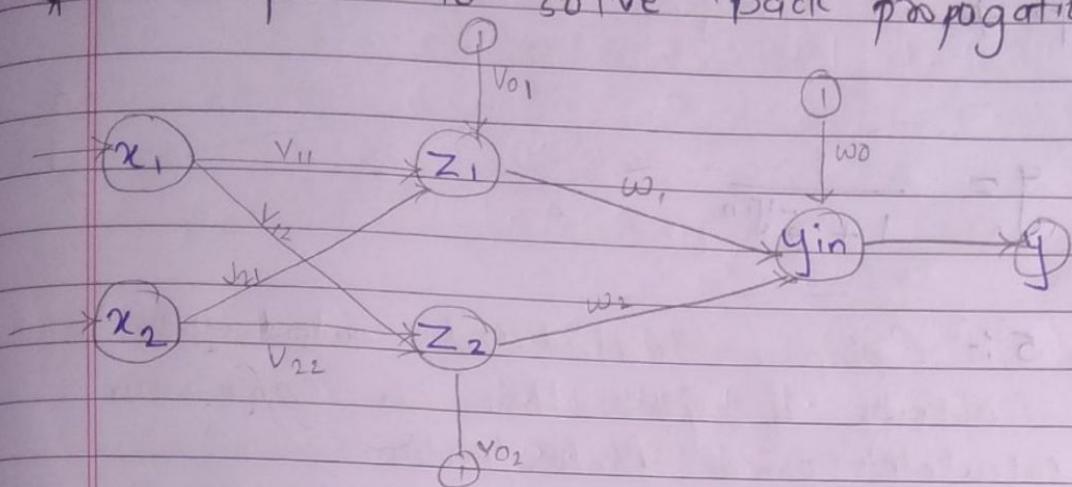
• The training of back propagation network is done in 3 stages:

- 1) The feed forward of input training pattern
- 2) The calculation & back propagation of the errors
- 3) Updation of weights

Note: The network associated with BPN is called BPN.

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* Steps to solve Back propagation Network



* Note:- In BPN we use binary sigmoidal activation function to calculate output

Step 1 :- Calculate net input at Z_1 layer (hidden layer)

$$Z_{in1} = v_{01} + x_1(v_{11}) + x_2(v_{21})$$

$$Z_{in2} = v_{02} + x_1(v_{12}) + x_2(v_{22})$$

Step 2 :- ~~$Z_1 = f(Z_{in1})$~~ & ~~$Z_2 = f(Z_{in2})$~~ Applying binary sigmoidal on Z_{in1} & Z_{in2} to calculate Z_1 & Z_2

$$Z_1 = \frac{1}{1 + e^{-Z_{in1}}}$$

$$Z_2 = \frac{1}{1 + e^{-Z_{in2}}}$$

Step 3 :- Calculate net input y_{in} at output layer

$$y_{in} = w_0 + z_1 w_1 + z_2 w_2$$

Step 4 :- Applying binary Sigmoid on y_{in} to calculate y .

$$y = \frac{1}{1 + e^{-y_{in}}}$$

Step 5 :- Compare Calculated output with target output if $y \neq t$ there is some error
 ∴ calculate error factors/portion.

$$\delta(\text{error}) = t - y \times f'(y_{in})$$

$$\text{where } f'(y_{in}) = f(y_{in}) \times (1 - f(y_{in}))$$

Step 6 :- Find the changes in weights between output layer & hidden layers.

