Blind Experiments with Finite Difference Methods

Patel Jaydipkumar Lalitkumar

Aerospace Engineering Department
Indian Institute of Technology, Bombay

Abstract- *This report consists of the explanation of results obtained on the application of finite difference methods to solve the one-dimensional advection equation, by varying Courant-Friedrichs-Lewy (CFL) numbers for five different initial conditions, using the Python software. Results show that high CFL values lead to instability, while low CFL values increase numerical diffusion. The comparative analysis gives insights into the different FDM schemes.*

Introduction

The Finite Difference Scheme(FDM) is **a numerical method used to solve partial differential equations by approximating derivatives with finite differences**. This method is particularly useful for solving complex equations that cannot be solved analytically.

First step towards approximate solution of Pde by FDM is to divide the physical domain into a finite number of points. Generate the grid. The intersection points of these grid form a set of finite number of grid points. One can approximate the governing Pde's at these grid points using different FDM schemes. Instead of attempting an exact solution which would be satisfied at each and every possible point of domain, apply an approximate solution of Pde's at a finite number of grid points as this approach is economical and viable where an analytical solution is difficult to obtain.

Finite Difference Schemes

***Forward Time Forward Space (FTFS) Scheme**:* The FTFS scheme is an explicit method where the time derivative is approximated by a forward difference, and the spatial derivatives are also approximated by a forward difference.

***Forward Time Central Space (FTCS) Scheme**:* The FTCS scheme is an explicit, the spatial derivatives are approximated using a central difference (taking points on both sides of the current point), while the time derivative uses a forward difference. This provides better accuracy compared to FTFS.

***Forward Time Backward Space (FTBS) Scheme**:* In the FTBS scheme, the time derivative is approximated by a forward difference, and the spatial derivatives are approximated by a backward difference (using points behind the current point).

***Lax-Wendroff (LW) Scheme**:* The Lax-Wendroff scheme is a higher-order explicit method that improves the accuracy of the solution by using both the forward difference in time and a combination of central differences for space. This method helps reduce numerical dispersion and dissipation, making it more accurate for solving problems with sharp gradients or rapid changes.

***Beam-Warming (BW) Scheme**:* The Beam-Warming scheme is a higher-order explicit method that uses a combination of forward time and backward space differences.

***Fromm (FR) Scheme**:* The Fromm scheme is a higher-order method that uses a weighted combination of forward and backward space differences for the spatial derivative, leading to improved accuracy over traditional central or forward/backward schemes. It is a second-order accurate scheme in both time and space.

***Implicit Scheme (Backward Time Central Space - BTCS)**:* The BTCS scheme is an implicit method where the time derivative is approximated by a backward difference, and the spatial derivative uses a central difference. Unlike explicit schemes, the BTCS scheme does not require a stability condition based on the time step, making it more stable for large time steps. However, it requires solving a system of equations at each time step, which makes it computationally more intensive.

Courant-Friedrichs-Lewy Number

$$\text{CFL} = \frac{c * \Delta t}{\Delta x};$$

c is wave speed, Δx *is step length and* Δt is time step

CFL ≤ 1: The CFL number is often considered to be safe for stability in explicit schemes. When the CFL number is less than or equal to 1, the numerical scheme is typically stable, meaning that information does not travel faster than the grid spacing can handle, and the solution does not become unstable.

CFL > 1: If the CFL number exceeds 1, the solution can become unstable. In this case, the numerical grid "moves ahead" of the information it is trying to track, causing oscillations or even divergence in the solution. This often leads to inaccurate or non-physical results.

Explicit-Schemes

In this kind of schemes, the solution of the next step is wholly depended on the previous results obtained. Requires small time steps to get stable solution. Computationally cheaper per time step. Less accurate for larger time step. It is easier to implement as the future solution is fully depended on past solution.

Implicit-Schemes

In this kind of schemes, the solution involves solving the system of having unknowns appearing on both sides. It allows to run the scheme for higher time steps also. Computationally it is expensive as it requires to solve the system of linear equation at each time step. Quite hard to implement as it requires matrix formulation to solve the linear equations.

$\boldsymbol{Explicit}$: $u(n+1) = u(n) + h.f\big(u(n)\big)$

$\boldsymbol{Implicit}$: $u(n+1) = u(n) + h.f(u(n+1))$

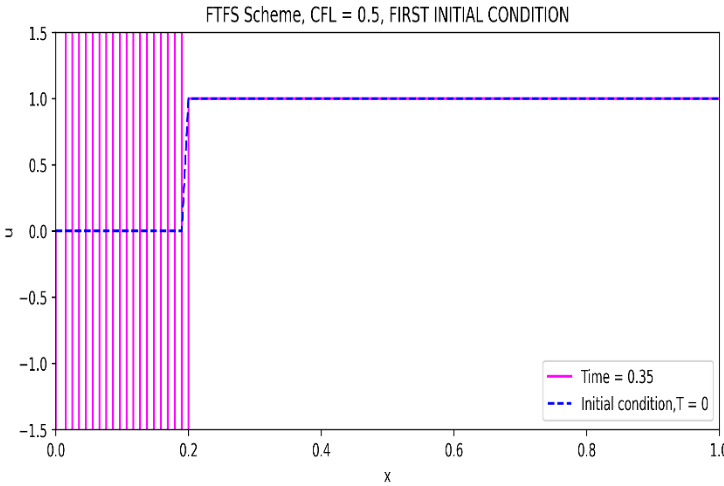
$; u(n) is\ the\ value\ of\ u\ at\ t = n$
$; u(n+1) is\ the\ value\ of\ u\ at\ t = (n+1)$

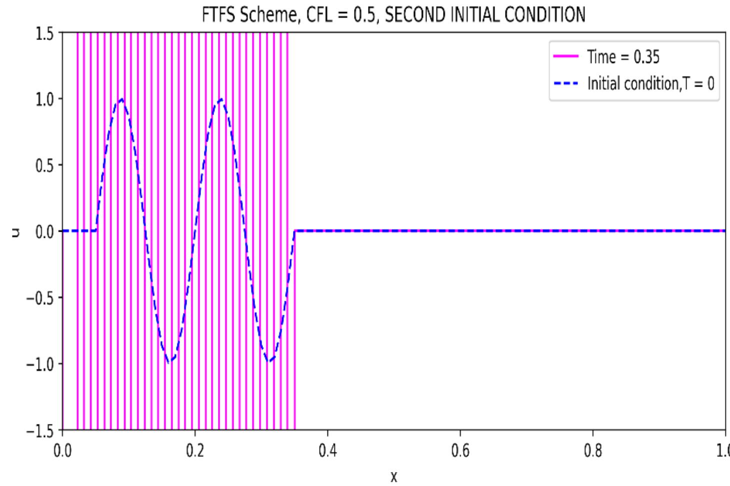
Results and Observations

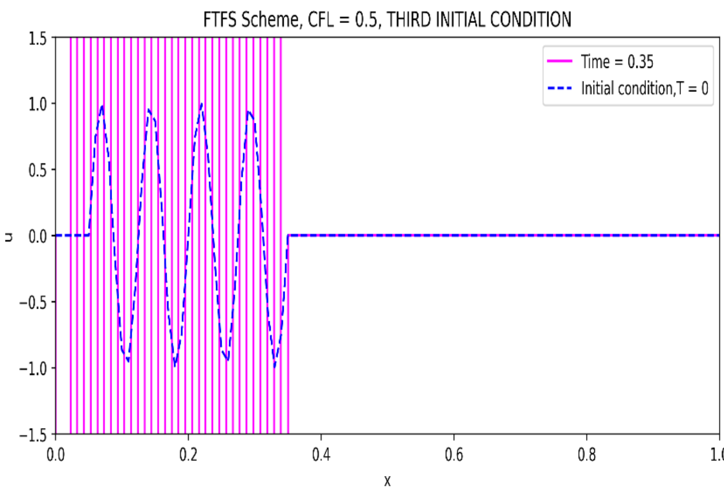
Forward Time Forward Space (FTFS) scheme

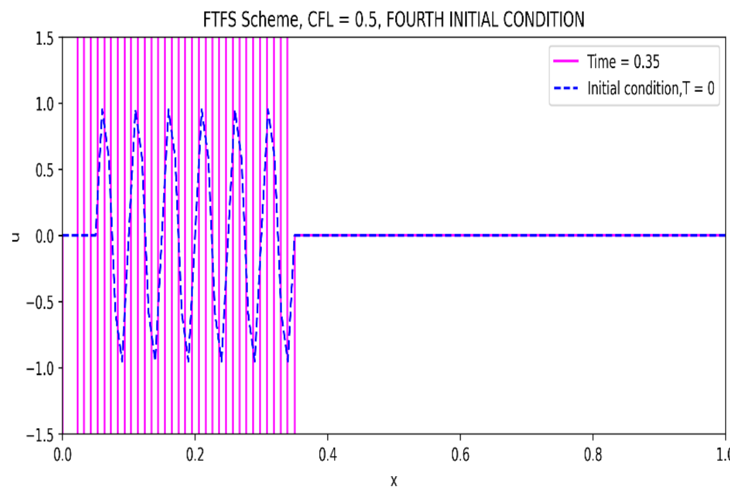
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Oscillations are observed in the plot, it proves that this scheme is not able to provide accurate and stable solution kind of same as numerical results though CFL number falls in stability criteria.	Kind of same solution as obtained before for same condition except CFL number.	Kind of same solution as obtained before for same condition except CFL number.
2	Initial solution having two periods	oscillations are observed, not an accurate and stable solution.	Kind of same solution as obtained before for same condition except CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
3	Initial solution having four periods	oscillations are observed, not an accurate and stable solution	Oscillations are still there but now it also results in the dissipation compared to previous case having same conditions except the CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
4	Initial solution having six periods	oscillations are observed, not an accurate and stable solution	Oscillations are still there but now it also results in the dissipation compared to previous case having same conditions except the CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
5	Initial solution having Gaussian curve	Less oscillatory solution compared to previous initial conditions but the solution faced dissipation or say diffusion, unstable and inaccurate solution.	Less oscillatory solution compared to previous initial conditions but the solution faced more dissipation or say diffusion, unstable and inaccurate solution.	Oscillations are still there accompanied with increment in diffusion, instabilities as well as inaccuracy in solution.

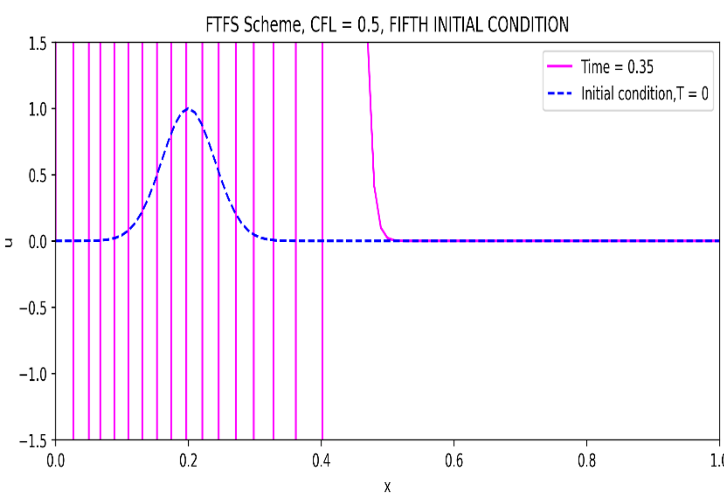
** For first initial condition boundary conditions are taken according to given function.

