Assignment-2 (Intro to robotics)

Pradeep Saini
18110120

Ans-1

To show: ~~to show~~ columns of rotation matrix R_0^1 are orthogonal.

$$R_0^1 = R_z R_y R_x$$

$$= \begin{bmatrix} \cos C & -\sin C & 0 \\ \sin C & \cos C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos B & 0 & \sin B \\ 0 & 1 & 0 \\ -\sin B & 0 & \cos B \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos A & -\sin A \\ 0 & \sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \cos C \cos B & -\sin C \cos A + \cos C \sin B \sin A & \sin C \sin A + \cos C \sin B \cos A \\ \sin C \cos B & \cos C \cos A + \sin C \sin B \sin A & -\cos C \sin A + \sin C \sin B \cos A \\ -\sin B & \cos B \sin A & \cos B \cos A. \end{bmatrix}$$

$$\begin{array}{ccc} \vec{C_1} & \vec{C_2} & \vec{C_3} \end{array}$$

If dot product of column vectors is zero, then they are orthogonal.

$$\vec{C_1} \cdot \vec{C_2} = \cos C \cos B \times (-\sin C \cos A + \cos C \sin B \sin A)$$
$$+ \sin C \cos B \times (\cos C \cos A + \sin C \sin B \sin A)$$
$$- \sin B \cos B \sin A$$

$$= -\sin C \cancel{\cos A \cos B} \cos C + \sin A \cos B \cos^2 C \sin B$$
$$+ \sin C \cancel{\cos A \cos B} \cos C + \sin^2 C \sin B \sin A \cos B$$
$$- \sin B \cos B \sin A$$

$$= \sin A \cos B \cos^2 C \sin B - \sin A \sin B \cos B (1 - \sin^2 C)$$

$$= \sin A \sin B \cos B \cos^2 C - \sin A \sin B \cos B \cos^2 C$$

$$= 0 \Rightarrow \vec{C_1} \,\&\, \vec{C_2} \text{ are orthogonal.}$$

similarly, $\vec{C_2}$ & $\vec{C_3}$ and $\vec{C_3}$ & $\vec{C_1}$ are orthogonal.

Ans-2) To show: $\det(R'_o) = 1$

$$\det(R'_o) = \det(R_z R_y R_x)$$

$$= \det(R_z) \times \det(R_y) \times \det(R_x)$$

$$= \det\left(\begin{bmatrix} \cos C & -\sin C & 0 \\ \sin C & \cos C & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \times \det\left(\begin{bmatrix} \cos B & 0 & \sin B \\ 0 & 1 & 0 \\ -\sin B & 0 & \cos B \end{bmatrix}\right) \times \det\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos A & -\sin A \\ 0 & \sin A & \cos A \end{bmatrix}\right)$$

$$= (\cos^2 C + \sin^2 C) \times (\cos^2 B + \sin^2 B) \times (\cos^2 A + \sin^2 A)$$

$$= 1$$

Hence shown

Scanned with CamScanner

Ans-5 To show $\Rightarrow$ $R\,S(a)\,R^T = S(Ra)$

where $a = [a_x \;\; a_y \;\; a_z]^T$

and R is rotation matrix.

Let b be a vector.

$$R\,S(a)\,R^T b = R\left(a \times R^T b\right) \quad \text{——} (1)$$ (using $S(a)p = a \times P$)

As $R \in SO(3)$ and a, b are vectors in $\mathbb{R}^3$

then, $R(a \times b) = Ra \times Rb$ (equation 2.5.8 from textbook)

Using this property in (1)

