设线段 AB 长度为 a , 易得:

$$AC=\sqrt{2}$$
 , $BM=rac{\sqrt{5}}{3}a$, $DM=BD-BM=5-rac{\sqrt{5}}{3}a$, $\sin \angle ABD=rac{\sqrt{5}}{5}$

观察并计算得:

$$\sin \angle AMD = \sin \angle BMC = \sin \left(\angle BAC + \angle ABD \right) = \sin \angle ABD \sin \angle BAC + \cos \angle ABD \cos \angle BAC$$
$$= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{5}}{5} + \frac{2\sqrt{5}}{5} \right)$$
$$= \frac{3\sqrt{10}}{10}$$

在 \triangle ACD 作 AC 边的高, 长度为 h, 则:

$$h = DM\sin\angle AMD = \frac{3\sqrt{10}}{10}(5 - \frac{\sqrt{5}}{3}a)$$

由此可得:

$$egin{aligned} S_{ riangle ACD} &= rac{AC \cdot h}{2} = rac{1}{2} \cdot \sqrt{2} a \cdot rac{3\sqrt{10}}{10} (5 - rac{\sqrt{5}}{3} a) \ &= rac{9}{10} imes rac{\sqrt{5}}{3} a (5 - rac{\sqrt{5}}{3} a) \end{aligned}$$

由算术-几何平均值不等式可得:

$$S_{ riangle ACD} \leq rac{9}{10} imes rac{(rac{\sqrt{5}}{3}a + 5 - rac{\sqrt{5}}{3}a)^2}{4} = rac{45}{8}$$
 (当且仅当 $rac{\sqrt{5}}{3}a = 5 - rac{\sqrt{5}}{3}a$,即 $a = rac{\sqrt{45}}{2}$ 时取相等)

综上,当 $AB=rac{\sqrt{45}}{2}$ 时, $S_{\triangle ACD}$ 面积最大为 $rac{45}{8}$