ASSIGNMENT 2 - RWIK
18110146

Q1)

$$
\begin{array}{cccc}
 & C_1 & C_2 & C_3 \\
R_0^1 = & \left[\begin{matrix} \hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 \end{matrix}\right. & \begin{matrix} \hat{j}_1 \cdot \hat{i}_0 \\ \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{k}_0 \end{matrix} & \left.\begin{matrix} \hat{k}_1 \cdot \hat{i}_0 \\ \hat{k}_1 \cdot \hat{j}_0 \\ \hat{k}_1 \cdot \hat{k}_0 \end{matrix}\right]
\end{array}
$$

where $\hat{i}_1, \hat{j}_1, \hat{k}_1$ are orthognal to eachother

i.e $\hat{i}_1 \cdot \hat{j}_1 = \hat{j}_1 \cdot \hat{k}_1 = \hat{k}_1 \cdot \hat{i}_1 = 0$

similarly, $\hat{i}_0 \cdot \hat{j}_0 = \hat{j}_0 \cdot \hat{k}_0 = \hat{k}_0 \cdot \hat{i}_0 = 0$

Checking for column space of R_0^1

$$C_1 \cdot C_2 = \hat{i}_1 \cdot \hat{i}_0 \cdot \hat{j}_1 \cdot \hat{i}_0 + \hat{i}_1 \cdot \hat{j}_0 \cdot \hat{j}_1 \cdot \hat{j}_0 + \hat{i}_1 \cdot \hat{k}_0 \cdot \hat{j}_1 \cdot \hat{k}_0$$

consists of $\hat{i}_1, \hat{j}_1$

$\hat{i}_1, \hat{j}_1$

$\hat{i}_1, \hat{j}_1$

All the 3 terms have $\hat{i}_1, \hat{j}_1$ in it. Terms can be rearranged in dot product. Thus in all the terms, we get $\hat{i}_1 \cdot \hat{j}_1 = 0$

Thus $C_1 \cdot C_2 = 0$

Similarly $C_2 \cdot C_3 = 0$ $\quad (\hat{j}_1 \cdot \hat{k}_1) = 0$

$C_3 \cdot C_1 = 0$ $\quad (\hat{i}_1 \cdot \hat{k}_1) = 0$

Therefore column space is orthogonal.

Q2) Lets take a vector $\vec{V}$ and two sets of basis B_1 and B_0.

Representation of $\vec{V}$ w.r.t B_1 is $\vec{V_1}$
Representation of $\vec{V}$ w.r.t B_0 is $\vec{V_0}$

As the vector is the same,

$$\det(\vec{V_1}) = \det(\vec{V_0})$$

Let R_0^1 be the rotation matrix.

$$\therefore \vec{V_0} = R_0^1 \vec{V_1}$$

Taking determinant on both sides,

$$\det(\vec{V_0}) = \det(R_0^1 \vec{V_1})$$

$$\Rightarrow \cancel{\det(\vec{V_0})} = \det(R_0^1) \cancel{\det(\vec{V_1})}$$

$$\Rightarrow \boxed{\det(R_0^1) = 1}$$

//

Q5) $R S(a) R^T = S(Ra)$

Since R is ~~orthogonal~~ rotation matrix i.e. it is orthogonal, give two vectors $\vec{x}$ and $\vec{y}$, we can say that

$$\cancel{R(a \times b} \quad R(\vec{x} \times \vec{y}) = R\vec{x} \times R\vec{y}$$

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$$R\,S(\vec{a})\,R^T = S(Ra)$$