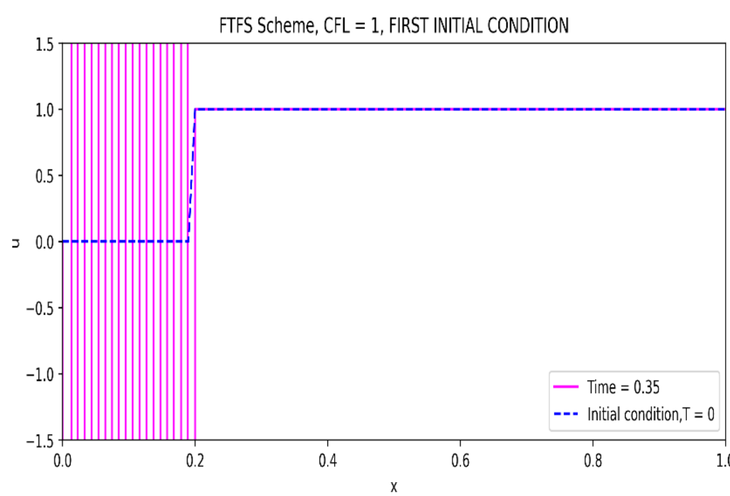

FTFS Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

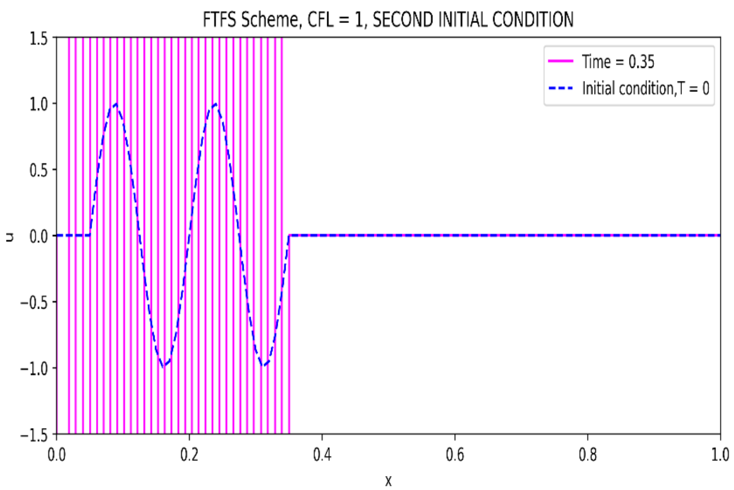

FTFS Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

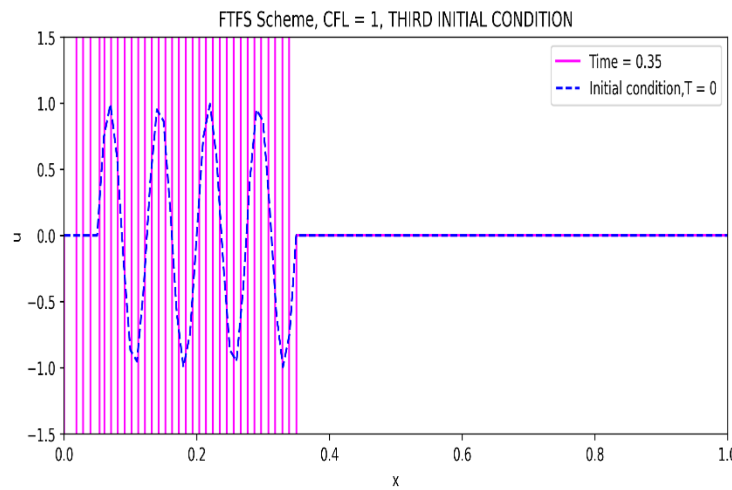

FTFS Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

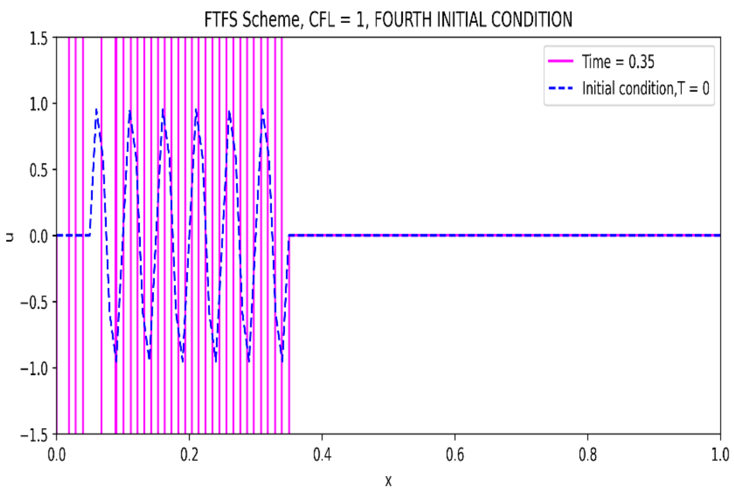

FTFS Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

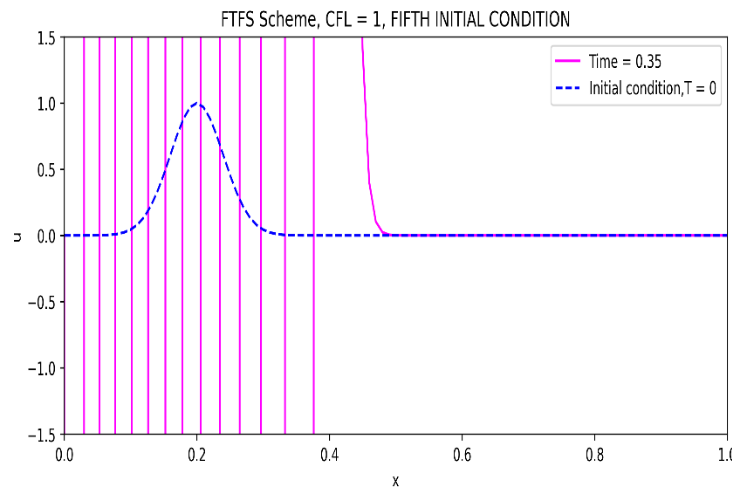

FTFS Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

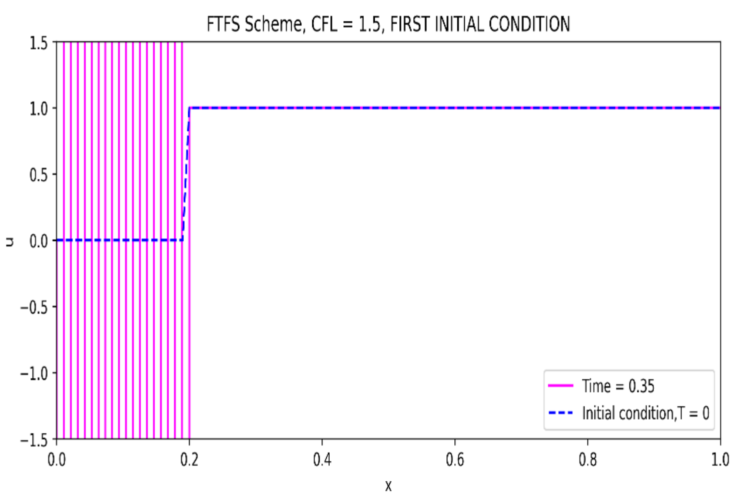

FTFS Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

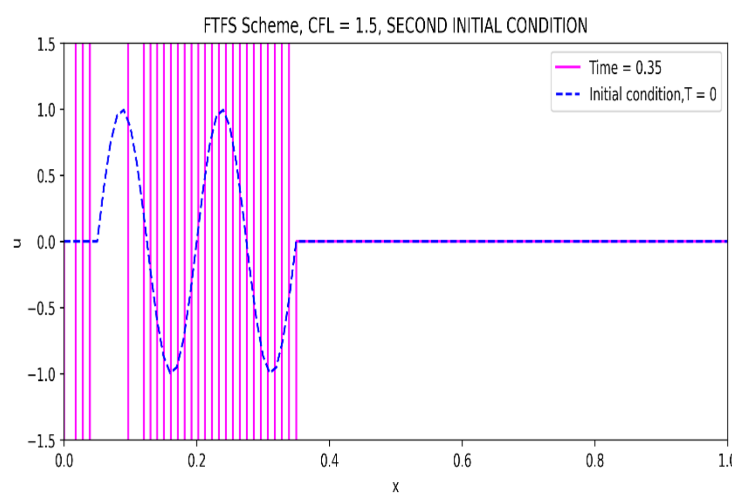

FTFS Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

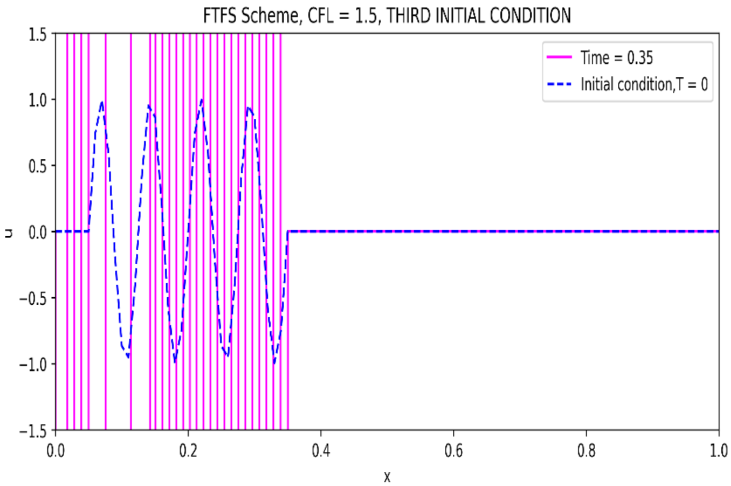

FTFS Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

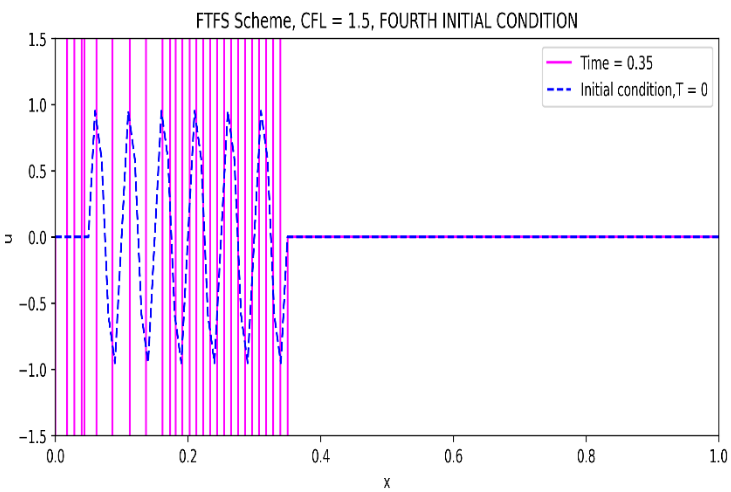

FTFS Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

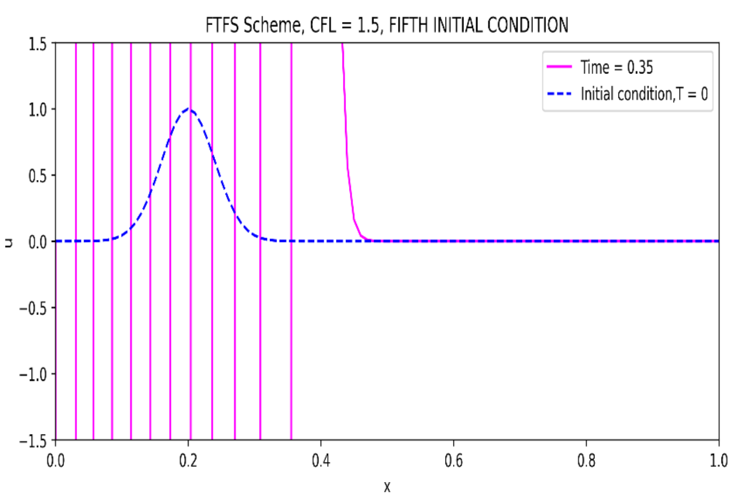

FTFS Scheme, CFL = 1, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FTFS Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FTFS Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

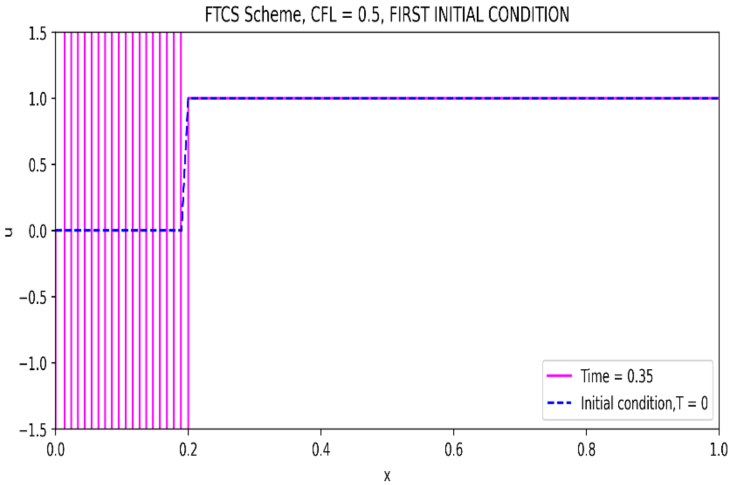

FTFS Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

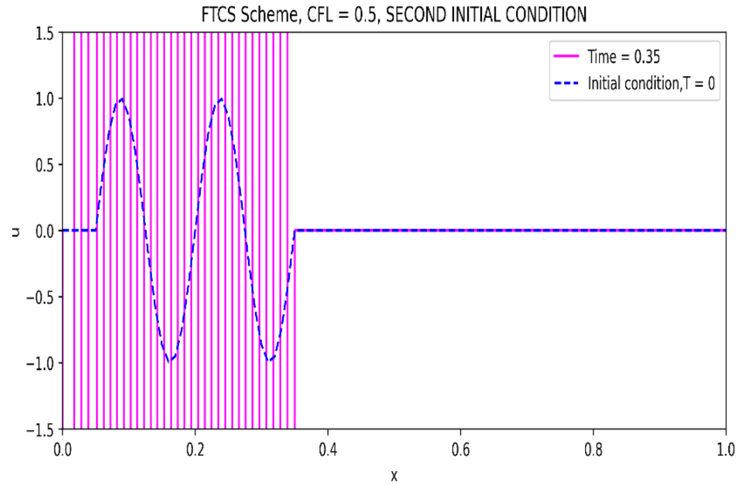

FTFS Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

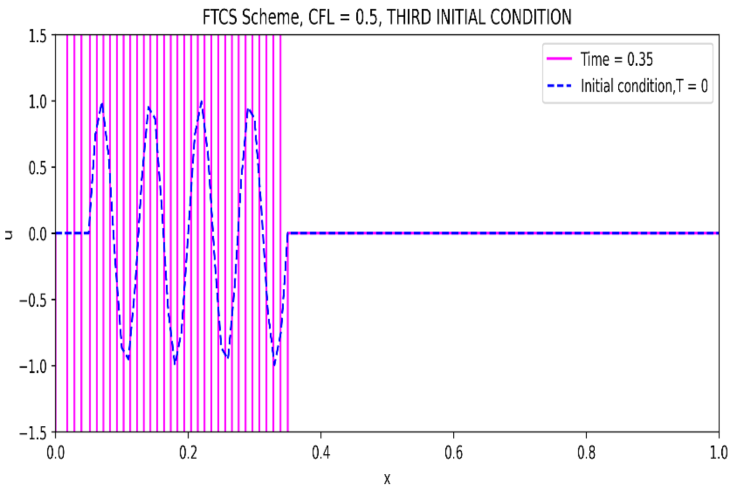

FTFS Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

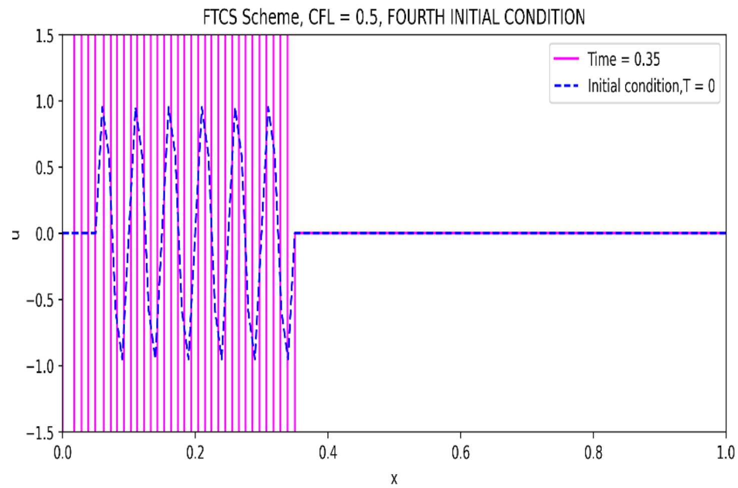
Forward Time Central Space (FTCS) scheme

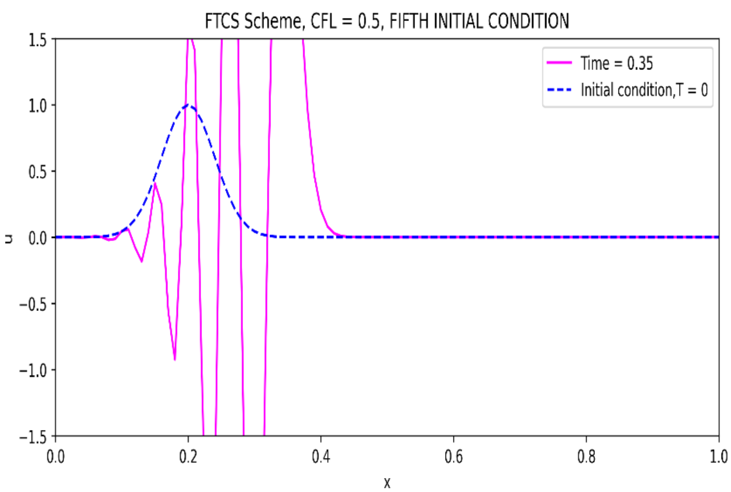
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Oscillations are observed in the plot, overall instability and in accuracy in solution. though CFL number falls in stability criteria.	Kind of same solution as obtained before for same condition except CFL number.	Kind of same solution as obtained before for same condition except CFL number.
2	Initial solution having two periods	oscillations are observed, not an accurate and stable solution.	Kind of same solution as obtained before for same condition except CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
3	Initial solution having four periods	oscillations are observed, not an accurate and stable solution	Oscillations are still there but now it also results in the dissipation compared to previous case having same conditions except the CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
4	Initial solution having six periods	oscillations are observed, not an accurate and stable solution	Oscillations are still there but now it also results in the dissipation compared to previous case having same conditions except the CFL number.	Oscillations are still there but got some reduction with considerable amount of diffusion, instabilities as well as inaccuracy in solution.
5	Initial solution having Gaussian curve	Observe less oscillations compared to FTFS scheme for same conditions, and it also observes the diffusion, overall instability and in accuracy in solution.	Kind of same solution as obtained before for same condition except CFL number.	Kind of same solution as obtained before for same condition except CFL number.