~~$$\Delta w_1 = \alpha \delta_1 \times z_1$$~~

~~$$\Delta w_2 = \alpha \delta_1 \times z_2$$~~

~~$$\Delta b_2 = \alpha \delta$$~~

Step 7 :- Compute the error portion between hidden layer & input layer

$$\delta_1 = \delta_{in1} \times f'(z_{in1}) \quad \text{where } \delta_{in1} = \delta_1 \times w_1$$

$$\delta_2 = \delta_{in2} \times f'(z_{in2}) \quad \text{where } \delta_{in2} = \delta_1 \times w_2 \quad \text{(from previous step)}$$

$$f'(z_{in1}) = f(z_{in1}) \times (1 - f(z_{in1}))$$

$$f'(z_{in2}) = f(z_{in2}) \times (1 - f(z_{in2}))$$

Step 8 :- Find the changes in weights between hidden layer & input layer

~~$$\Delta v_{11} = \alpha \delta_1 \times x_1$$~~

~~$$\Delta v_{21} = \alpha \delta_1 \times x_2$$~~

~~$$\Delta v_{01} = \alpha \delta_1$$~~

$$\Delta w_{12} = \alpha s_2 x_1$$

$$\Delta v_{22} = \alpha s_2 x_2$$

$$\Delta v_{02} = \alpha s_2$$

Step 9: Compute the final weights of the network

$$v_{11}(\text{new}) = v_{11}(\text{old}) + \Delta v_{11}$$

$$v_{12}(\text{new}) = v_{12}(\text{old}) + \Delta v_{12}$$

$$v_{21}(\text{new}) = v_{21}(\text{old}) + \Delta v_{21}$$

$$v_{22}(\text{new}) = v_{22}(\text{old}) + \Delta v_{22}$$

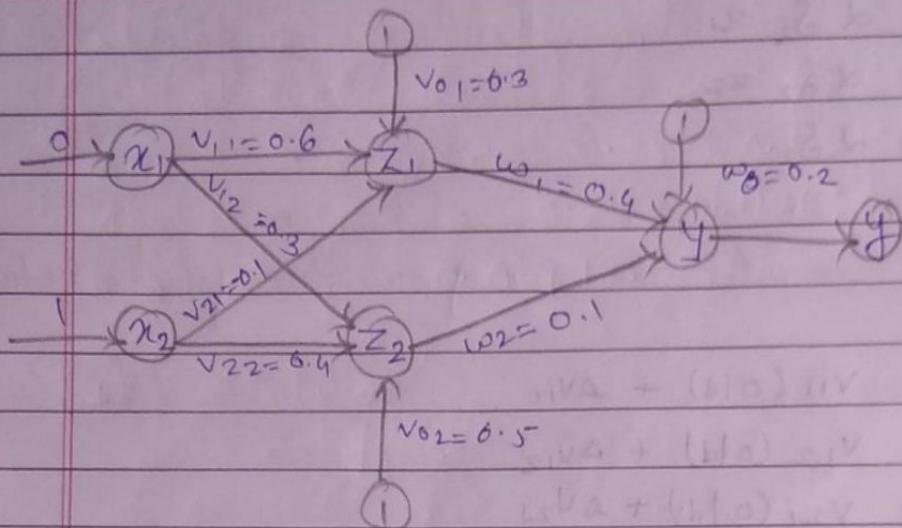
$$v_{01}(\text{new}) = v_{01}(\text{old}) + \Delta v_{01}$$

$$v_{02}(\text{new}) = v_{02}(\text{old}) + \Delta v_{02}$$

$$w_1(\text{new}) = w_1(\text{old}) + \Delta w_1$$

$$w_2(\text{new}) = w_2(\text{old}) + \Delta w_2$$

$$w_0(\text{new}) = w_0(\text{old}) + \Delta w_0$$



$$\text{Sol} \quad [v_{11} \ v_{21} \ v_{01}] = [0.6 \ -0.1 \ 0.3]$$

$$[v_{12} \ v_{22} \ v_{02}] = [-0.3 \ 0.4 \ 0.5]$$

$$[w_1 \ w_2 \ w_0] = [0.4 \ 0.1 \ -0.2]$$

① Calculate net input at Z layers (Computational)

$$\begin{aligned} Z_{in1} &= v_{01} + x_1 \times v_{11} + x_2 \times v_{21} \\ &= 0.3 + 0(0.6) + (1)(-0.1) \\ &= 0.3 + 0 + (-0.1) \\ &= 0.3 - 0.1 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} Z_{in2} &= v_{02} + x_1 \times v_{12} + x_2 \times v_{22} \\ &= 0.5 + 0(-0.3) + (1)(0.4) \\ &= 0.5 - 0 + 0.4 \\ &= 0.9 \end{aligned}$$