We know $S(a)p = a \times p$

We take a arbitrary vector,

$$R\,S(\vec{a})\,R^T\vec{b} = R(\vec{a} \times R^T\vec{b})$$

$$= R\vec{a} \times R R^T \vec{b}$$

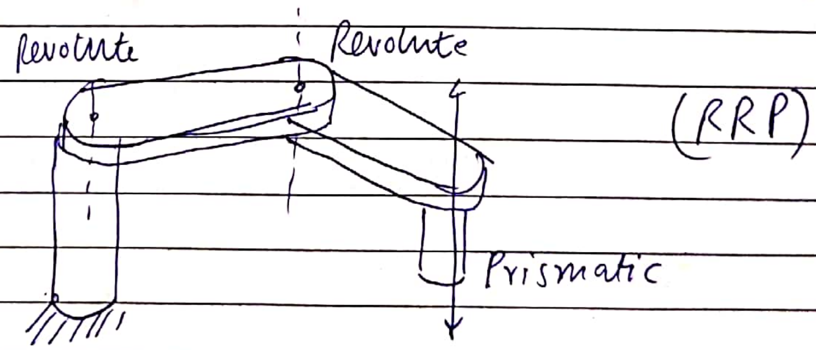
$$= R\vec{a} \times \vec{b}$$

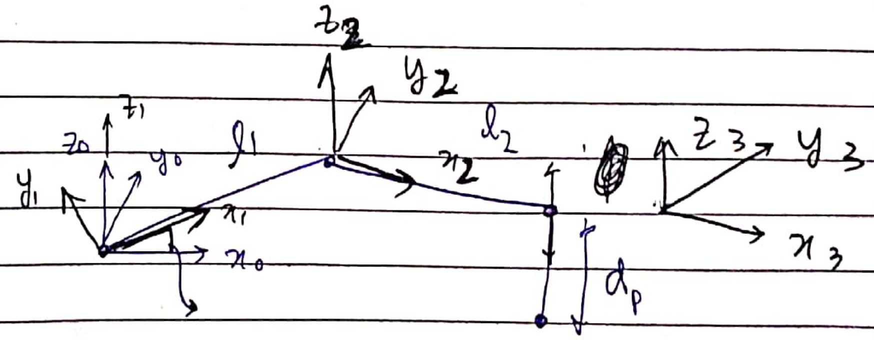
$$R\,S(\vec{a})\,R^T\vec{b} = S(R\vec{a})\,\vec{b}$$

from LHS, RHS we can say that

$$\boxed{R\,S(\vec{a})\,R^T = S(R\vec{a})}$$

Q6)





$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

(x_2, y_2, z_2) and (x_3, y_3, z_3) are parallel axes systems &

$d_p \rightarrow$ distance moved by prismatic joint

$$* \quad H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\hookrightarrow q_3 = 0^\circ$

$$* \quad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

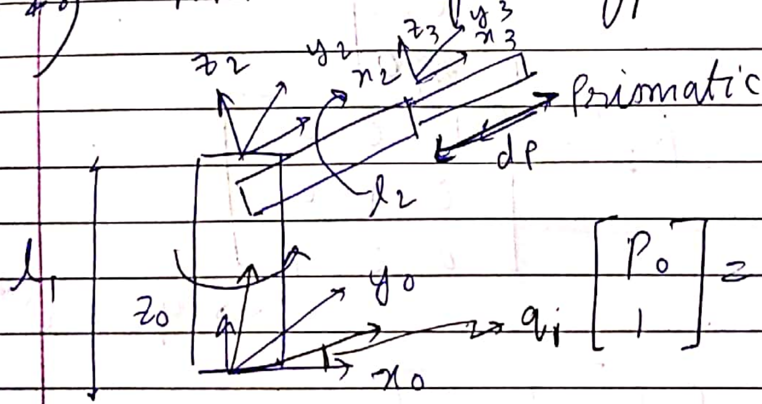
$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ d_p \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Q ~~7~~8) RRP stanford type.



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_i^{i+1} = \begin{bmatrix} R_i^{i+1} & d_i^{i+1} \\ 0 & 1 \end{bmatrix}$$

$$R_2^3 = I, \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

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$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\pi/_2 & -s\pi/_2 \\ 0 & s\pi/_2 & c\pi/_2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c \rightarrow$ cosine

$s \rightarrow$ sine.

$$= \begin{bmatrix} cq_2 & -sq_2 & 0 \\ 0 & 0 & -1 \\ sq_2 & cq_2 & 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} d_p \\ 0 \\ 0 \end{bmatrix}$$

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Using DH parameters:

Q1b)
$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} \cancel{a_1 c_1 + a_2 c_{12}} \\ \cancel{a_1 s_1 + a_2 s_{12}} \\ \end{bmatrix}$$

O_3 is prismatic

$$O_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 \end{bmatrix}$$

d_3 ↳ movement of prismatic joint.