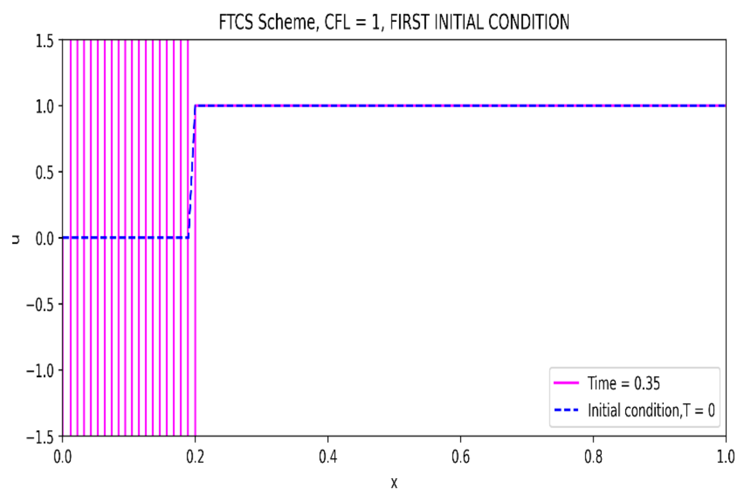

FTCS Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

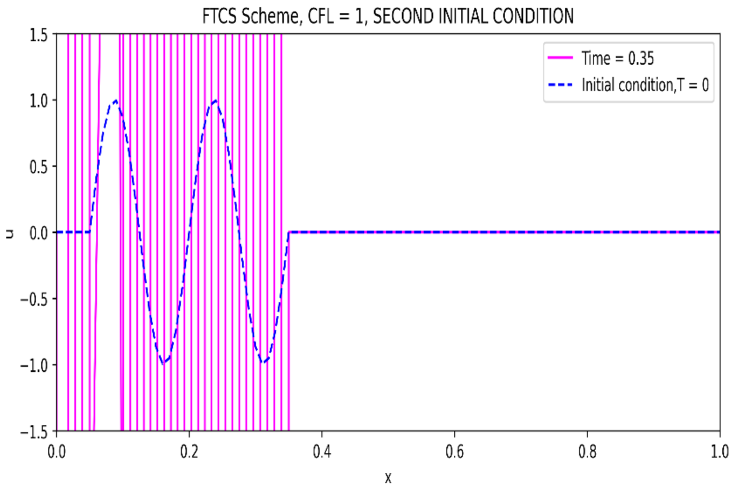

FTCS Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

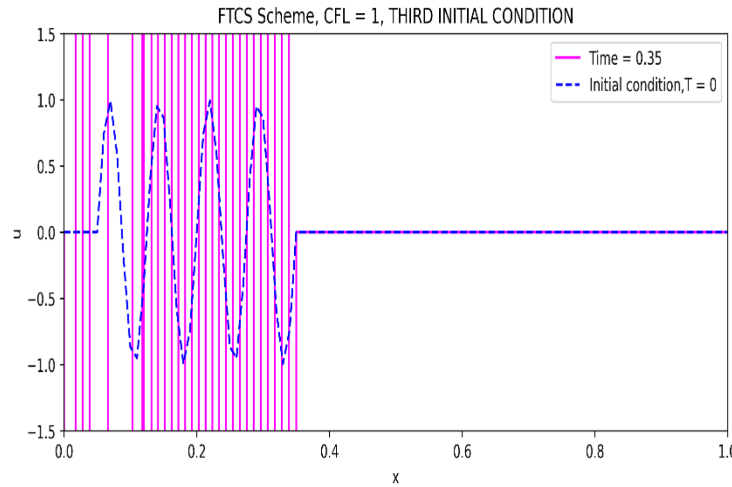

FTCS Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

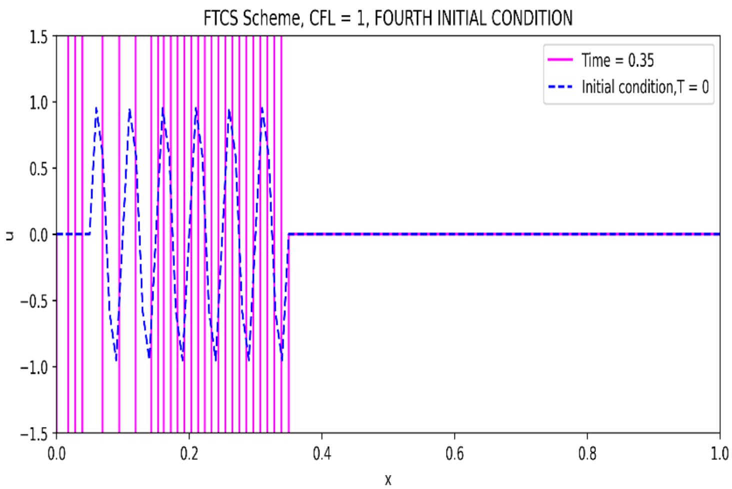

FTCS Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

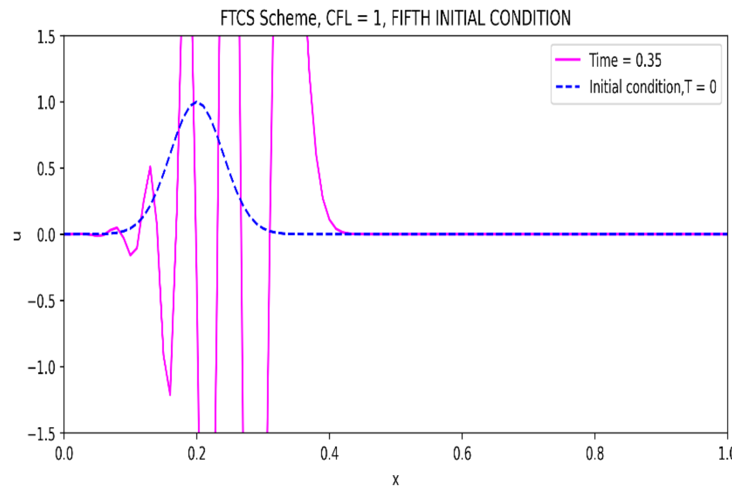

FTCS Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

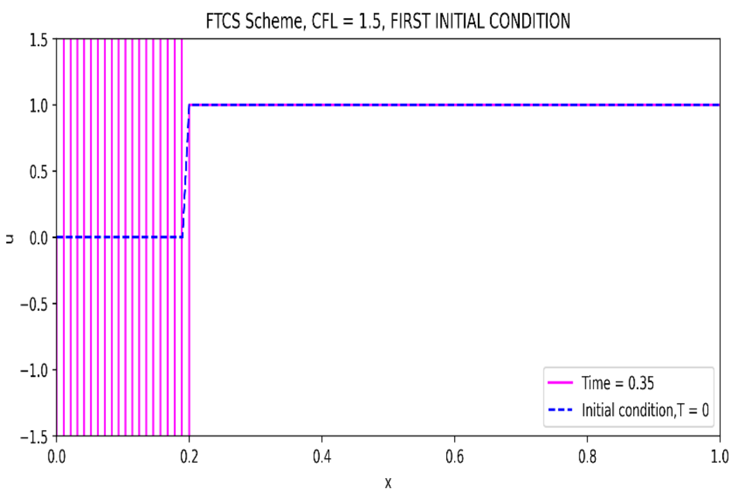

FTCS Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

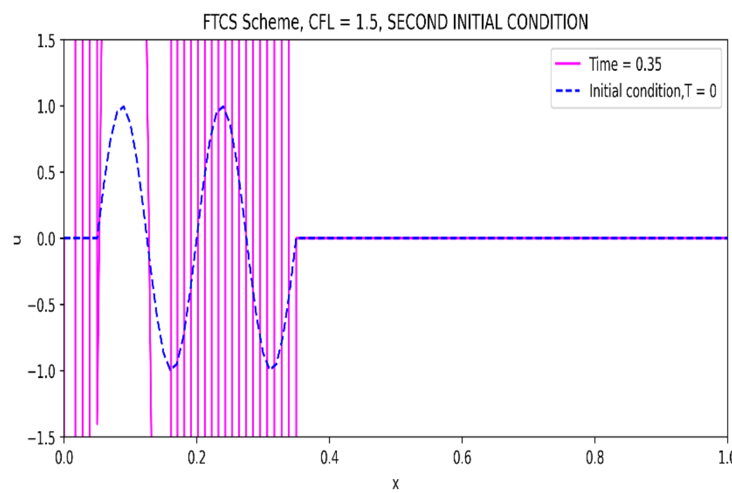

FTCS Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

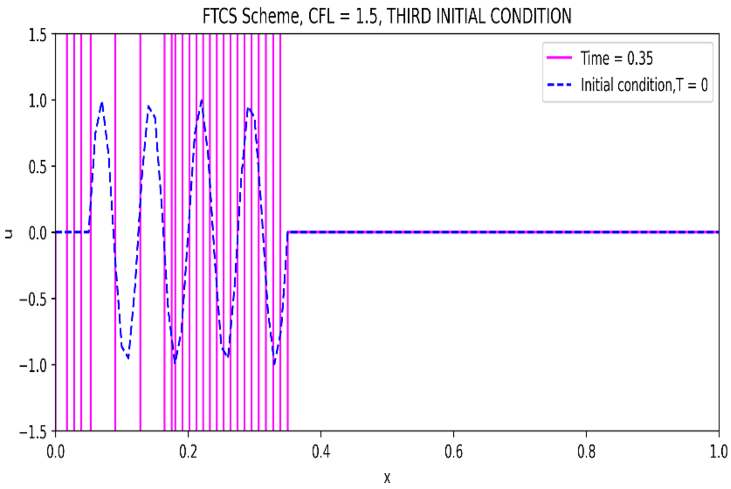

FTCS Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

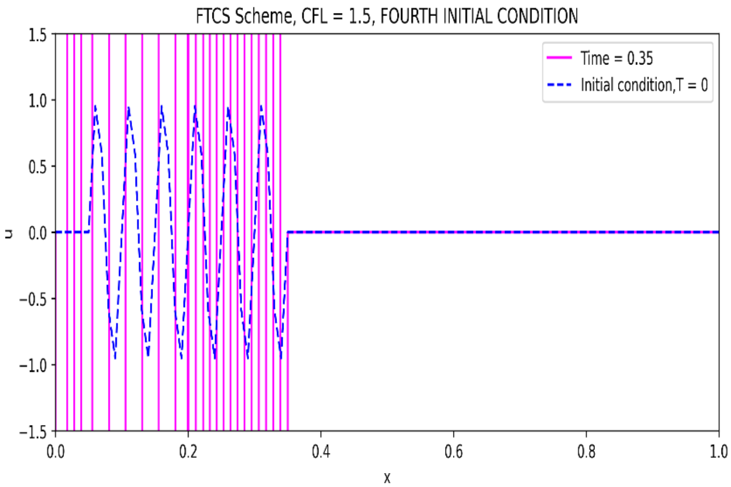

FTCS Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

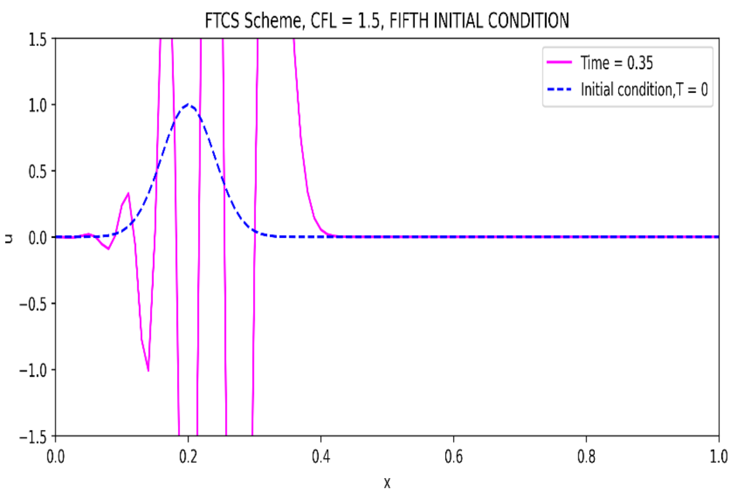

FTCS Scheme, CFL = 1, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u


FTCS Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u


FTCS Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

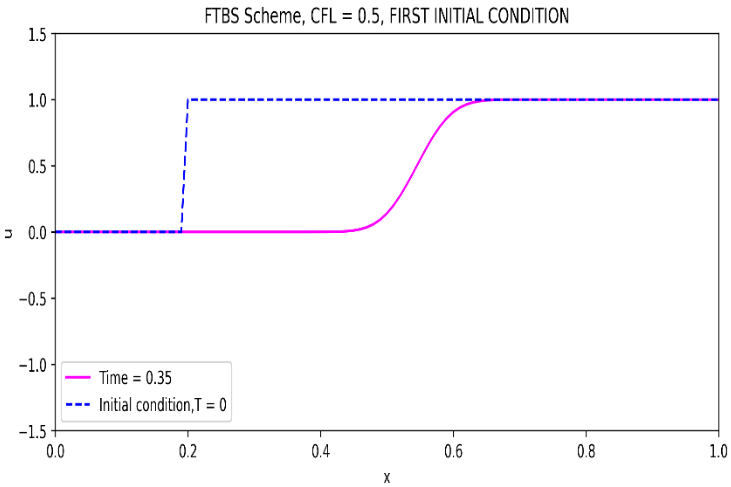

FTCS Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

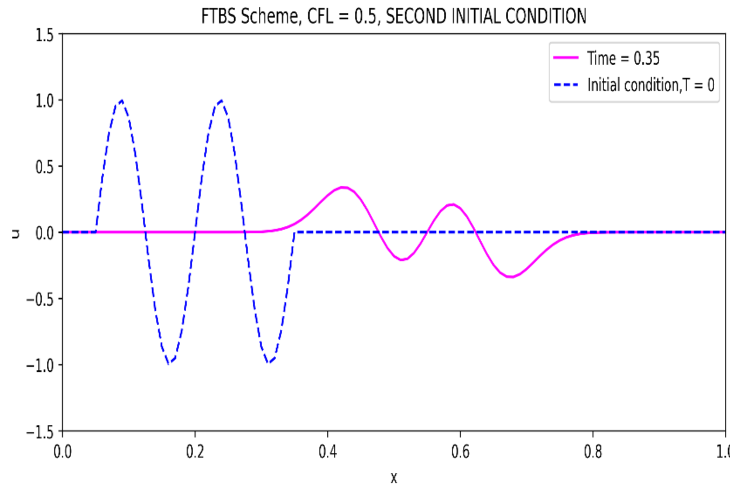

FTCS Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

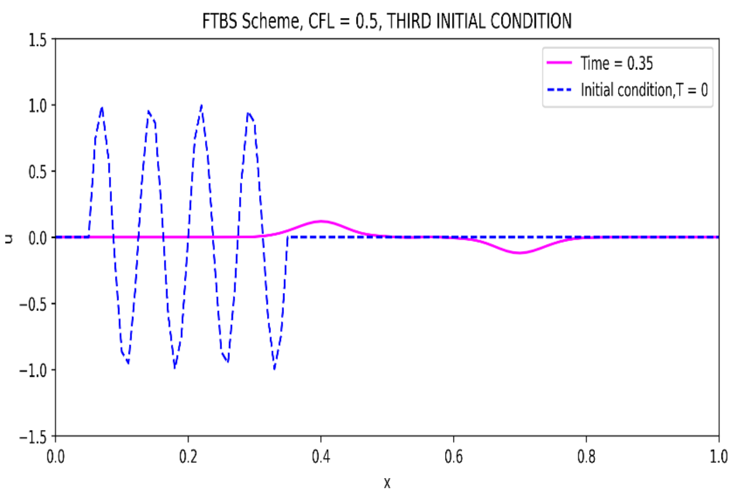

FTCS Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

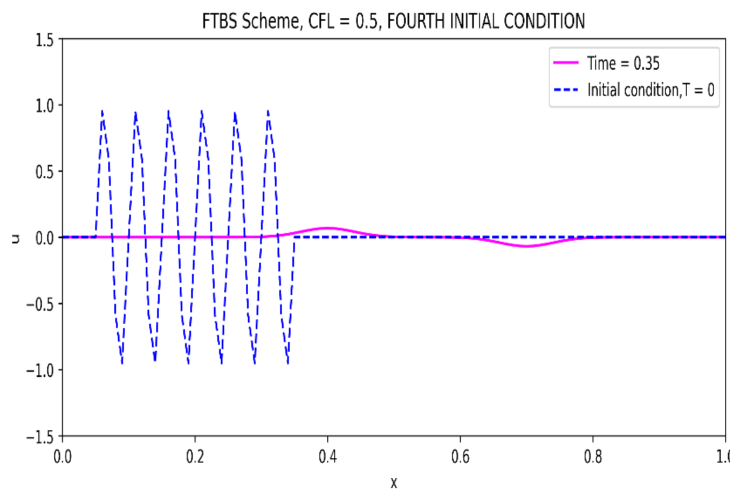
Forward Time Backward Space (FTBS) scheme

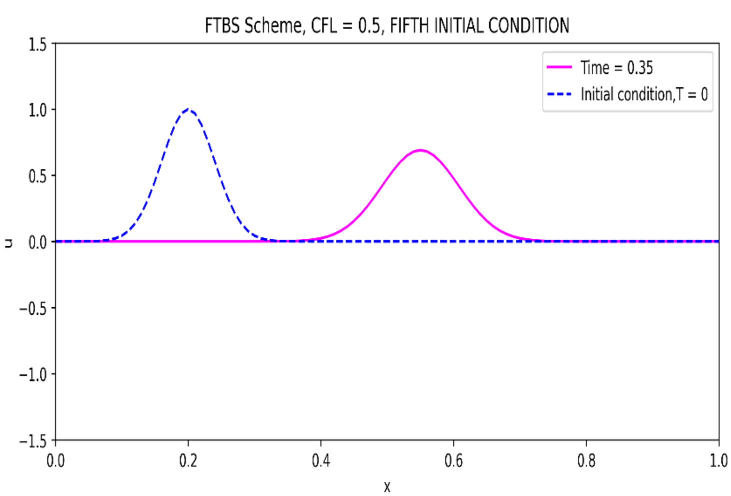
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Smooth solution, no diffusion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Increment in CFL number results in oscillations, unstable and inaccurate solution.
2	Initial solution having two periods	Smooth solution observed accompanied with dispersion, kind of accurate and stable solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
3	Initial solution having four periods	Observed more dispersion compared to same initial condition.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
4	Initial solution having six periods	Observed more dispersion compared to same initial condition.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
5	Initial solution having Gaussian curve	Smooth solution, accurate and stable.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.