② Applying Binary Sigmoidal Function to calculate Z_1 & Z_2 as:

$$Z_1 = \frac{1}{1+e^{-Z_{in1}}} = \frac{1}{1+e^{-0.2}} = \frac{1}{1.8187} = 0.5498$$

$$Z_2 = \frac{1}{1 + e^{-\frac{1}{402}}} = \frac{1}{1 + e^{-0.9}} = \frac{1}{1.4065} = 0.7109$$

③ Calculate net input i.e. y_{in} -

$$\begin{aligned} y_{in} &= w_0 + Z_1 \times w_1 + Z_2 \times w_2 \\ &= -0.2 + 0.5498 \times (0.4) + (0.7109)(0.1) \\ &= -0.2 + 0.2199 + 0.07109 \\ &= 0.09099 \approx 0.09101 \end{aligned}$$

④ Applying Sigmoidal function to calculate y

$$y = \frac{1}{1 + e^{-y_{in}}} = \frac{1}{1 + e^{-0.09101}} = \frac{1}{1.9130} = 0.5227$$

But the target output is given as 1 and we are obtaining calculated output as ~~0.5227~~ from the network as 0.5227 which means there is some error ($t \neq y$).

⑤ Compute error portion (δ_1)

$$\delta_1 = (t - y) \times f'(y_{in}) = 0.4773 \times 0.2495 = 0.11908$$

$$\begin{aligned} f'(y_{in}) &= f(y_{in}) \times (1 - f(y_{in})) \\ &= f(y_{in}) \times 0.4773 \\ &= 0.5227 \times 0.4773 \end{aligned}$$

$$= 0.24944$$

Using error portion δ_1

⑥ Calculate the changes between weight

$$\Delta w_1 = \alpha \delta_1 \times z_1 = 0.25 \times 0.1191 \times 0.5498 = 0.0164$$

$$\Delta w_2 = \alpha \delta_1 \times z_2 = 0.25 \times 0.1191 \times 0.7109 = 0.021178$$

$$\Delta w_3 = \alpha \delta_1 = 0.02978.$$

⑦ Calculating error portion between hidden output layer.

$$\delta_1 = \delta_{in1} \times f'(z_{in1})$$

$$\delta_2 = \delta_{in2} f'(z_{in2})$$

$$f'(z_{in1}) = f(z_{in1}) \times (1 - f(z_{in1}))$$

$$= 0.5498 \times 0.4502$$

$$= 0.2475$$

$$f'(z_{in2}) = f(z_{in2}) \times (1 - f(z_{in2}))$$

$$= 0.7109 \times 0.2891$$

$$= 0.2055$$

$$\delta_1 = 0.2475 \times 0.1191 \times 0.2055 = 0.0295 \times 0.4502$$

$$\delta_2 = 0.1191 \times 0.2055 = 0.0295 \times 0.1 = 0.00295$$

⑧ Calculate the weight changes

$$\Delta v_{11} = \alpha \times \delta_1 \times x_1 = 0$$

$$\Delta v_{21} = \alpha \times \delta_1 \times x_2 = 0.00295$$

$$\Delta v_{01} = \alpha \times \delta_1 = 0.00295$$

$$\Delta v_{12} = \alpha \times \delta_2 \times x_1 = 0$$

$$\Delta v_{22} = \alpha \times \delta_2 \times x_2 = 0.00295 \times 0.0006125$$

$$\Delta v_{02} = \alpha \times \delta_2 = 0.0006125$$

a) Compute final weight

$$v_{11}(\text{new}) = v_{11}(\text{old}) + \Delta v_{11}, \\ = 0.6 + 0 = 0.6,$$

$$v_{12}(\text{new}) = v_{12}(\text{old}) + \Delta v_{12} \\ = -0.3 + 0 = -0.3$$

$$v_{21}(\text{new}) = v_{21}(\text{old}) + \Delta v_{21} \\ = -0.1 + 0.00295 = -0.09705$$

$$v_{22}(\text{new}) = v_{22}(\text{old}) + \Delta v_{22} \\ = 0.4 + 0.006125 = 0.4006125$$

