

设线段 AB 长度为 a ，易得：

$$AC = \sqrt{2}, \quad BM = \frac{\sqrt{5}}{3}a, \quad DM = BD - BM = 5 - \frac{\sqrt{5}}{3}a, \quad \sin \angle ABD = \frac{\sqrt{5}}{5}$$

观察并计算得：

$$\begin{aligned} \sin \angle AMD &= \sin \angle BMC = \sin (\angle BAC + \angle ABD) = \sin \angle ABD \sin \angle BAC + \cos \angle ABD \cos \angle BAC \\ &= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{5}}{5} + \frac{2\sqrt{5}}{5} \right) \\ &= \frac{3\sqrt{10}}{10} \end{aligned}$$

在 $\triangle ACD$ 作 AC 边的高，长度为 h ，则：

$$h = DM \sin \angle AMD = \frac{3\sqrt{10}}{10} \left(5 - \frac{\sqrt{5}}{3}a \right)$$

由此可得：

$$\begin{aligned} S_{\triangle ACD} &= \frac{AC \cdot h}{2} = \frac{1}{2} \cdot \sqrt{2}a \cdot \frac{3\sqrt{10}}{10} \left(5 - \frac{\sqrt{5}}{3}a \right) \\ &= \frac{9}{10} \times \frac{\sqrt{5}}{3}a \left(5 - \frac{\sqrt{5}}{3}a \right) \end{aligned}$$

由算术-几何平均值不等式可得：

$$S_{\triangle ACD} \leq \frac{9}{10} \times \frac{\left(\frac{\sqrt{5}}{3}a + 5 - \frac{\sqrt{5}}{3}a \right)^2}{4} = \frac{45}{8} \quad (\text{当且仅当 } \frac{\sqrt{5}}{3}a = 5 - \frac{\sqrt{5}}{3}a, \text{ 即 } a = \frac{\sqrt{45}}{2} \text{ 时取相等})$$

综上，当 $AB = \frac{\sqrt{45}}{2}$ 时， $S_{\triangle ACD}$ 面积最大为 $\frac{45}{8}$