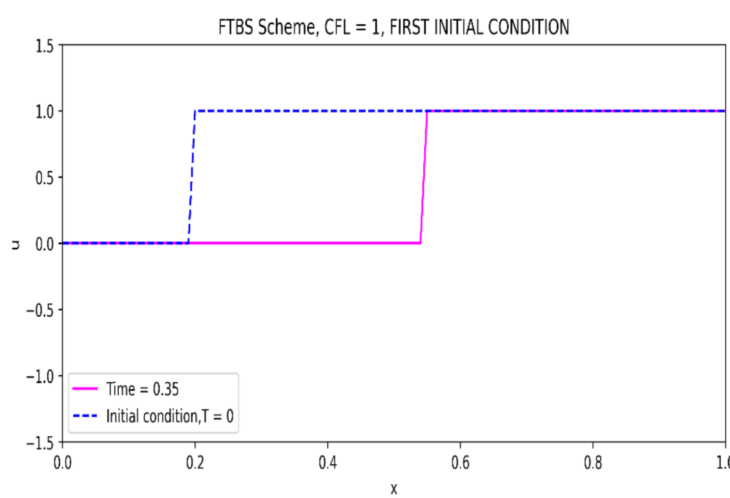

FTBS Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

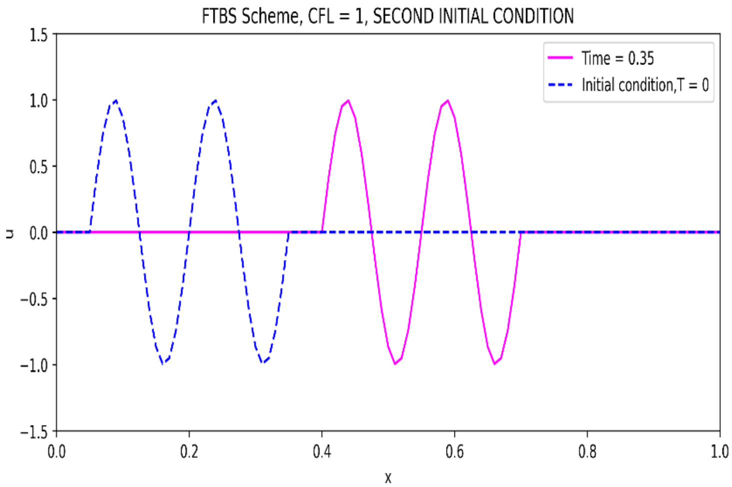

FTBS Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

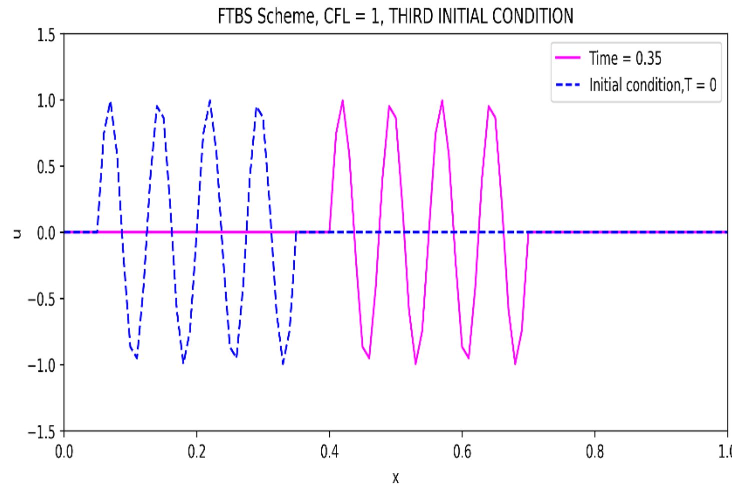

FTBS Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

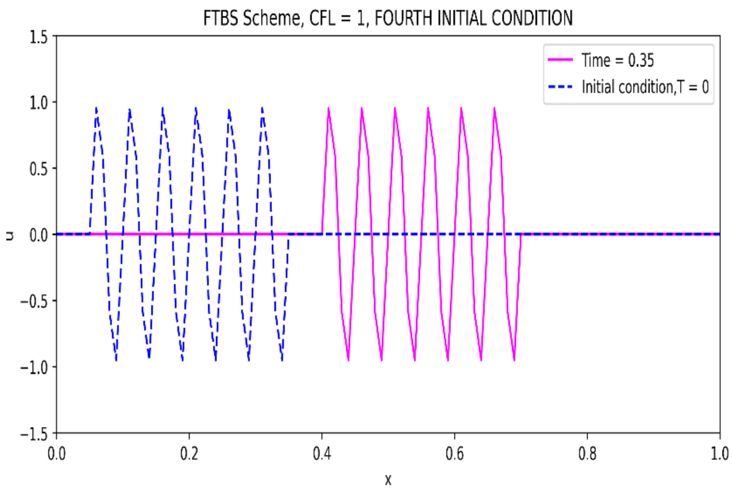

FTBS Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

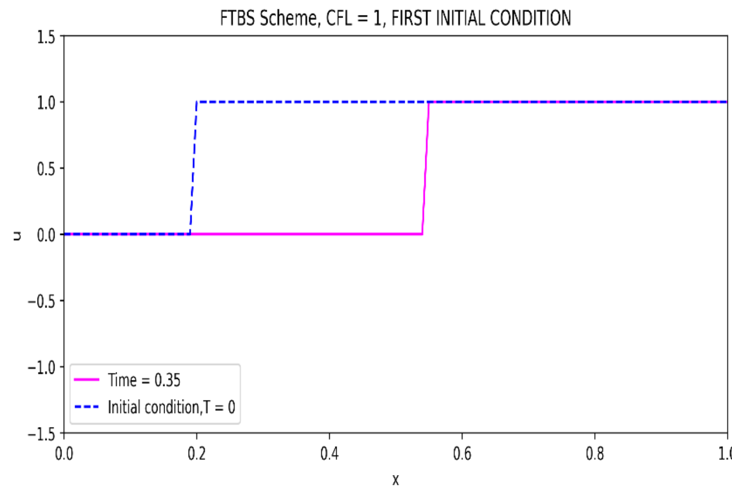

FTBS Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

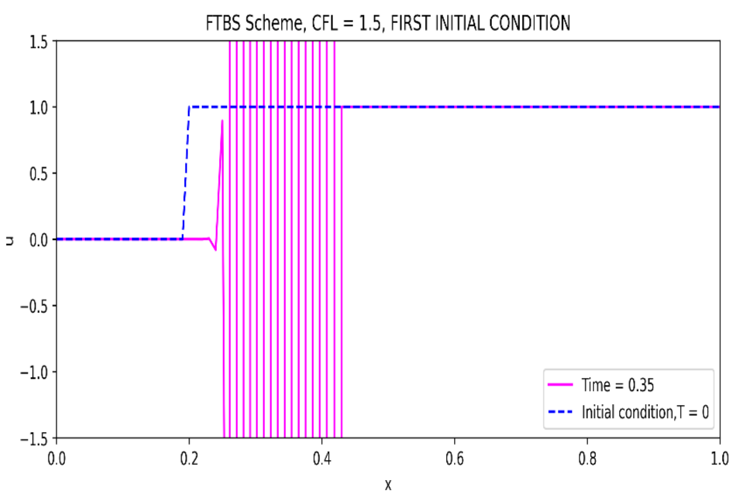

FTBS Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

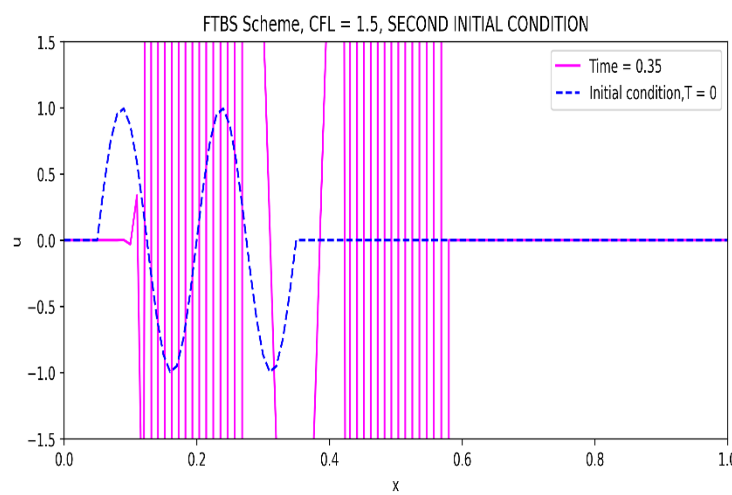

FTBS Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

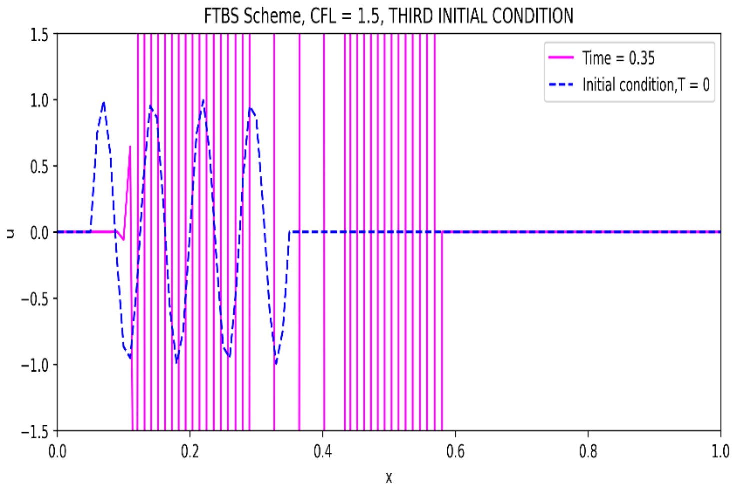

FTBS Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

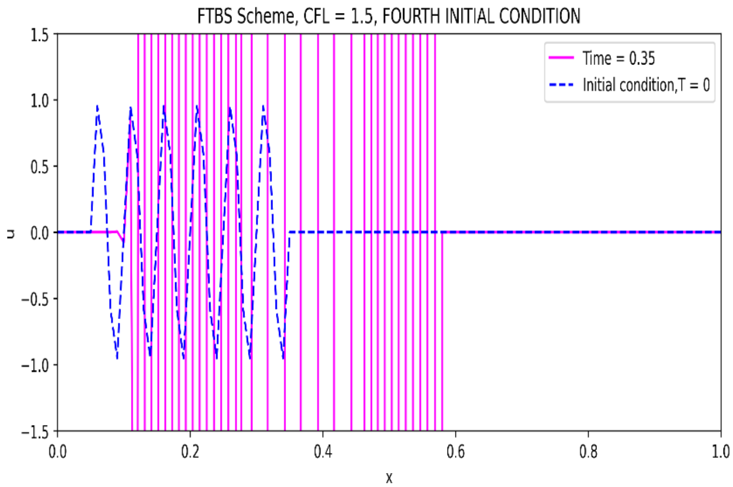

FTBS Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

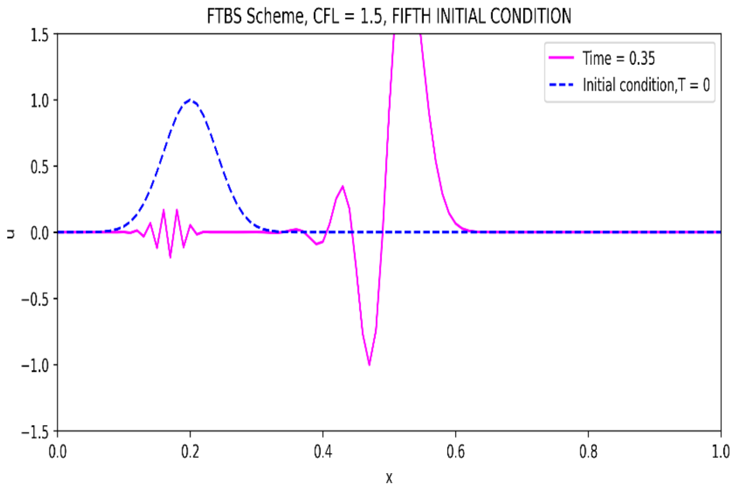

FTBS Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FTBS Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FTBS Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

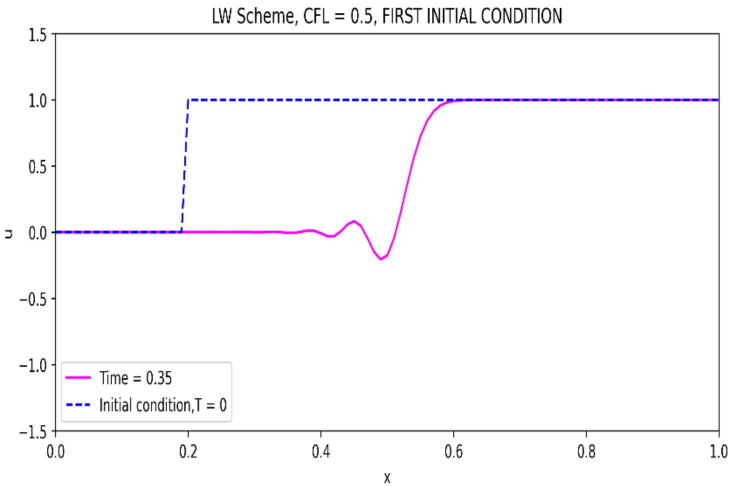

FTBS Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

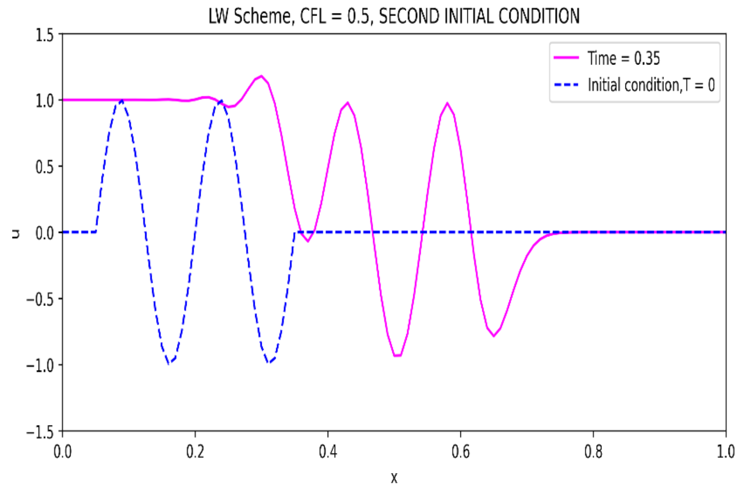

FTBS Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

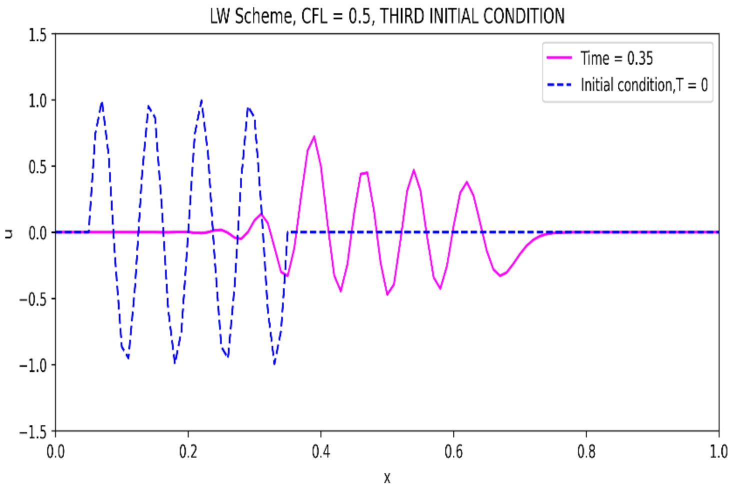

FTBS Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

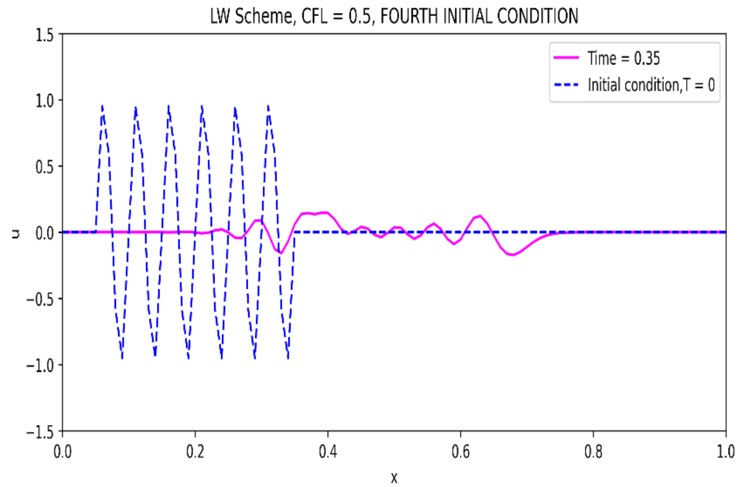
Lax-Wendroff (LW) scheme

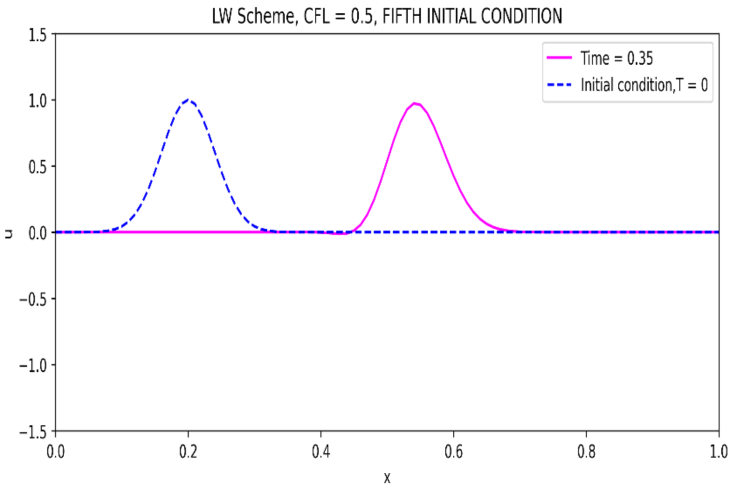
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Increment in CFL number results in oscillations, unstable and inaccurate solution.
2	Initial solution having two periods	Smooth solution observed with less dispersion compared to FTBS, kind of accurate and stable solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
3	Initial solution having four periods	Observed less dispersion compared to compared to FTBS, kind of stable solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
4	Initial solution having six periods	Observed less dispersion compared to FTBS.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.
5	Initial solution having Gaussian curve	Perfectly follows the numerical solution, highly stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Oscillations and diffusion in solution, unstable and inaccurate solution.

