ME 639 - INTRODUCTION TO ROBOTICS

Assignment - 3

Name : Jaydeep Ramnani
Roll No.: 18110071

Task 1.

Singular configurations or Singularities are robot configurations which are restricted to move in some directions or in other words, they lose one or more of their degrees of freedom. The singularity occurs when we try to move through the origin in Cartesian space as at this point, the joint space velocity becomes infinity for one joint.

Yes, we can find the singular configurations of a robot by analyzing the Jacobian matrix of a manipulator. The singularity occurs whenever we cannot find the inverse of the Jacobian or in other words, the determinant of the Jacobian is equal to zero.

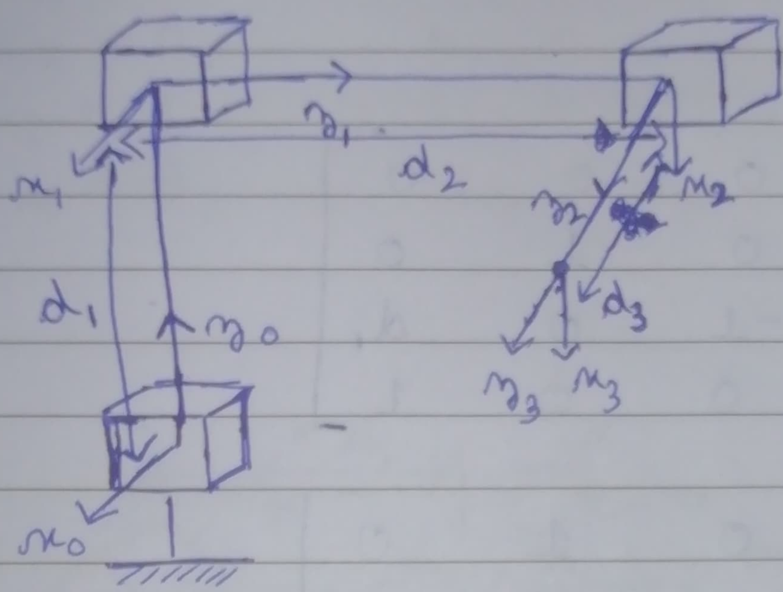
$\rightarrow det(J) = 0$

Task 7.

The Planar elbow manipulator (directly driven 2R manipulator) and the remotely driven manipulator differ in their equations of motion as when the second joint is remotely driven through a gearing mechanism or timing belt, we are able to eliminate the Coriolis forces. We still remain with the centrifugal forces coupling the two joints.

Now, when we compare these both with the five bar linkage manipulator, then we notice an important difference. The variables defined g1 and g2 are dependent on q1 and q2 respectively only, but are independent of q2 and q1 respectively. Thus, the complicated equation of motion becomes simple enough and there is minimal interaction between the two angles. Thus, it is advantageous to others and popular amongst industries.

5.



THREE LINK CARTESIAN ROBOT

We have defined the co-ordinate frames, and according to it, the DH parameters will be,

Link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	$90°$	d_2	$90°$
3	0	$0°$	d_3	$-90°$

For some, $a_i, \alpha_i, d_i, \theta_i$, the homogeneous transformation A_i is defined as,

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, the overall transformation is,