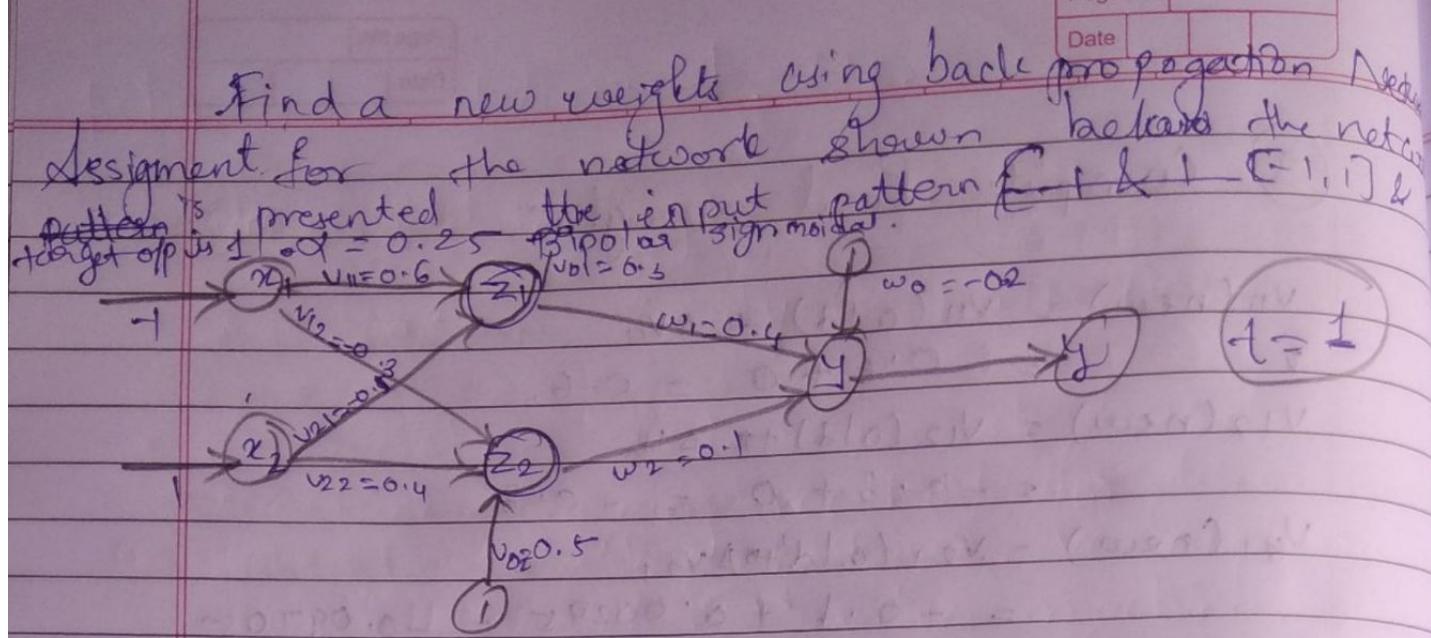
$$v_{01}(\text{new}) = v_{01}(\text{old}) + \Delta v_{01} \\ = 0.3 + 0.00295 = 0.30295$$

$$v_{02}(\text{new}) = v_{02}(\text{old}) + \Delta v_{02} \\ = 0.5 + 0.006125 = 0.5006125$$

$$w_1(\text{new}) = w_1(\text{old}) + \Delta w_1 \\ = 0.4 + 0.0164 = 0.00656 \quad 0.4164$$

$$w_2(\text{new}) = w_2(\text{old}) + \Delta w_2 \\ = 0.1 + 0.02117 = 0.00217$$

$$w_0(\text{new}) = w_0(\text{old}) + \Delta w_0 \\ = -0.2 + 0.02978 \\ = -0.65956 - 0.17022$$