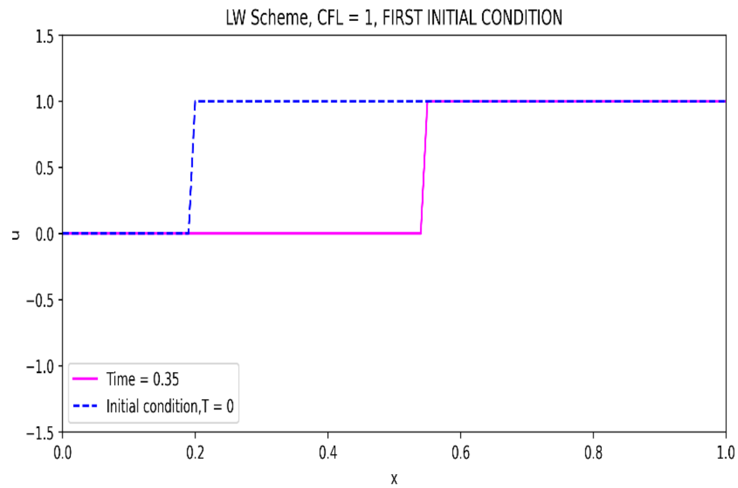

LW Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

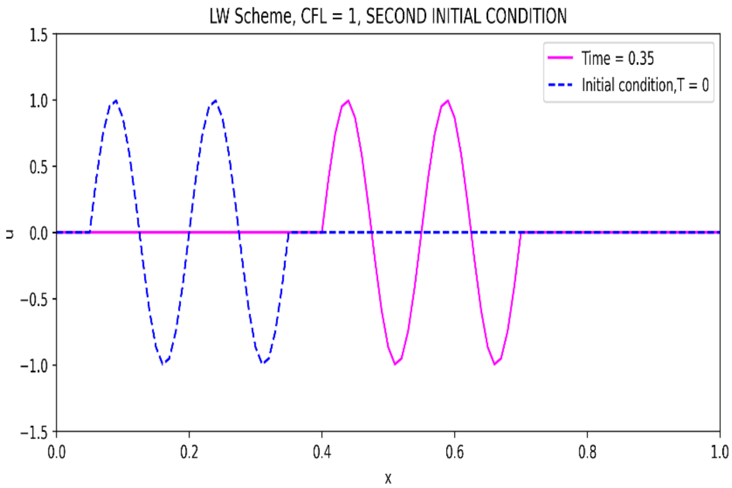

LW Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

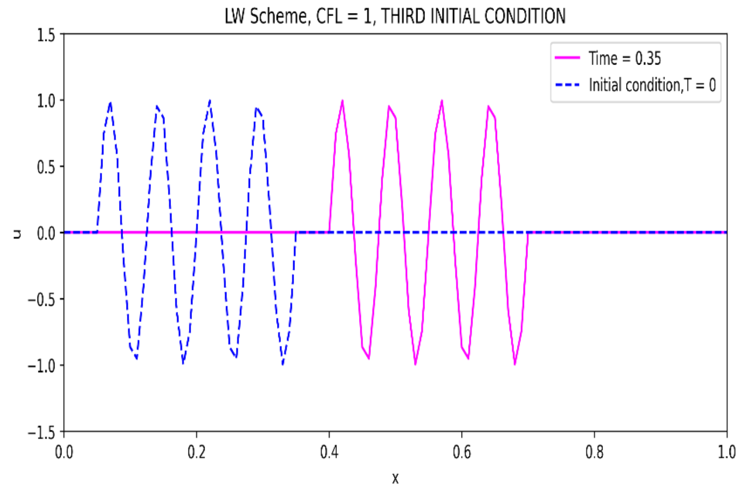

LW Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

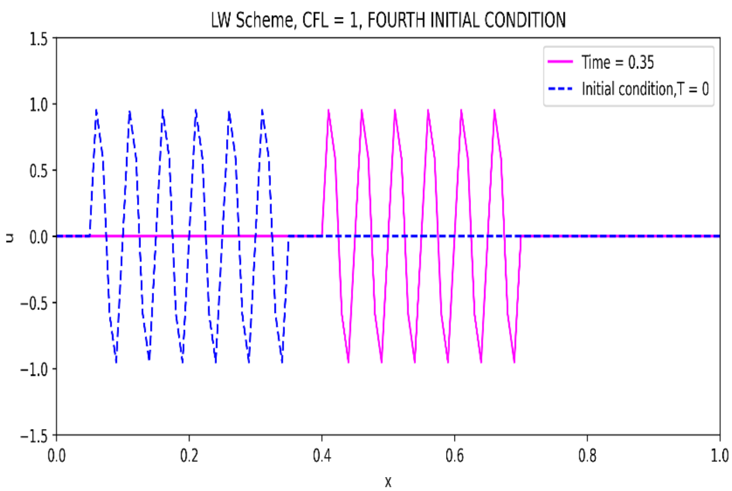

LW Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

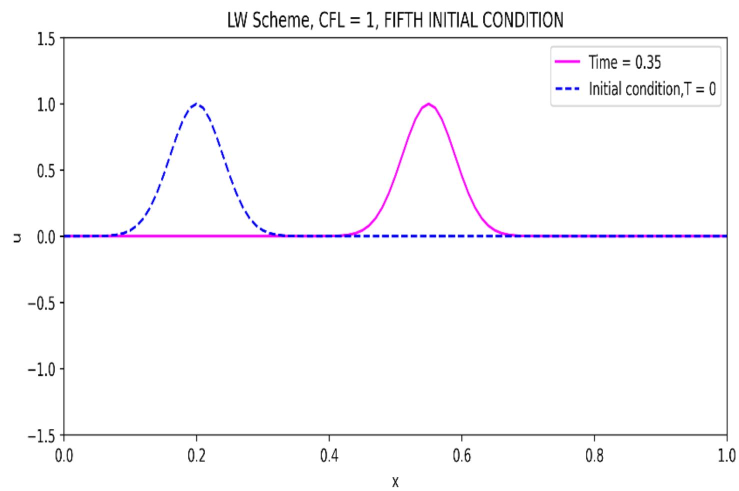

LW Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

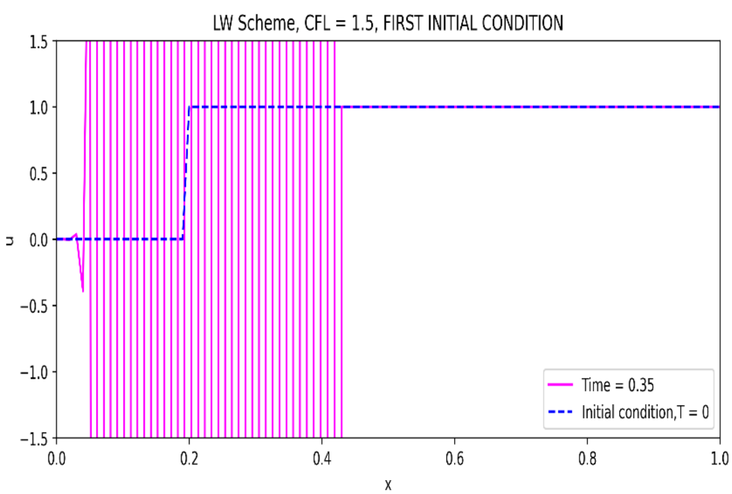

LW Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

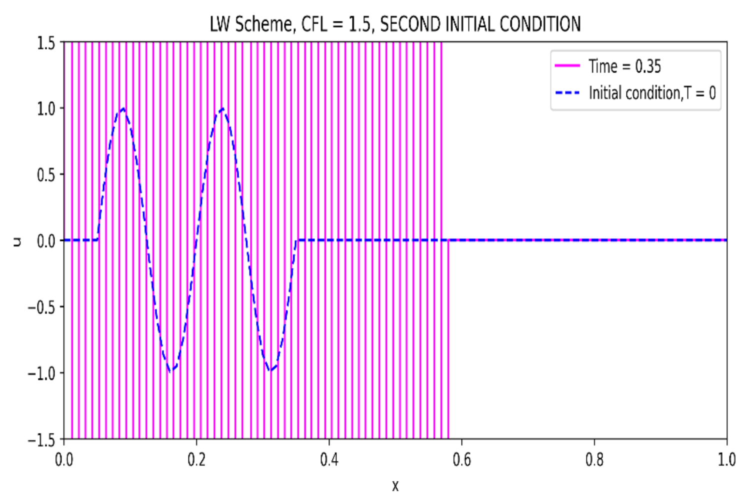

LW Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

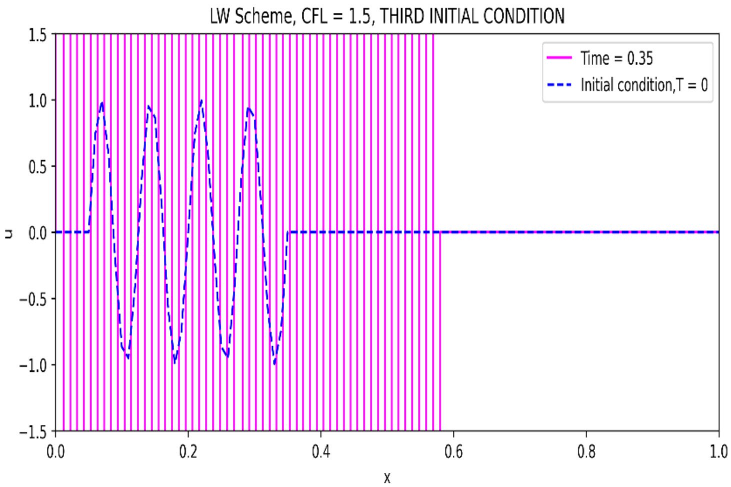

LW Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

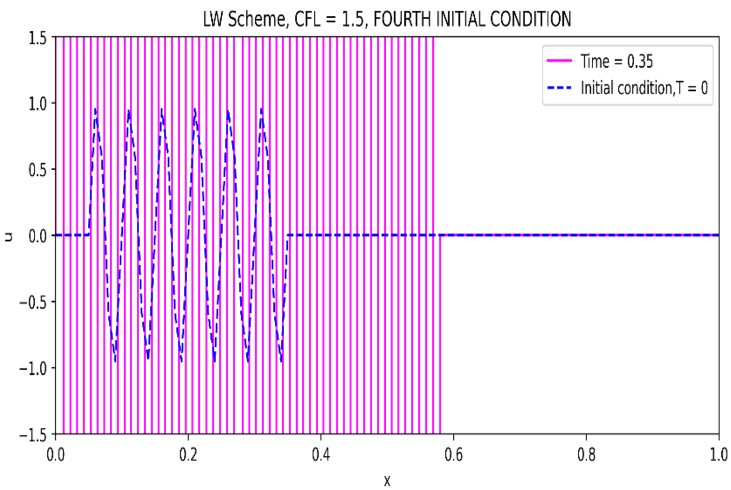

LW Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

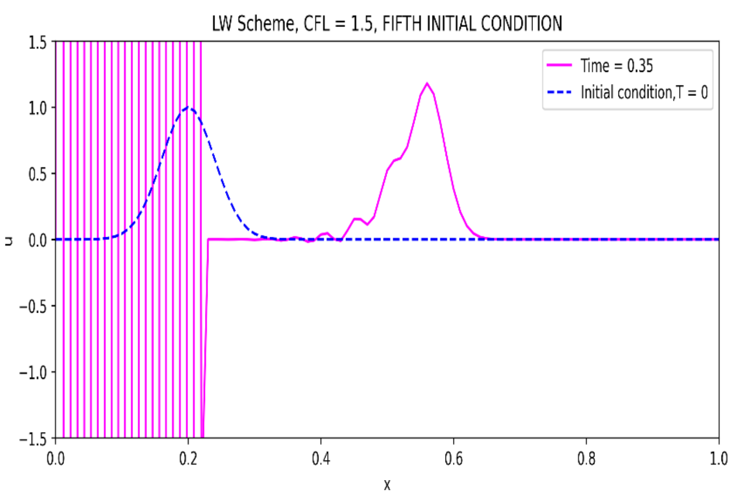

LW Scheme, CFL = 1, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u


LW Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u


LW Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

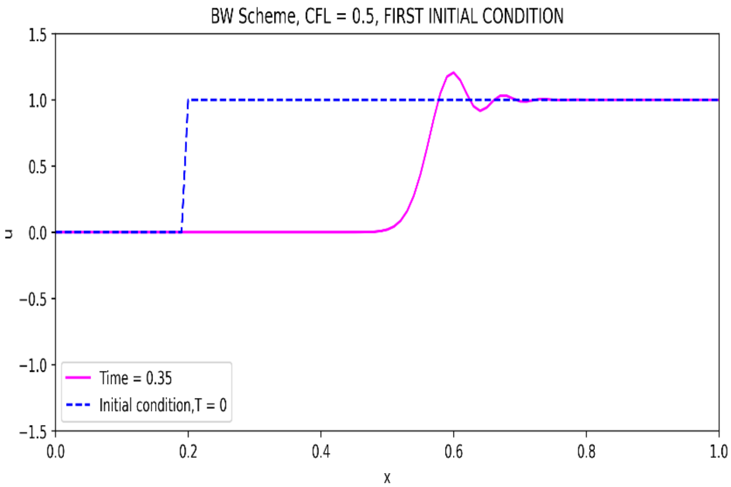

LW Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

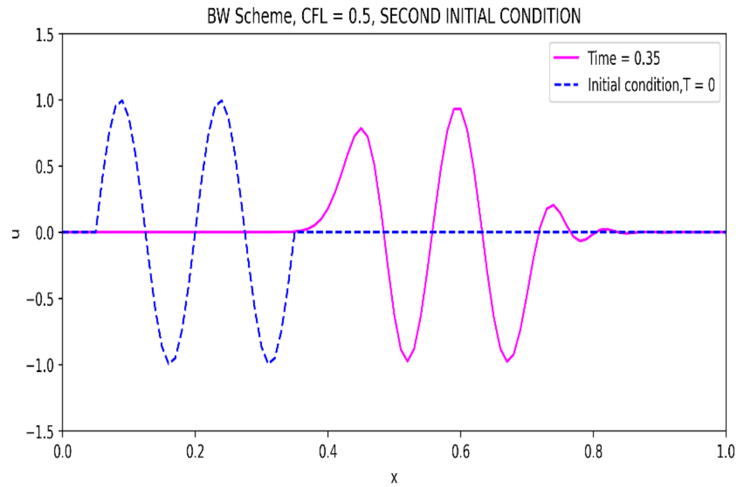

LW Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

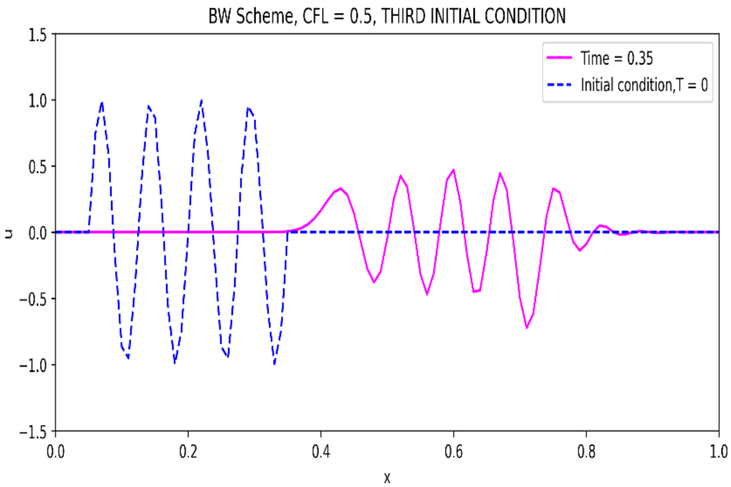

LW Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

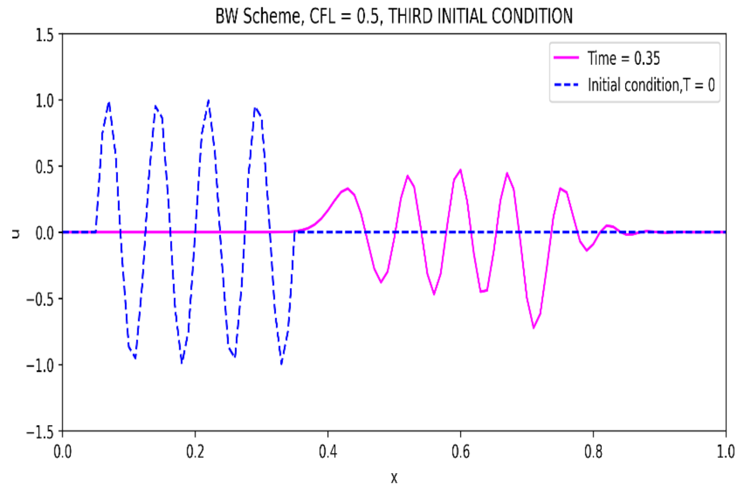
Beam-Warming (BW) scheme

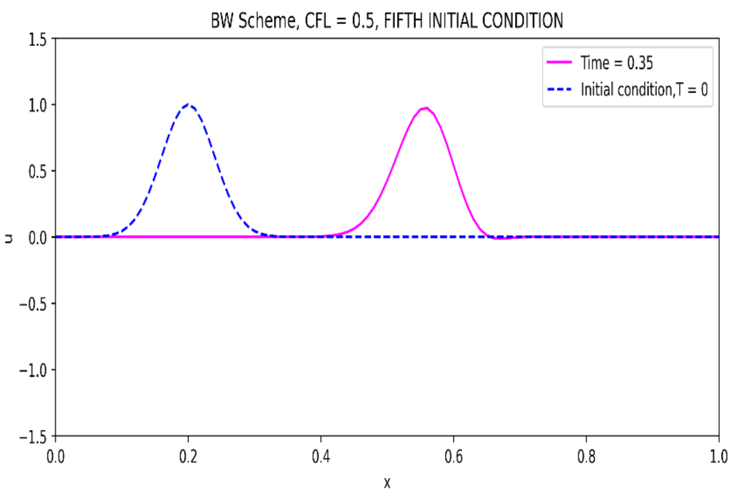
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Smooth solution, but observe dispersion quite stable and accurate solution.
2	Initial solution having two periods	Smooth solution observed with less dispersion compared to LW, kind of accurate and stable solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Smooth solution, but observe dispersion quite stable and accurate solution.
3	Initial solution having four periods	Observed less dispersion compared to compared to LW, kind of stable solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Smooth solution, but observe dispersion quite stable and accurate solution. .
4	Initial solution having six periods	Observed less dispersion compared to LW.	Perfectly follows the numerical solution, highly stable and accurate solution.	Smooth solution, but observe dispersion quite stable and accurate solution.
5	Initial solution having Gaussian curve	Perfectly follows the numerical solution, highly stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Smooth solution, but observe dispersion quite stable and accurate solution.