$$T_0^3 = A_1 A_2 A_3$$

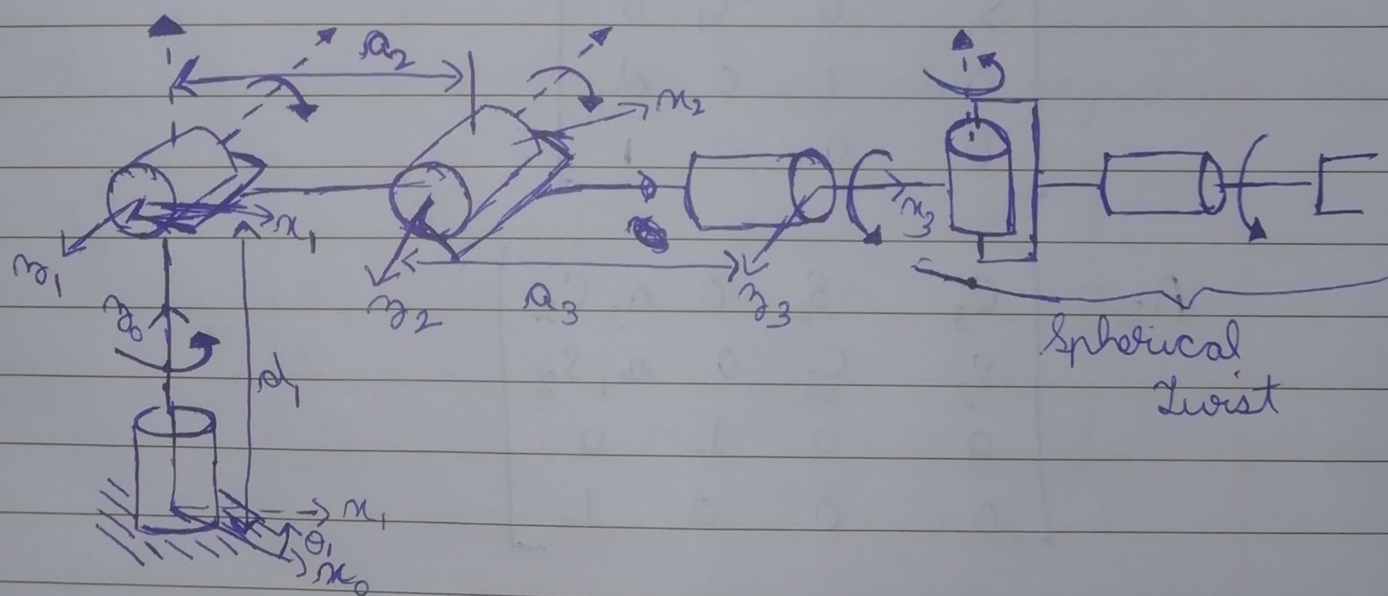
After performing calculations, we get,

$$T_0^3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, the forward kinematic equations are,

$r_{11} = 0$ | $r_{21} = -1$ | $r_{31} = 0$
$r_{12} = 0$ | $r_{22} = 0$ | $r_{32} = -1$
$r_{13} = 1$ | $r_{23} = 0$ | $r_{33} = 0$
$d_x = d_3$ | $d_y = d_2$ | $d_z = d_1$

6.



THREE-LINK ARTICULATED MANIPULATOR ATTACHED TO A SPHERICAL WRIST

According to the co-ordinate frames defined, the DH parameters are,

Link	a_i	α_i	d_i	θ_i
1	0	90°	d_1	θ_1
2	a_2	0°	0	θ_2
3	a_3	0°	0	θ_3
4	0	-90°	0	θ_4
5	0	90°	0	θ_5
6	0	0°	d_6	θ_6

Defining homogeneous transformations for each link,

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$T_3^0 = A_1 A_2 A_3$$

$$= \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & a_2 C_1 C_2 + a_3 C_1 C_{23} \\ S_1 C_{23} & S_1 S_{23} & -C_1 & a_2 S_1 C_2 + a_3 S_1 C_{23} \\ S_{23} & C_{23} & 0 & a_2 S_2 + a_3 S_{23} + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For spherical wrist, using equation (3.15) from pdf of book,

$$T_6^3 = A_4 A_5 A_6 = \begin{bmatrix} C_4 S_5 C_6 - S_4 S_6 & -C_4 S_5 S_6 - S_4 C_6 & C_4 S_5 & C_4 S_5 d_6 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & S_4 S_5 & S_4 S_5 d_6 \\ -S_5 C_6 & S_5 S_6 & C_5 & C_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, the forward kinematic equations are,

$$T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

where

$$r_{11} = C_1 (C_5 C_6 C_{234} - S_6 S_{234}) - S_1 S_5 C_6$$

$$r_{12} = S_1 S_5 S_6 - C_1 (C_5 C_6 C_{234} + C_6 S_{234})$$

$$r_{13} = C_1 C_{234} S_5 + S_1 C_5$$

$$d_x = a_2 C_1 C_2 + a_3 C_1 C_{23} + d_6 (C_1 S_5 C_{234} + S_1 C_5)$$

P.T.O.

$r_{21} = S_1 (C_5 C_6 C_{234} - S_6 S_{234}) + C_1 S_5 S_6$

$r_{22} = -C_1 S_5 S_6 - S_1 (S_6 C_5 C_{234} + C_6 S_{234})$

$r_{23} = S_1 S_5 C_{234} - C_1 C_5$

$d_y = (S_1 S_5 C_{234} - C_1 C_5) d_6 + a_2 S_1 C_2 + a_3 S_1 C_{23}$

$r_{31} = C_5 C_6 S_{234} + S_6 C_{234}$

$r_{32} = + C_6 C_{234} - C_5 S_6 S_{234}$

$r_{33} = S_5 S_{234}$

$d_z = a_2 S_2 + a_3 S_{23} + d_1 + d_6 S_5 S_{234}$.