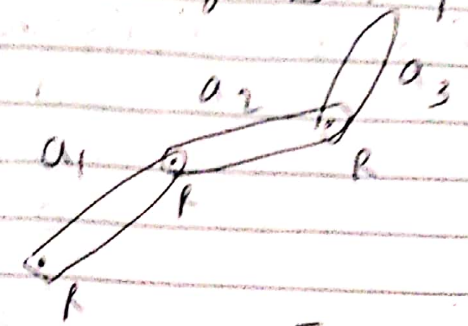
$z_0 = z_1 = k$

$k = [0 \ 0 \ 1]^T$

$z_2 = -k.$

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q13) Using DH parameters,



$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

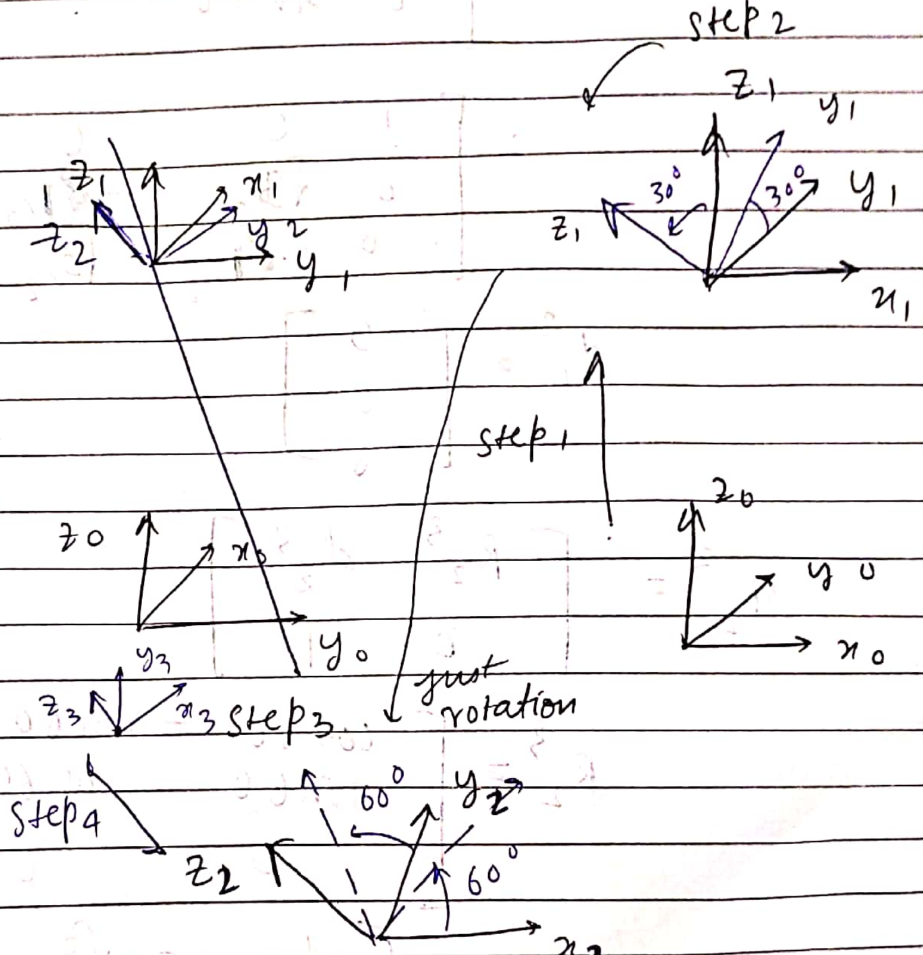
$$\cancel{J = \begin{bmatrix} a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ -a_1 c_1 - a_2 c_{12} - a_3 c_{123} \\ \\ \end{bmatrix}}$$

$\Rightarrow$

$$J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} - a_3 S_{123} & -a_2 S_{12} - a_3 S_{123} & -a_3 S_{123} \\ a_1 C_1 + a_2 C_{12} + a_3 C_{123} & a_2 C_{12} + a_3 C_{123} & +a_3 C_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q9) There are a total of 4 transformations,

- Drone Base → 10 mtr
- Drone Rotation about x → 30°
- Drone Rotation about new z → 60°
- obstacle 3 mtrs z axis



$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_4 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_0^1 = I$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3^4 = I \qquad d_3^4 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_1^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & -\sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{H_0^1 H_1^2 H_2^3 H_3^4 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & -\sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & \sqrt{3}/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & -\sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.59 \\ 1 \end{bmatrix}$$

Task 10

There are many types of gearboxes used in the field of robotics, here are a few examples for the same.

1. Planetary gearhead:
   They are used in power trains. They are suited for rotating prime mover like electrical motors. They have the ability to produce high gear ratios. They consist of planet gears. It provides high torques while being very compact. Planetary gears are used in helicopters, automobiles, marine applications etc.
2. Cycloid Drives:
   Cycloid drives are used in cranes, boats, CNC machines etc. It consists of a planet wheel which moves in a wobly cycloidal manner given input motion . It produces high reduction capacity. They are used when there is a heavy cam wheel.

Drones have propellers which require high speed i.e. high RPM to help them hover and move. Gear boxes are generally used to increase the torque to the system. But in drones we only require high speed propellers. So a simple gear will suffice i.e. gear connecting the motor and the propellers.