* Revision

- Perception - @ we calculate output on the basis of net input value. @ we calculate new weight & bias values as follows $[w_i(\text{new}) = \alpha \times t \times x_i + w_i(\text{old})]$.

w_1	w_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b

- Aadine - @ we calculate error factor using error factor we calculate weight & bias value. $[w_i(\text{new}) = w_i(\text{old}) + \alpha(t - y_{in})x_i]$

Note:- For each on every iteration we calculate mean square error

w_1	w_2	t	$t - y_{in}$	Δw_1	Δw_2	Δb	w_1	w_2	b	$\frac{1}{2}(t - y)^2$

Unsupervised learning:

Association Analysis with Apriori Algorithm

- Association Rule is a rule, based on machine learning method for discovering interest relations between variables in large data sets. It involves machine learning models to analyze data for patterns & for co-occurrences in a database.

If there are 2 products A & B if we purchase product A then product B so we write $A \Rightarrow B$ (A then B) where A is called as Antecedent & B is called as consequent 2 measurement units are used to measure association between itemsets.

- Support - It denotes percentage of transaction that contains both A & B.
- Confidence - % of transaction containing A which also contains B.

Example:-

The rule is Milk \Rightarrow Bread.

So, formula for support (Milk \Rightarrow Bread) =

$$= \frac{\text{no of tuples containing both milk & bread}}{\text{total no. of tuple}}$$

$$\text{Confidence}_{(\text{Milk} \Rightarrow \text{Bread})} = \frac{\text{no. of tuples contain both milk & bread}}{\text{no. of tuples containing milk}}$$

Suppose

No. of tuples containing Milk & Bread = 4

Total No. of Tuples = 5

No. of tuples containing Milk is 4

$S = \frac{4}{5} = 80\%$. There is 80% of support (Milk \Rightarrow Bread)

$C = \frac{4}{4} = 100\%$. There is 100% of confidence.

Q. Trans-id

Item Sets

01 Bag, Uniform, Water Bottle

02 Book, Bag, Uniform

03 Bag, Uniform, Pen

04 Bag, Pen, Book

05 Uniform, WaterBottle, Bag,

06 Bag, Pen, Book

07 WaterBottle, Uniform, Bag.

08 Books, WaterBottle, Bag

09 Uniform, WaterBottle, Pen

10 Pen, Uniform, Book

12/5/22

Find Association Rule with 30% support & 75% confidence.

Solⁿ

part 1 :- (Iteration 1)

Elements	Frequency	Support
Bag	8	$(8/10)^* 100 = 80\%$.
Uniform	7	$(7/10)^* 100 = 70\%$.
Waterbottle	5	$(5/10)^* 100 = 50\%$.
Book	5	$(5/10)^* 100 = 50\%$.
Pen	5	$(5/10)^* 100 = 50\%$.

Since all elements are having support percentage more than 30%.
 Therefore all the elements will pass to second iteration.

~~Part 2~~ (2nd Iteration).

Item Set	Frequency / Support	Support %.
Bag, Uniform	5	$(5/10) * 100 = 50\%$
Bag, WaterBottle	4	$(4/10) * 100 = 40\%$
Bag, Book	4	$(4/10) * 100 = 40\%$
Bag, Pen	3	$(3/10) * 100 = 30\%$
Uniform, WaterBottle	4	$(4/10) * 100 = 40\%$
Uniform, Book	2	$(2/10) * 100 = 20\%$
Uniform, Pen	3	$(3/10) * 100 = 30\%$
WaterBottle, Book	1	$(1/10) * 100 = 10\%$
WaterBottle, Pen	1	$(1/10) * 100 = 10\%$
Book, Pen	3	$(3/10) * 100 = 30\%$

So the combination of item sets having support percentage more than 30% will pass on to 3rd iteration.