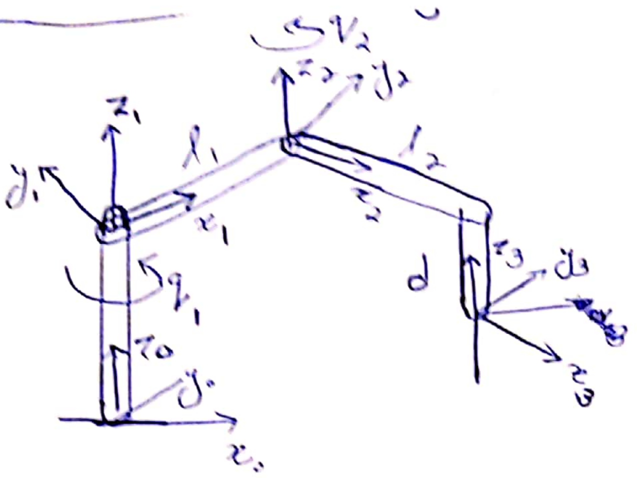
$$R\,S(a)\,R^T b = (Ra) \times (R R^T b)$$

$$= (Ra) \times b$$ (using $R R^T = I$)

$$= S(Ra)\, b$$ (using $a \times p = S(a)p$)

$\therefore$ $R\,S(a)\,R^T = S(Ra)$

Ans-6) Scara RRP configuration-



$$R_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 \\ s_{q_1} & c_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} C_0 & -S_0 & 0 \\ S_0 & C_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I, \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ -d \end{bmatrix}$$

(d is ⊕ve for upward translation and ⊖ve for downward translation)

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & l_1 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

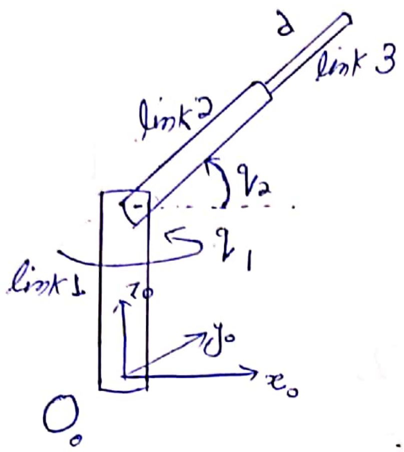
$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_0 \\ 1 \end{bmatrix} \quad \text{where} \quad P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

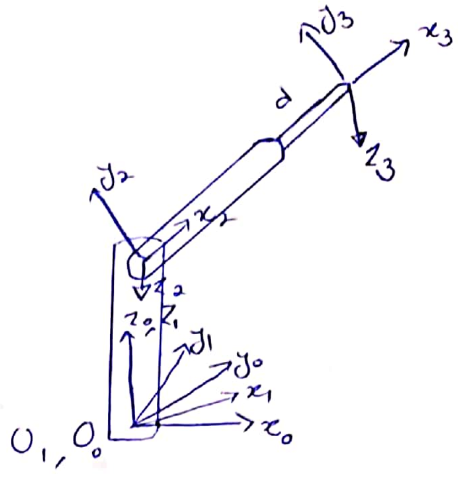
$$= H_0^1 H_1^2 \begin{bmatrix} l_2 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= H_0^1 \begin{bmatrix} l_2 c_{q_2} + l_1 \\ l_2 s_{q_2} \\ d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_1 c_{q_1} + l_2 c_{q_1} c_{q_2} - l_2 s_{q_1} s_{q_2} \\ l_1 s_{q_1} + l_2 s_{q_1} c_{q_2} + l_2 c_{q_1} s_{q_2} \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} \\ d \\ 1 \end{bmatrix}$$

Ans-8| Stanford type RRP configuration -





$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} C_0 & -S_0 & 0 \\ S_0 & C_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I, \quad d_2^3 = \begin{bmatrix} l_2+d \\ 0 \\ 0 \end{bmatrix}$$

Simplified $R_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ 0 & 0 & -1 \\ Sq_2 & Cq_2 & 0 \end{bmatrix}$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad H_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