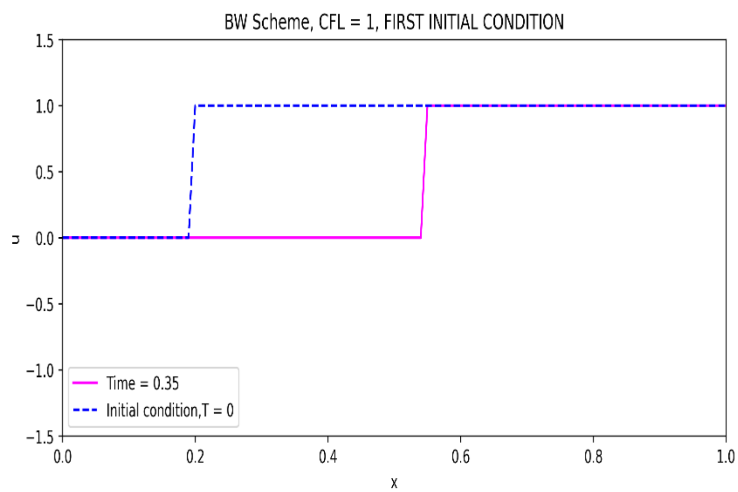

BW Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

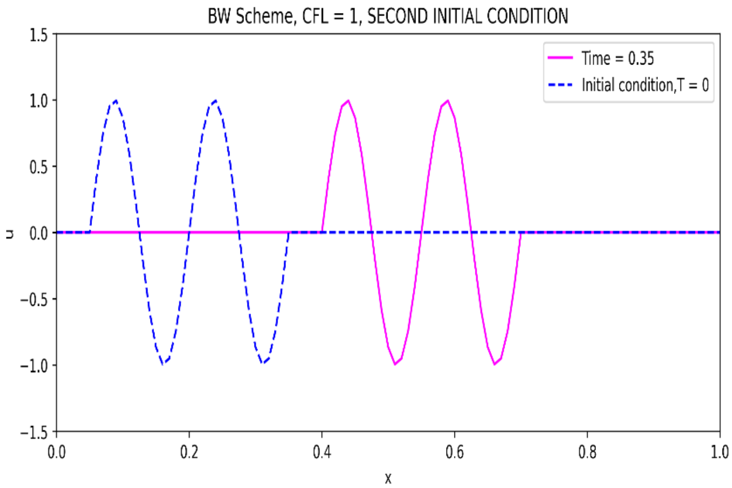

BW Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

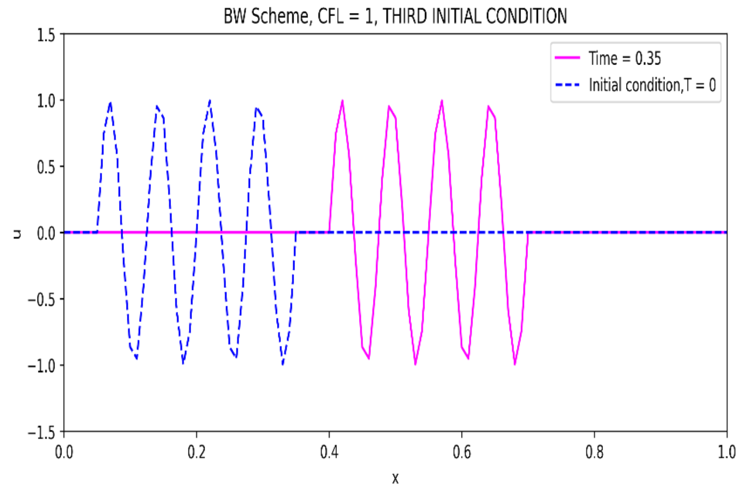

BW Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

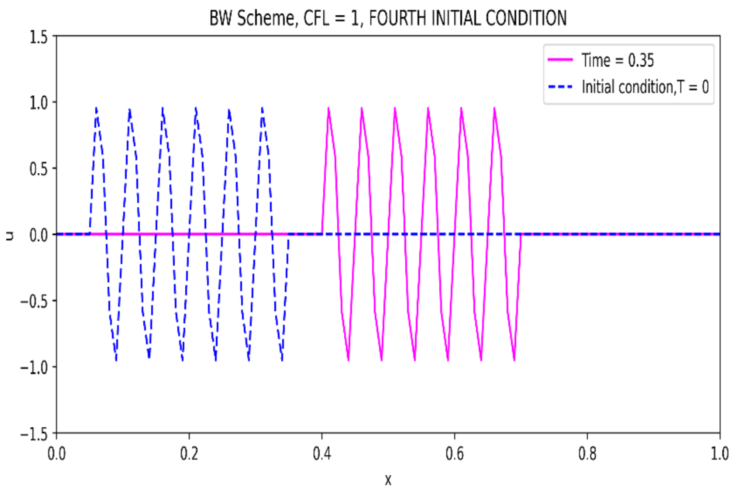

BW Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

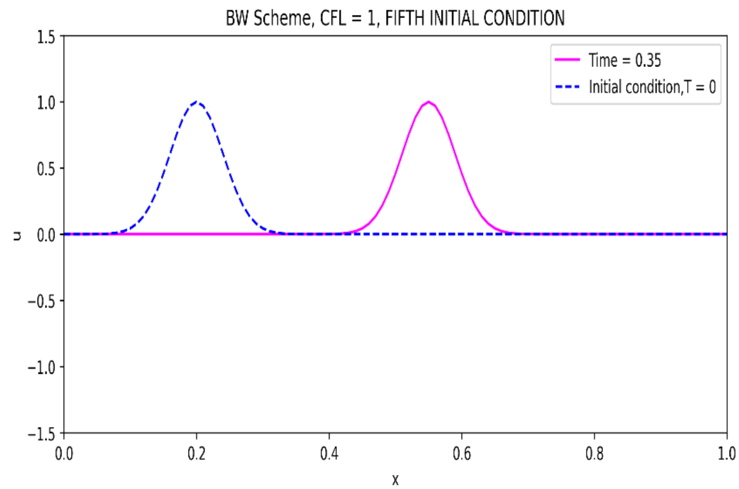

BW Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

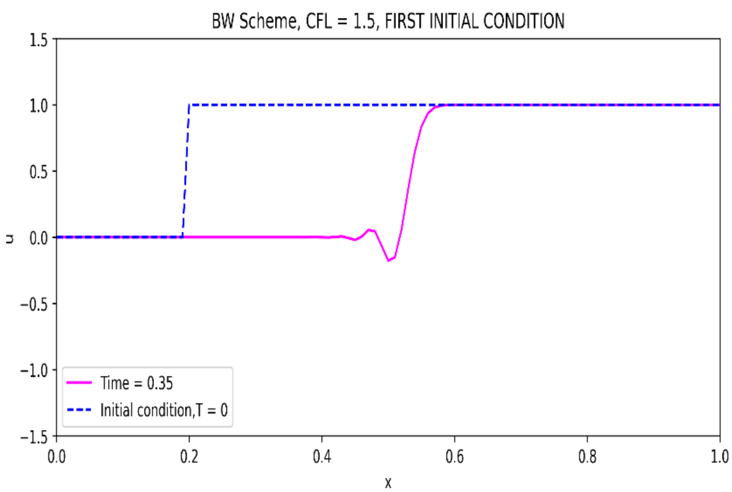

BW Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

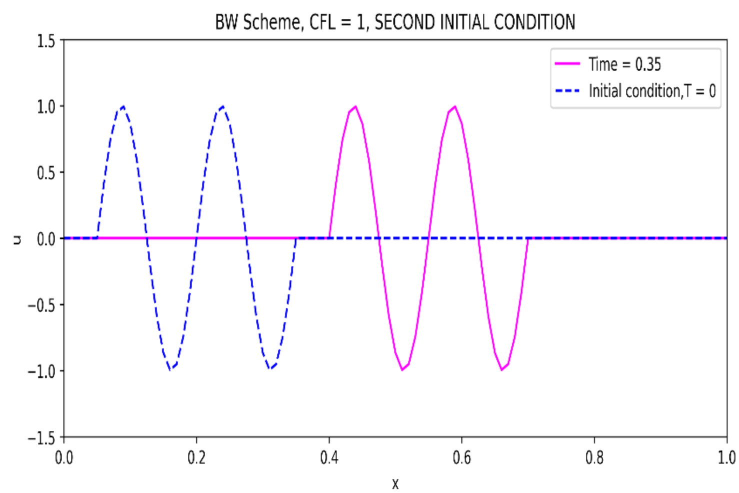

BW Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

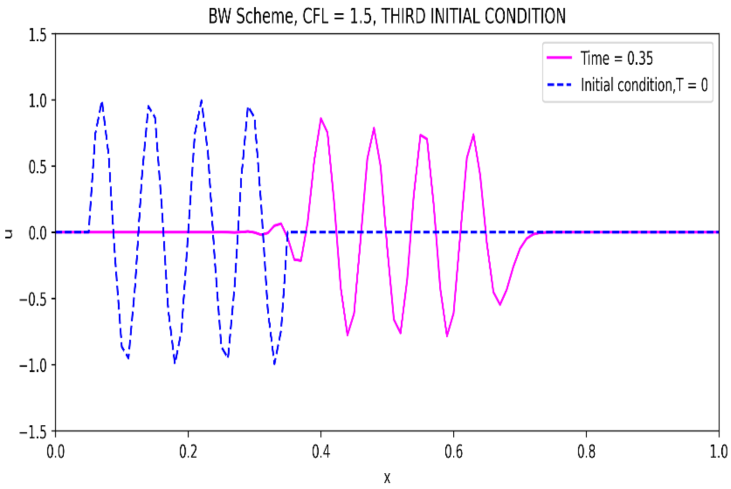

BW Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BW Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

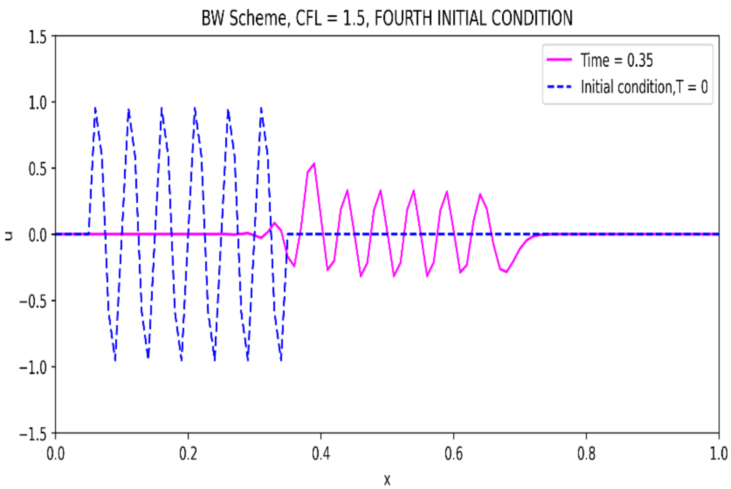

BW Scheme, CFL = 1, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BW Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

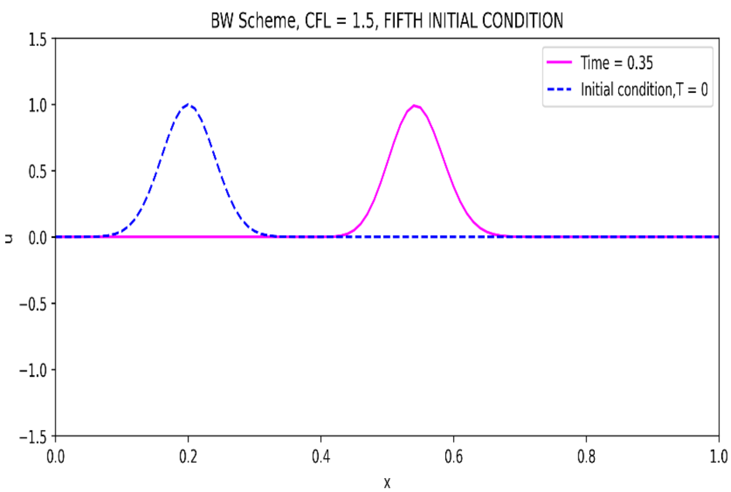

BW Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x



BW Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x



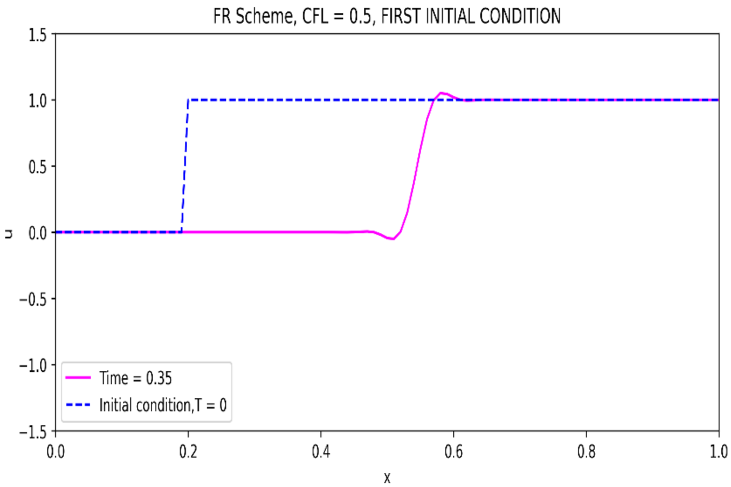
BW Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

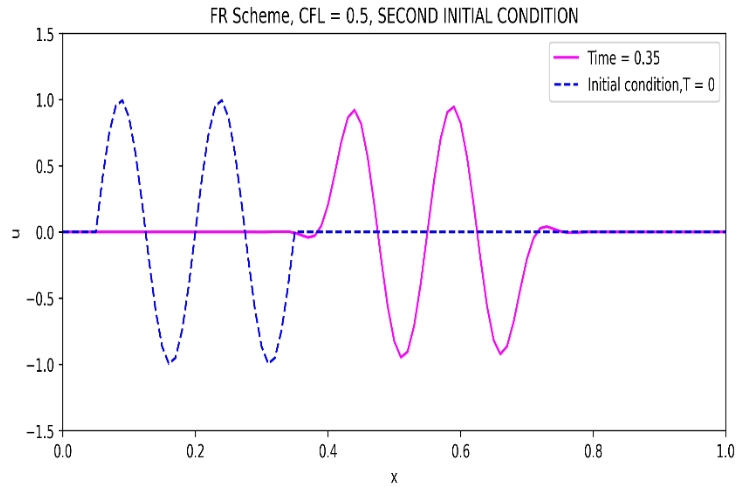


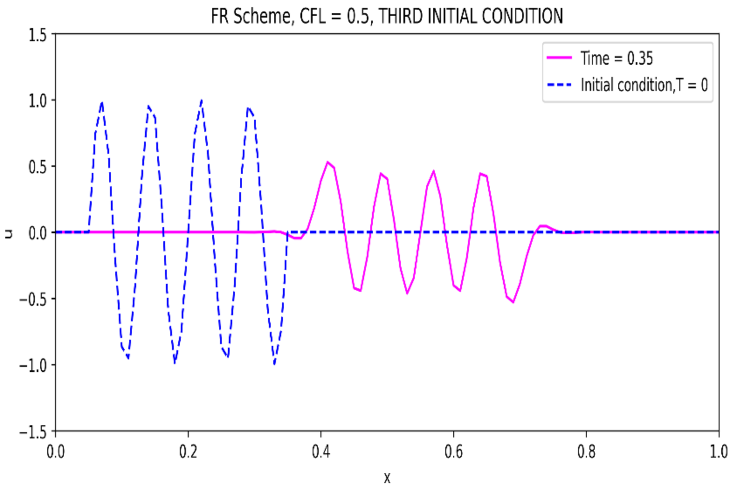
BW Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

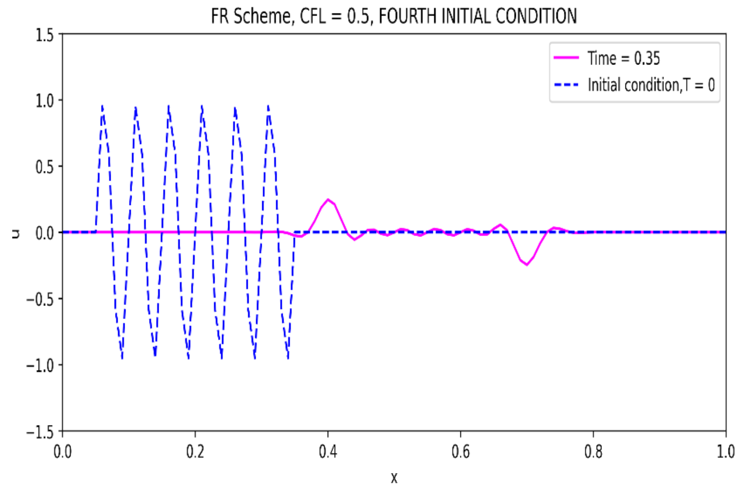
Fromm (FR) scheme

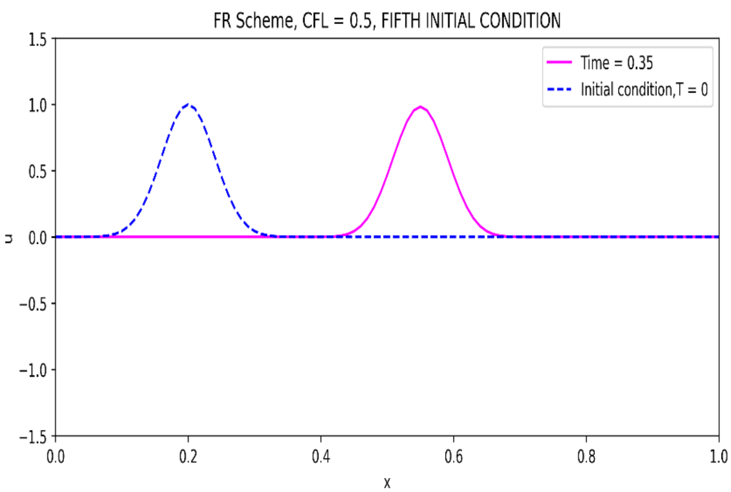
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Very much oscillatory solution, unstable and inaccurate.
2	Initial solution having two periods	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Very much oscillatory solution, unstable and inaccurate.
3	Initial solution having four periods	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Very much oscillatory solution, unstable and inaccurate.
4	Initial solution having six periods	Smooth solution, but observe dispersion quite stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Very much oscillatory solution, unstable and inaccurate.
5	Initial solution having Gaussian curve	Perfectly follows the numerical solution, highly stable and accurate solution.	Perfectly follows the numerical solution, highly stable and accurate solution.	Very much oscillatory solution in initial phase, but also follows original solution later on but still unstable and in accurate solution.

