ECS 323: Control Systems

Planar VTOL System

Design Study

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1. Design Study Description

In this design study we need to design a control system for the given planar VTOL system with the parameters:

- $M_c = 2$ kg
- $J_c = 0.009$ kg m^2
- $m_l = 0.3$ kg
- $m_r = 0.3$ kg
- $d = 0.28$ m
- $\mu = 0.21$ kg s^{-1}

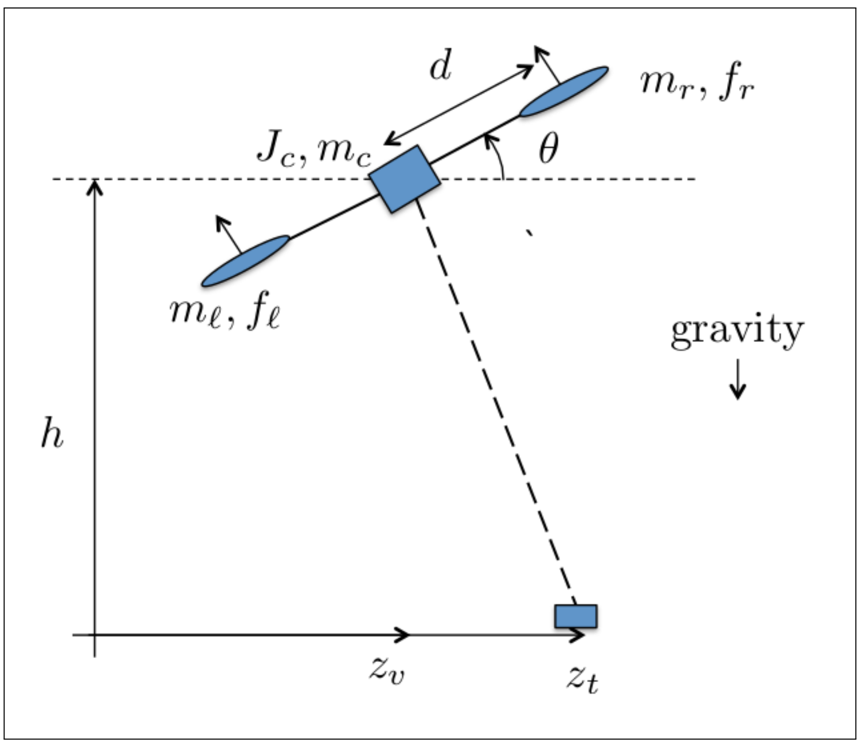

Figure 1: Planar VTOL System

2. Kinetic Energy

The postions of the various components of the VTOL are given by:

$$\begin{aligned}\mathbf{p_c} &= (z_v,\ h)\\ \mathbf{p_l} &= (z_v - d\cos\theta,\ h - d\sin\theta)\\ \mathbf{p_r} &= (z_v + d\cos\theta,\ h + d\sin\theta)\end{aligned}$$

So, the velocities can be written as:

$$\begin{aligned}\mathbf{v_c} &= (\dot{z}_v,\ \dot{h})\\ \mathbf{v_l} &= (\dot{z}_v + d\dot{\theta}\sin\theta,\ \dot{h} - d\dot{\theta}\cos\theta)\\ \mathbf{v_r} &= (\dot{z}_v - d\dot{\theta}\sin\theta,\ \dot{h} + d\dot{\theta}\cos\theta)\end{aligned}$$

Kinetic energy of the centerpod is given by:

$$K_{pod} = \frac{1}{2}m_c\mathbf{v}_c^T\mathbf{v}_c + \frac{1}{2}\boldsymbol{\omega}_c^T J_c\boldsymbol{\omega}_c = \frac{1}{2}m_c(\dot{z}_v^2 + \dot{h}^2) + \frac{1}{2}J_c\dot{\theta}^2 \tag{1}$$

Kinetic energy of the left and right rotors is given by:

$$\begin{aligned}K_{rotors} &= \frac{1}{2}m_l\mathbf{v}_l^T\mathbf{v}_l + \frac{1}{2}m_r\mathbf{v}_r^T\mathbf{v}_r\\ &= \frac{1}{2}m_l(\dot{z}_v + d\dot{\theta}\sin\theta)^2 + \frac{1}{2}m_l(\dot{h} - d\dot{\theta}\cos\theta)^2\\ &\quad + \frac{1}{2}m_r(\dot{z}_v - d\dot{\theta}\sin\theta)^2 + \frac{1}{2}m_l(\dot{h} + d\dot{\theta}\cos\theta)^2\\ &= \frac{1}{2}(m_l + m_r)(\dot{z}_v^2 + \dot{h}^2) + \frac{1}{2}(m_l + m_r)d^2\dot{\theta}^2\\ &\quad + (m_l - m_r)(\dot{z}_v\sin\theta - \dot{h}\cos\theta)d\dot{\theta}\end{aligned} \tag{2}$$

Now, the total kinetic energy of the VTOL will be given by the sum of 1 and 2:

$$K_V = K_{pod} + K_{rotors} \tag{3}$$