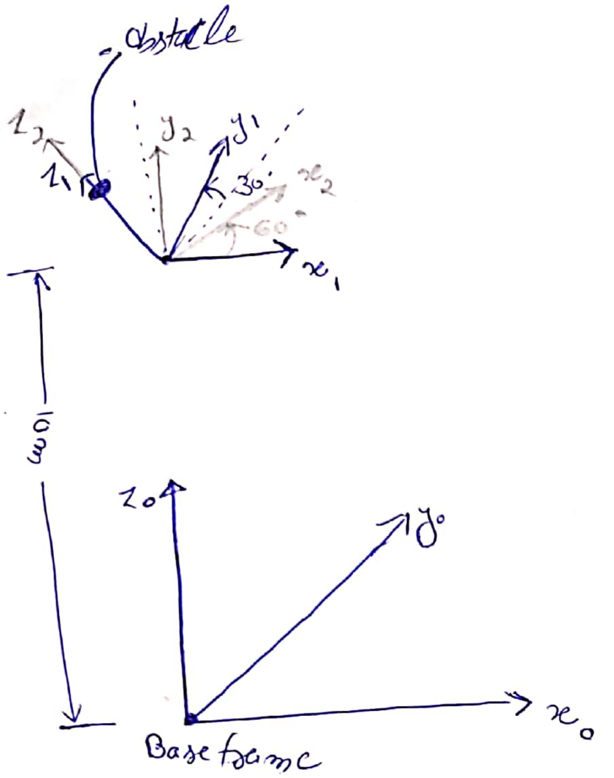
$$H_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_{q_2} & c_{q_2} & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2+d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \qquad \text{where } P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= H_0^1 H_1^2 \begin{bmatrix} l_2+d \\ 0 \\ 0 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} (l_2+d)c_{q_2} \\ 0 \\ (l_2+d)s_{q_2} + l_1 \\ 1 \end{bmatrix} = \begin{bmatrix} (l_2+d)\, c_{q_1} c_{q_2} \\ (l_2+d)\, s_{q_1} c_{q_2} \\ (l_2+d)\, s_{q_2} + l_1 \\ 1 \end{bmatrix}$$

Ans - 9)



Point in drone frame (obstacle) $P_2 = [0 \ , 0 \quad 3]^T$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° \\ 0 & \sin 30° & \cos 30° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.87 & -0.5 \\ 0 & 0.5 & 0.87 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.87 & 0 \\ 0.87 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.87 & -0.5 & 0 \\ 0 & 0.5 & 0.87 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -0.87 & 0 & 0 \\ 0.87 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.87 & -0.5 & 0 \\ 0 & 0.5 & 0.87 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.61 \\ 1 \end{bmatrix}$$

$\therefore P_0 = [0 \quad -1.5 \quad 12.61]^T$

Ans-11 $J = [J_1 \; J_2 \; \ldots\ldots \; J_n]$

where $J_i = \begin{cases} \begin{bmatrix} z_{i-1}(O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{if joint } i \text{ is revolute} \\ \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{if joint } i \text{ is prismatic} \end{cases}$

$z_{i-1} = R_0^{i-1} \hat{k}$

$O_i = d_0^i$

for RRP scara manipulator, (refer Q.6 for frame diagram)

$$J_3 = \begin{bmatrix} z_0 (O_3 - O_0) & z_1(O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix} \quad \text{——} \; (A)$$

here $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = z_1$, $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $O_1 = \begin{bmatrix} l_1 c_{q_1} \\ l_1 s_{q_1} \\ 0 \end{bmatrix}$

$$z_2 = R_0^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = R_0^1 R_1^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= R_0^1 \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} \\ d \end{bmatrix}$$

~~putting all the above values in equation~~

$\cancel{J =}$

$$Z_0 \times (O_3 - O_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} - 0 \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} - 0 \\ d - 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_{q_1} - l_2 s_{(q_1+q_2)} \\ l_1 c_{q_1} + l_2 c_{(q_1+q_2)} \\ 0 \end{bmatrix}$$

$$Z_1 \times (O_3 - O_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} - l_1 c_{q_1} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} - l_1 s_{q_1} \\ d - 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 s_{(q+q_2)} \\ l_2 c_{(q_1+q_2)} \\ 0 \end{bmatrix}$$

putting all of the above values in eq^n (A)