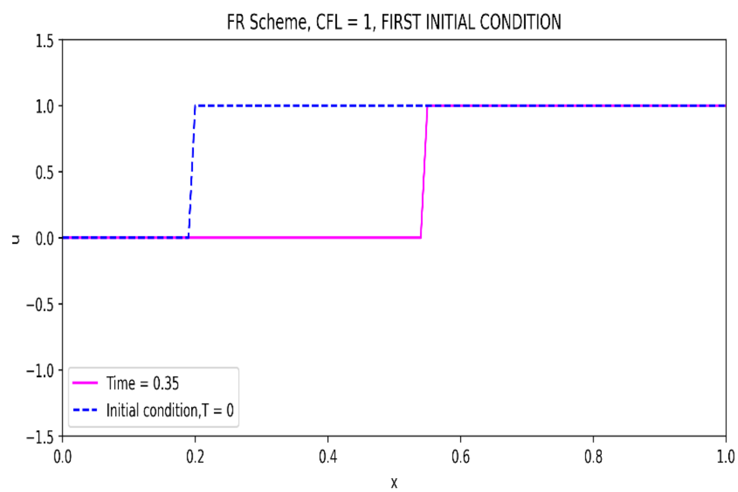

FR Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

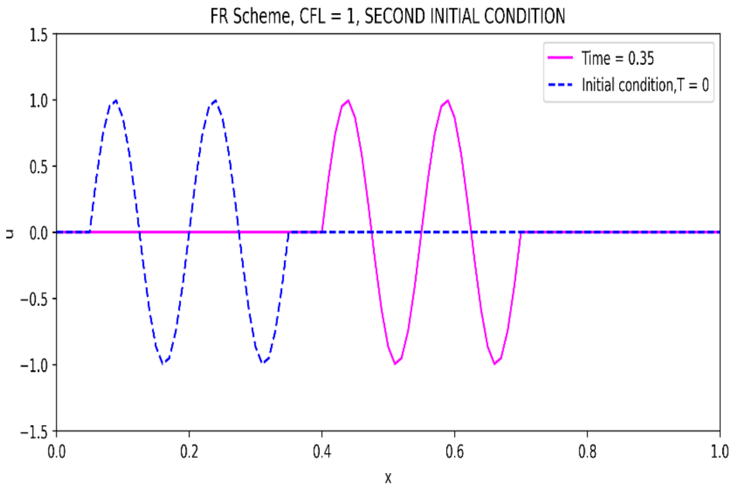

FR Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

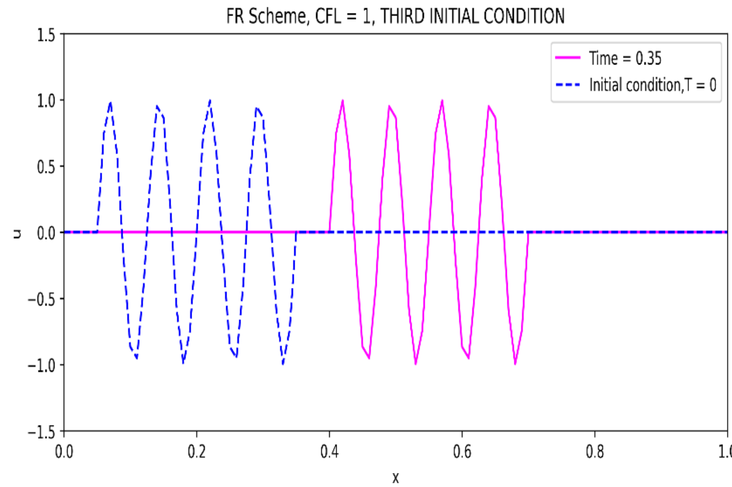

FR Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

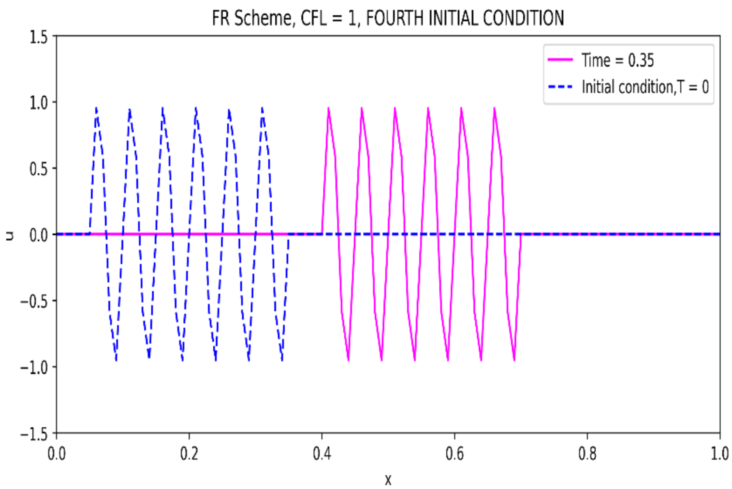

FR Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

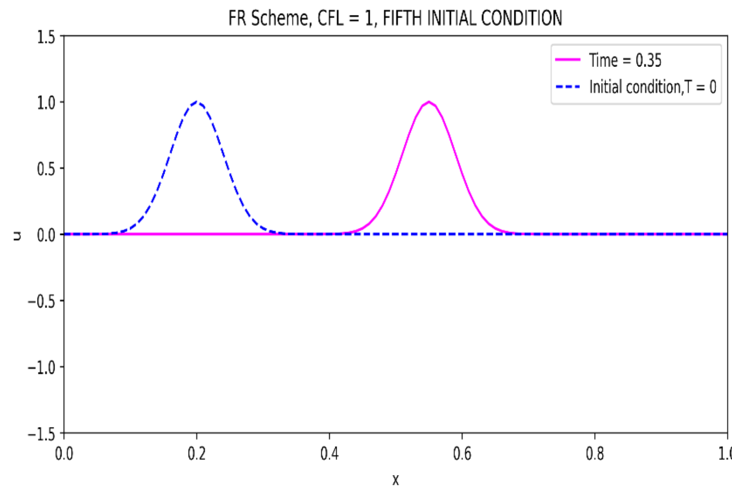

FR Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

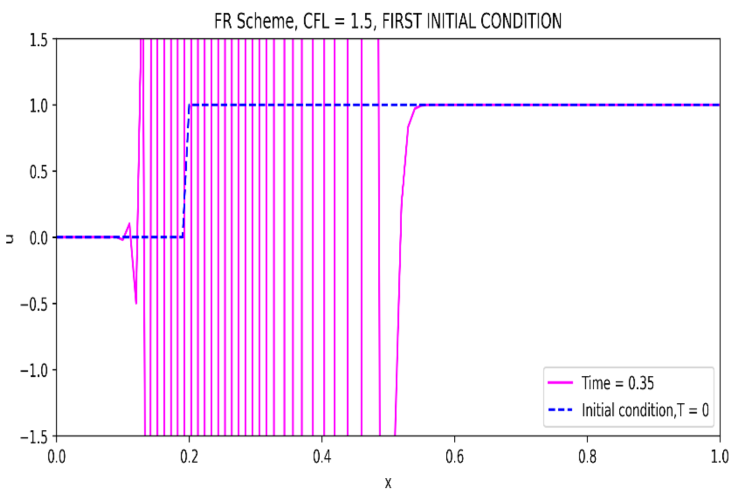

FR Scheme, CFL = 1, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

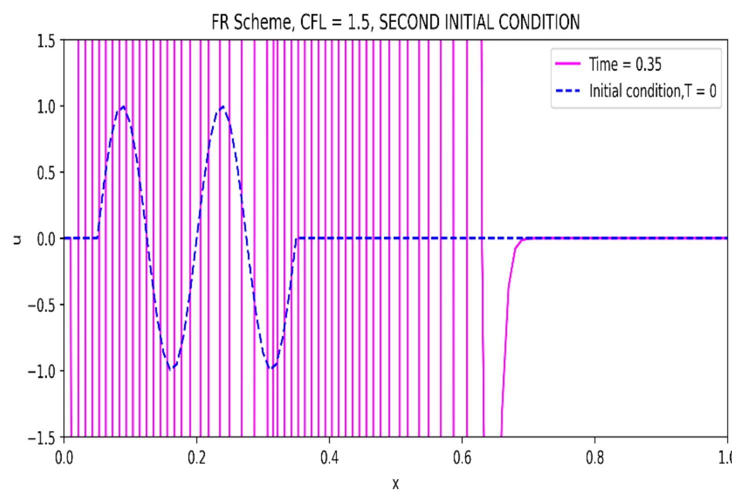

FR Scheme, CFL = 1, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

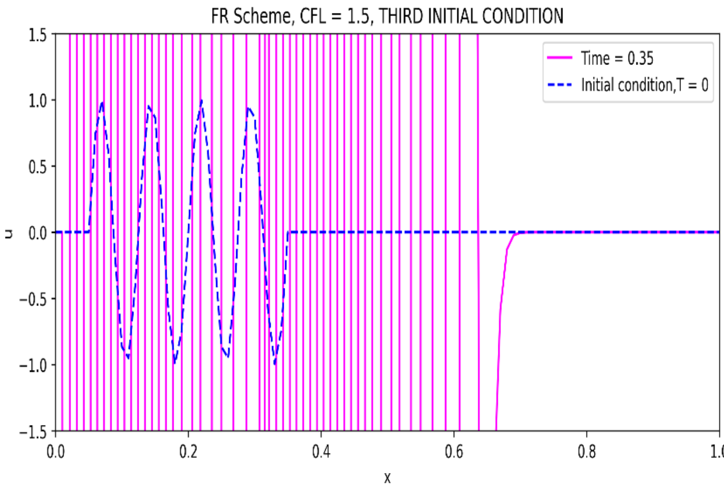

FR Scheme, CFL = 1, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

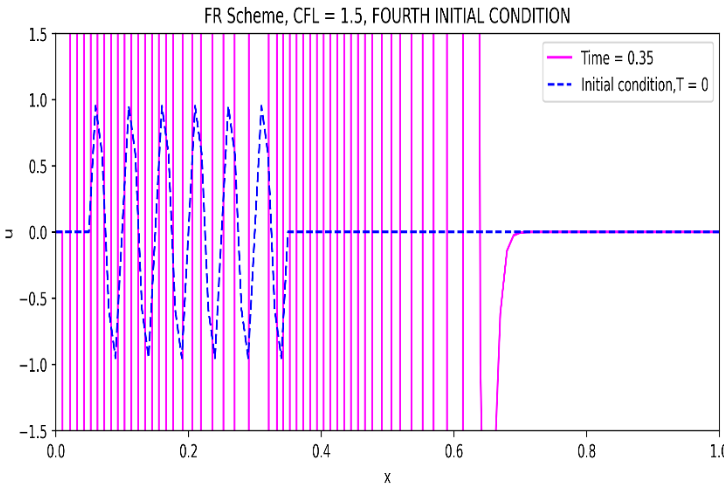

FR Scheme, CFL = 1, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

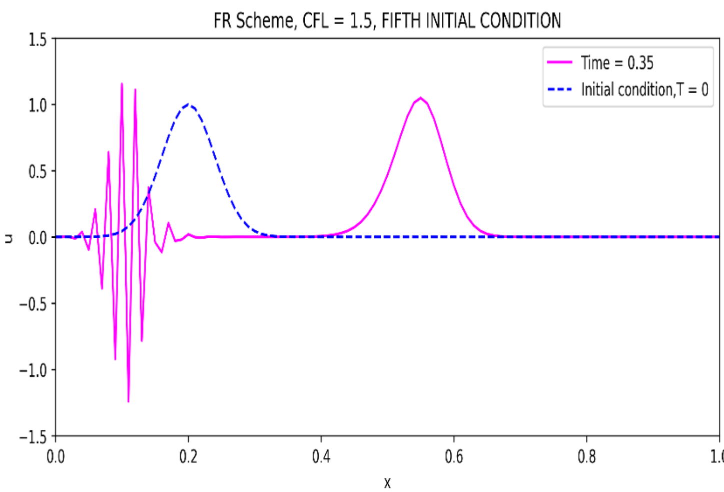

FR Scheme, CFL = 1, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FR Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


FR Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

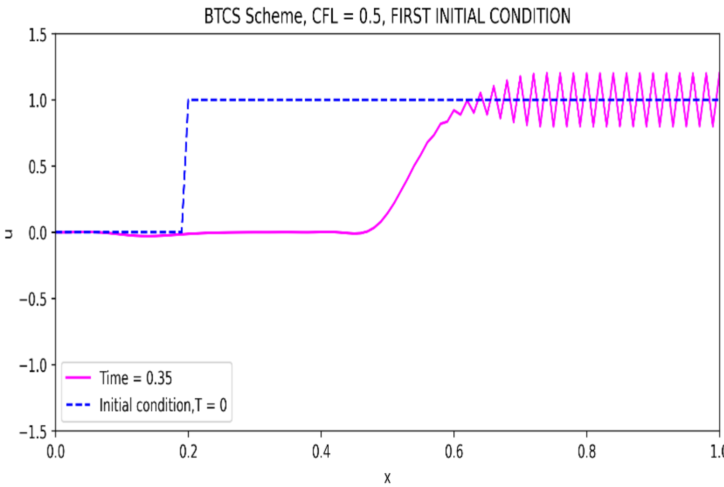

FR Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

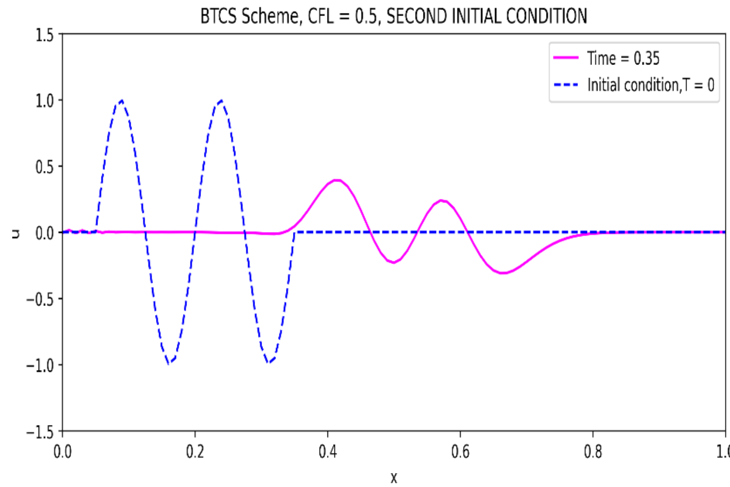

FR Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

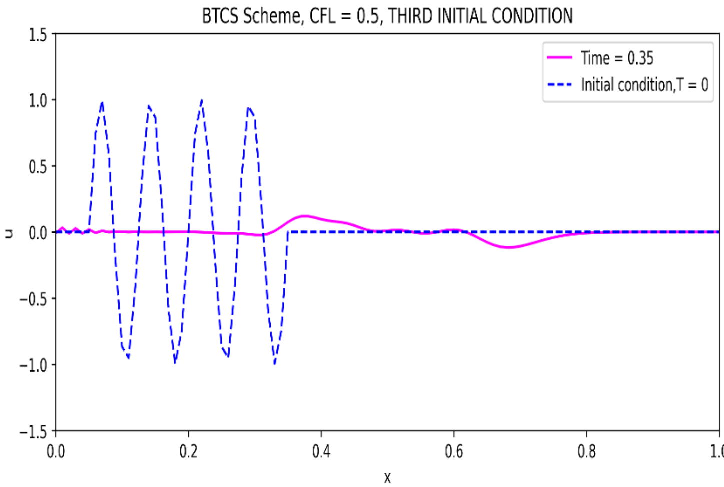

FR Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

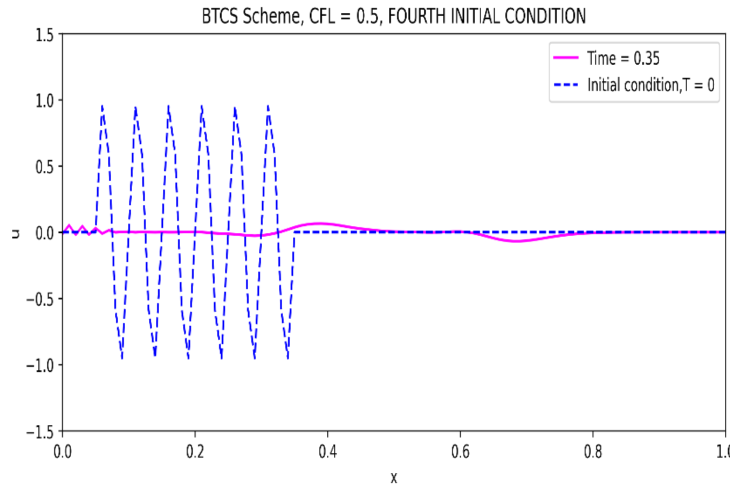
Implicit scheme, Backward Time Central Space (BTCS) scheme