Part 3

Phaser - (3rd Iteration)

ItemSet	Frequency	Support %
Bag, Uniform, WaterBottle	3	30%
Bag, Uniform, Book	1	10%
Bag, WaterBottle, Book	1	10%
Bag, WaterBottle, Pen	0	0%
Bag, Uniform, Pen	1	10%
Bag, Book, Pen	2	20%
Uniform, WaterBottle, Pen	1	10%
Uniform, Pen, Book	1	10%

Since only {Bag, Uniform, WaterBottle} is having the minimum support 30%. therefore this dataset will pass to next iteration.

Part 2 - 4th Iteration

$$x = \{ \text{Bag, Uniform, WaterBottle} \}$$

Subsets of $\{ \}$ x

1. $\{\text{Bag}\}$

2. $\{\text{Uniform}\}$

3. $\{\text{WaterBottle}\}$

4. $\{\text{Bag, Uniform}\}$

5. $\{\text{Bag, WaterBottle}\}$

6. $\{\text{Uniform, WaterBottle}\}$

7. $\{\text{Bag, Uniform, WaterBottle}\}$ x

8. $\{\text{Bag, Uniform, WaterBottle}\}$ x

We never consider null set & the whole set.

$\{ \text{Bags} \} \Rightarrow \{ \text{Uniform, WaterBottle} \}$

$$= \frac{P(\text{Bag} \cap \text{Uniform} \cap \text{WaterBottle})}{P(\text{Bag})}$$

$$= \frac{3}{7} \times \frac{25}{100}$$

$$= 37.5 \quad \times$$

$\{ \text{Uniform} \} \Rightarrow \{ \text{Bag, WaterBottle} \}$

$$= \frac{P(\text{Bag} \cap \text{Uniform} \cap \text{WaterBottle})}{P(\text{Uniform})}$$

$$= \frac{3}{7} \times 100$$

$$= 42.85\% \quad \times$$

$\{ \text{WaterBottle} \} \Rightarrow \{ \text{Bag, Uniform} \}$

$$= \frac{P(\text{Bag} \cap \text{Uniform} \cap \text{WaterBottle})}{P(\text{WaterBottle})}$$

$$= \frac{3}{5} \times 100$$

$$= 60\% \quad \times$$

$\{ \text{Uniform, WaterBottle} \} \Rightarrow \{ \text{Bag} \}$

$$= \frac{P(\text{B} \cap \text{U} \cap \text{W})}{P(\text{Uniform, WaterBottle})}$$

$$= \frac{3}{4} \times 100$$

$$= 75\%$$

$\{ \text{Bag, WaterBottle} \} \Rightarrow \{ \text{Uniform} \}$

$$= \frac{P(B \cap U \cap W)}{P(\text{Bag} \cap \text{Waterbottle})}$$

$$= \frac{3}{4} \times 100$$

$$= 75\%$$

$\{ \text{Bag, Uniform} \} \Rightarrow \{ \text{WaterBottle} \}$

$$= \frac{P(B \cap U \cap W)}{P(\text{Bag} \cap \text{Uniform})}$$

$$= \frac{3}{5} \times 100$$

$$= 60\% \quad \times$$

Q. Find support & confidence for
the rule $\{\text{Bag}\} \rightarrow \{\text{Uniform}\}$

$$\text{support} = \frac{P(B \cap U)}{P(B)} = \frac{3}{10} \times 100 = 30\%$$

$$\text{confidence} = \frac{P(B \cap U)}{P(U)} = \frac{3}{8} \times 100 = 62.5\%$$