$$J = \begin{bmatrix} -l_1 s_{q_1} - l_2 s_{(q_1+q_2)} & -l_2 s_{(q_1+q_2)} & 0 \\ l_1 c_{q_1} + l_2 c_{(q_1+q_2)} & l_2 c_{(q_1+q_2)} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Ans-13) For planar RRR configuration —

$$R^3_2 = \begin{bmatrix} c_{q_3} & -s_{q_3} & 0 \\ s_{q_3} & c_{q_3} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d^3_2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R^2_1 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d^2_1 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R^1_0 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 \\ s_{q_1} & c_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d^1_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 c_{q_1} \\ l_1 s_{q_1} \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = H^3_0 P_3 = H^1_0 H^2_1 H^3_2 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & l_1 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_3} & -s_{q_3} & 0 & l_2 \\ s_{q_3} & c_{q_3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & l_1 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 c_{q_3} + l_2 \\ l_3 s_{q_3} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 c_{q_2} + l_3 c_{q_2} c_{q_3} - l_3 s_{q_2} s_{q_3} + l_1 \\ l_2 s_{q_2} + l_3 s_{q_2} c_{q_3} + l_3 s_{q_3} c_{q_2} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 + l_2 c_{q_2} + l_3 c_{(q_2+q_3)} \\ l_2 s_{q_2} + l_3 s_{(q_2+q_3)} \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} + l_3 c_{(q_1+q_2+q_3)} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} + l_3 s_{(q_1+q_2+q_3)} \\ 0 \\ 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_2 = R_0^2 k$$

$$= R_0^1 R_1^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 \\ s_{q_1} & c_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, $z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 \times (O_3 - O_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 c_{q_1} + l_2 c_{(q_1+q_2)} + l_3 c_{(q_1+q_2+q_3)} \\ l_1 s_{q_1} + l_2 s_{(q_1+q_2)} + l_3 s_{(q_1+q_2+q_3)} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_{q_1} - l_2 s_{(q_1+q_2)} - l_3 s_{(q_1+q_2+q_3)} \\ l_1 c_{q_1} + l_2 c_{(q_1+q_2)} + l_3 c_{(q_1+q_2+q_3)} \\ 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 c_{(q_1+q_2)} + l_3 c_{(q_1+q_2+q_3)} \\ l_2 s_{(q_1+q_2)} + l_3 s_{(q_1+q_2+q_3)} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 s_{(q_1+q_2)} - l_3 s_{(q_1+q_2+q_3)} \\ l_2 c_{(q_1+q_2)} + l_3 c_{(q_1+q_2+q_3)} \\ 0 \end{bmatrix}$$

Similarly $z_2 \times (O_3 - O_2) = \begin{bmatrix} -l_3 s_{(q_1+q_2+q_3)} \\ l_3 c_{(q_1+q_2+q_3)} \\ 0 \end{bmatrix}$

$$\therefore \quad J = \begin{bmatrix} -l_1 S_{q_1} - l_2 S_{(q_1+q_2)} - l_3 S_{(q_1+q_2+q_3)} & -l_2 S_{(q_1+q_2)} - l_3 S_{(q_1+q_2+q_3)} & -l_3 S_{(q_1+q_2+q_3)} \\ l_1 C_{q_1} + l_2 C_{(q_1+q_2)} + l_3 C_{(q_1+q_2+q_3)} & l_2 C_{(q_1+q_2)} + l_3 C_{(q_1+q_2+q_3)} & l_3 C_{(q_1+q_2+q_3)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Ans-10 Different Gearboxes used with motors in a robotic application:

① Planetary gearbox –

- Used for reduction of high RPM electric motors to high-torque low RPM applications.
- Used in precision instruments as it provides very high accuracy.
- Can give high number of gear ratios
- Difficult manufacturing and material handling.

② Worm gearbox –

- Produces high torques from high speed motors at low cost.
- A worm gear is meshed with spur gears.
- highly used in musical instruments
- have low performance due to high friction & axial stresses.

③ Twin-motor gearbox –

- Two motors are used and two outputs of different gear ratios can be obtained.
- Overheats easily even at low voltages.
- Gear ratio options at different outputs allows to perform two tasks simultaneously.