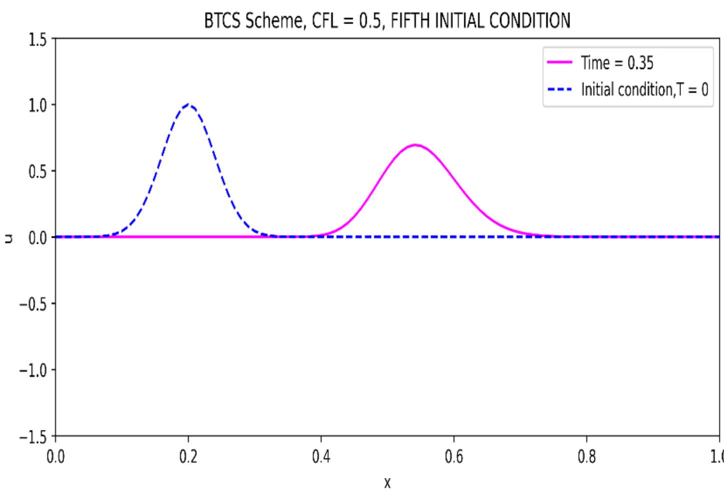
SR No.		CFL = 0.5	CFL = 1	CFL = 1.5
1	Discontinuous initial solution	Initially the solution was smooth and stable but later on lots of oscillations and inaccuracy observed.	Initially the solution was smooth and stable but later on more oscillations and inaccurate compared to 0.5 CFL number.	Initially the solution was smooth and stable but later on the solution becomes very much inaccurate and unstable.
2	Initial solution having two periods	Smooth and less diffused solution but kind of dispersed solution.	Smooth and less diffused solution but kind of dispersed solution.	Smooth and less diffused solution but kind of dispersed solution.
3	Initial solution having four periods	Very much diffused solution very much inaccurate and not stable.	Very much diffused solution very much inaccurate and not stable.	Very much diffused solution very much inaccurate and not stable.
4	Initial solution having six periods	Very much diffused solution very much inaccurate and not stable.	Very much diffused solution very much inaccurate and not stable.	Very much diffused solution very much inaccurate and not stable.
5	Initial solution having Gaussian curve	Kind of smooth and sort of accurate and stable solution.	Kind of smooth and sort of accurate and stable solution.	Kind of smooth and sort of accurate and stable solution but got diffused.

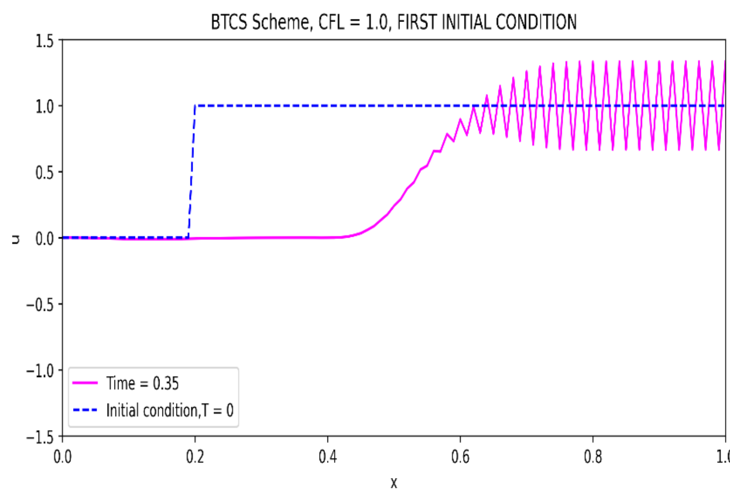

BTCS Scheme, CFL = 0.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

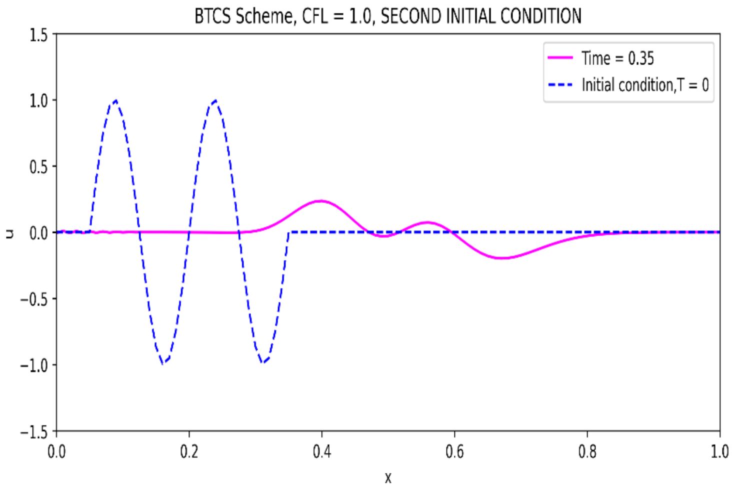

BTCS Scheme, CFL = 0.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

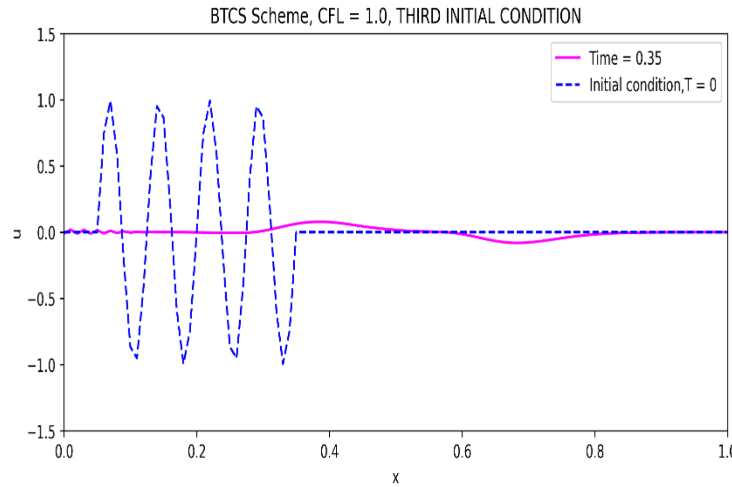

BTCS Scheme, CFL = 0.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

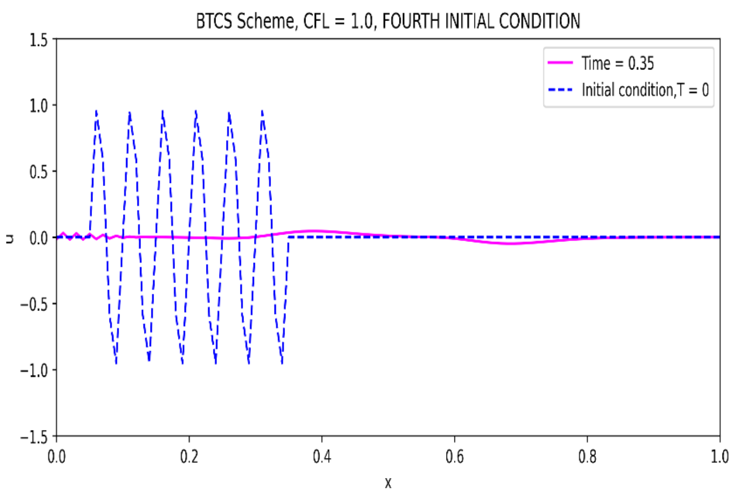

BTCS Scheme, CFL = 0.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

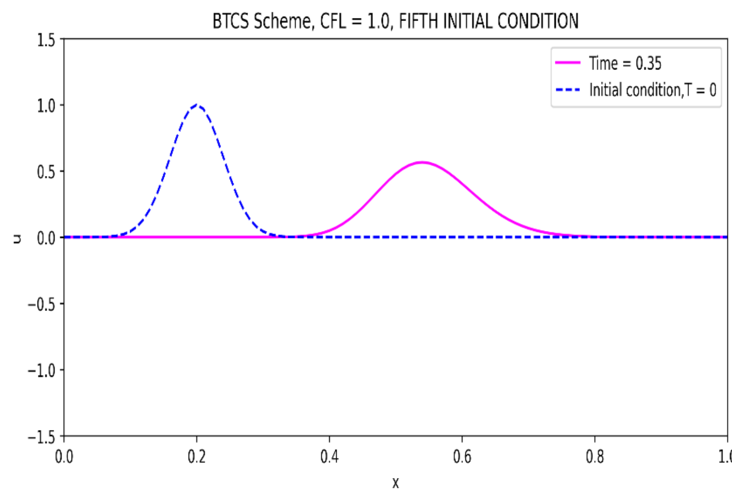

BTCS Scheme, CFL = 0.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

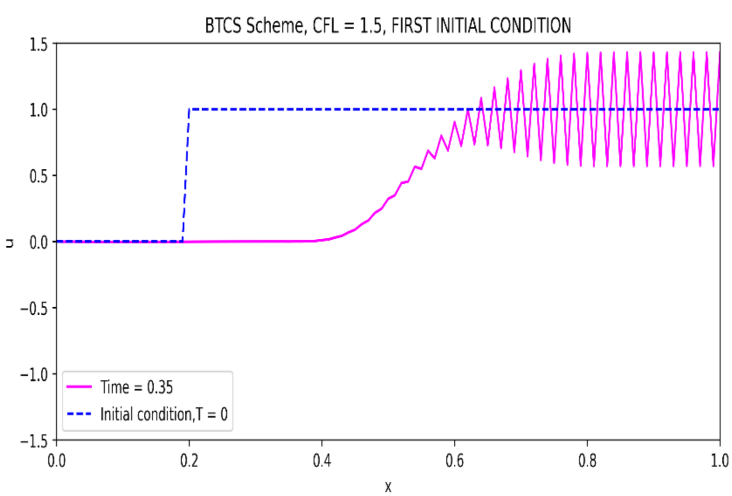

BTCS Scheme, CFL = 1.0, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
x
u

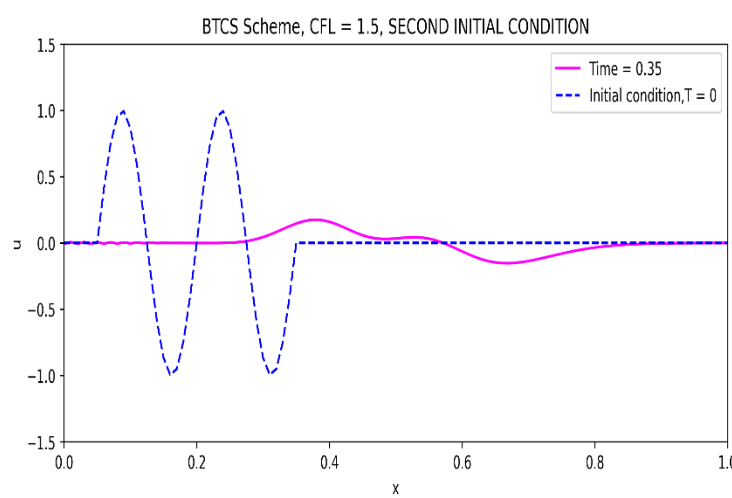

BTCS Scheme, CFL = 1.0, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

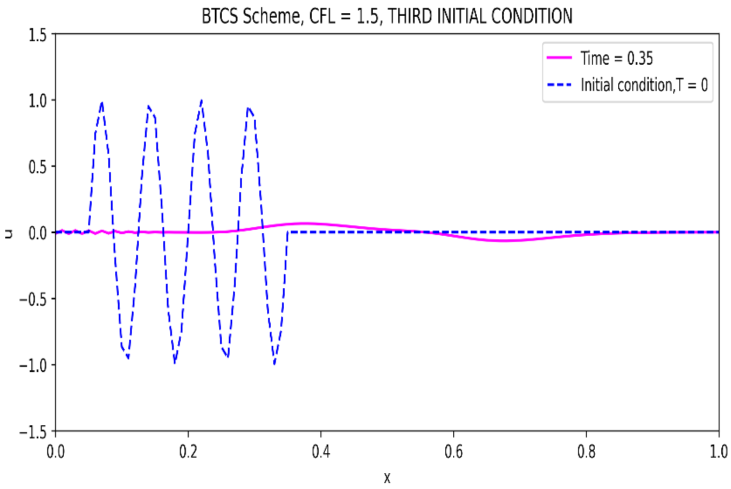

BTCS Scheme, CFL = 1.0, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

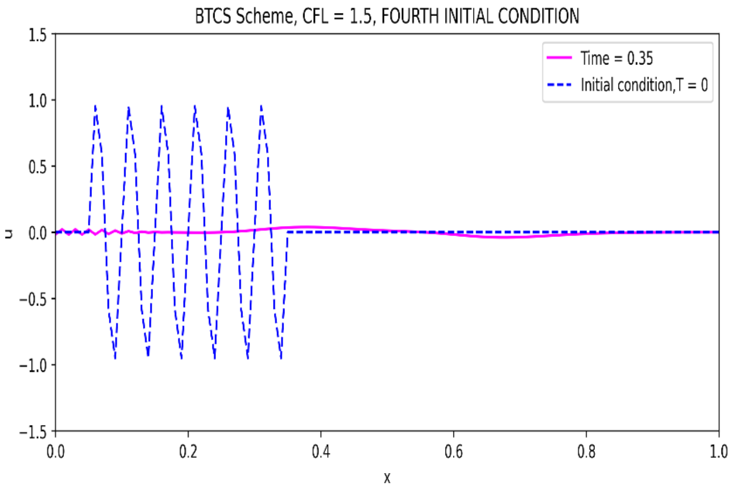

BTCS Scheme, CFL = 1.0, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

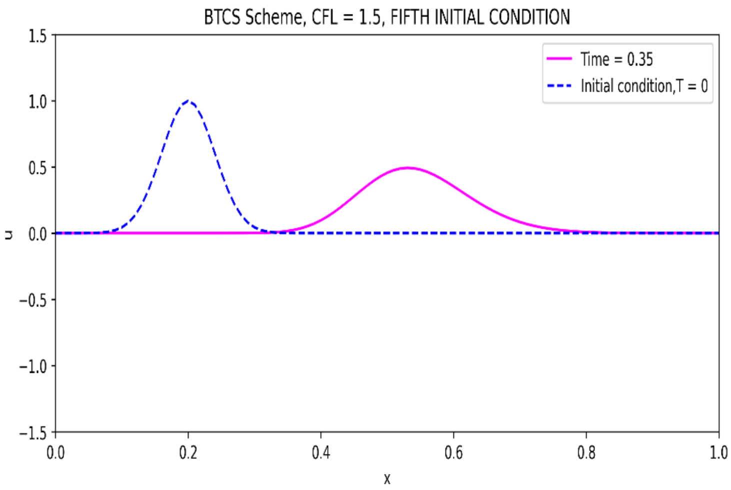

BTCS Scheme, CFL = 1.0, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BTCS Scheme, CFL = 1.5, FIRST INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BTCS Scheme, CFL = 1.5, SECOND INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BTCS Scheme, CFL = 1.5, THIRD INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BTCS Scheme, CFL = 1.5, FOURTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x


BTCS Scheme, CFL = 1.5, FIFTH INITIAL CONDITION
Time = 0.35
Initial condition,T = 0
u
x

References

- Introduction to CFD (Nptel), Arnab Roy, IIT Kharagpur
- Difference between Explicit and Implicit Schemes---cfd-online
- Simulating Fluid Flows Using Python---Tanmay Agrawal