8. For deriving the dynamic equations of 2R manipulator, we use the DH joint variables as generalized co-ordinates. Thus,

$$v_{c_1} = J_{v_{c_1}} \dot{q}$$

where,

$$J_{v_{c_1}} = \begin{bmatrix} -l_c \, \cancel{\sin q_1} \, S_{q_1} & 0 \\ l_{c_1} \, \cancel{\cos} \, C_{q_1} & 0 \\ 0 & 0 \end{bmatrix}.$$

Similarly, $v_{c_2} = J_{v_{c_2}} \dot{q}$

where, $$J_{v_{c_2}} = \begin{bmatrix} -l_1 S_{q_1} - l_{c_2} S_{q_1 q_2} & -l_{c_2} S_{q_1 q_2} \\ l_1 C_{q_1} + l_{c_2} C_{q_1 q_2} & l_{c_2} C_{q_1 q_2} \\ 0 & 0 \end{bmatrix}$$

Hence, the translational kinetic energy is,

$$\frac{1}{2} m_1 v_{c_1}^T v_{c_1} + \frac{1}{2} m_2 v_{c_2}^T v_{c_2} = \frac{1}{2} \dot{q} \{ m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} \} \dot{q}$$

Calculating angular velocities, we get

$$\omega_1 = \dot{q}_1 k_1 \quad , \quad \omega_2 = \dot{q}_2 k_2$$

(base inertial frame)

Now, as ω_i is aligned with z-axes for each frame, the rotational kinetic energy reduces to $\frac{1}{2} I_i \omega_i^2$, where I_i is the moment of inertia about an axis through mass centre of that link parallel to its z-axis. Hence, the total rotational kinetic energy of system is,

$$\frac{1}{2} \dot{q}^T \left\{ I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\} \dot{q}$$

Now, the total kinetic energy $D(q)$ will be,

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

Thus, we get

$$d_{11} = m_1 l_{c_1}^2 + m_2 (l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2} \cos q_2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2 (l_{c_2}^2 + l_1 l_{c_2} \cos q_2) + I_2$$

$$d_{22} = m_2 l_{c_2}^2 + I_2$$

We can ~~so~~ now compute Christoffel symbols, as follows,

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c_2} S_{q_2} = h$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$$

$$c_{122} = c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Calculating the potential energy of each link,

$$P_1 = m_1 g l_{c_1} S_{q_1}$$

$$P_2 = m_2 g (l_1 S_{q_1} + l_{c_2} S_{q_1 q_2})$$

Therefore, total potential energy is,

$$P = P_1 + P_2$$

$$= (m_1 l_{c_1} + m_2 l_1) g \, S_{q_1} \cancel{\sin q_1} + m_2 l_{c_2} g S_{q_1 q_2}$$

Thus,

$$g_1 = \frac{\partial P}{\partial q_1} = (m_1 l_{c_1} + m_2 l_1) g C_{q_1} + m_2 l_{c_2} g C_{q_1 q_2}$$

$$g_2 = \frac{\partial P}{\partial q_2} = m_2 l_{c_2} g C_{q_1 q_2}$$

Finally, the dynamical equations of system can be calculated and are:,

$$\Rightarrow \tau_1 = d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 + c_{121} \dot{q}_1 \dot{q}_2 + c_{211} \dot{q}_2 \dot{q}_1 + c_{22} \dot{q}_2^2 + g_1$$

$$\Rightarrow \tau_2 = d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + c_{112} \dot{q}_1^2 + g_2$$

10. If we are given $D(q)$ and $V(q)$, the potential energy $P = V(q)$ and the kinetic energy is

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q} = \frac{1}{2} \sum_{i,j} d_{ij}(q) \dot{q}_i \dot{q}_j$$

Now, we move on to deriving Euler-Lagrange equations for such a system. Thus,

$$L = K - P = \frac{1}{2} \sum_{i,j} d_{ij}(q) \dot{q}_i \dot{q}_j - V(q)$$

Now, $\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj} \dot{q}_j$ (joint velocity)

Further, its derivative with time is,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}\dot{q}_j$$

$$= \sum_j d_{kj}\ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

Now,

$$\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial P}{\partial q_k} = \tau_k$$

(joint position)

Thus, for each $k = 1, \ldots, n$, the Euler-Langrange equations can be written

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j + \frac{\partial P}{\partial q_k} = \tau_k$$

By interchanging the order of summation and taking advantage of symmetry, we can show

$$\sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i}\right\}\dot{q}_i\dot{q}_j = \frac{1}{2}\sum\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right\}\dot{q}_i\dot{q}_j$$

Subtracting $\frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j$ from both sides

we get

$$\sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i \dot{q}_j$$

$$= \sum_{i,j} c_{ijk} \dot{q}_i \dot{q}_j$$

where, $\underbrace{c_{ijk}}_{\text{Christoffel symbols}} = \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\}$

Christoffel symbols

Finally, let $g_k = \frac{\partial P}{\partial q_k}$

Thus, we can write Euler-lagrange equations as

$$\Rightarrow \sum_{j=1}^{n} d_{kj}(q) \ddot{q}_j + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ijk}(q) \dot{q}_i \dot{q}_j + g_k(q)$$

$$= \tau_k$$

$$\forall k = 1, \ldots, n$$