REPORT

Numerical Solution of Flow through CD Nozzle using van Leer Flux Vector Splitting Method

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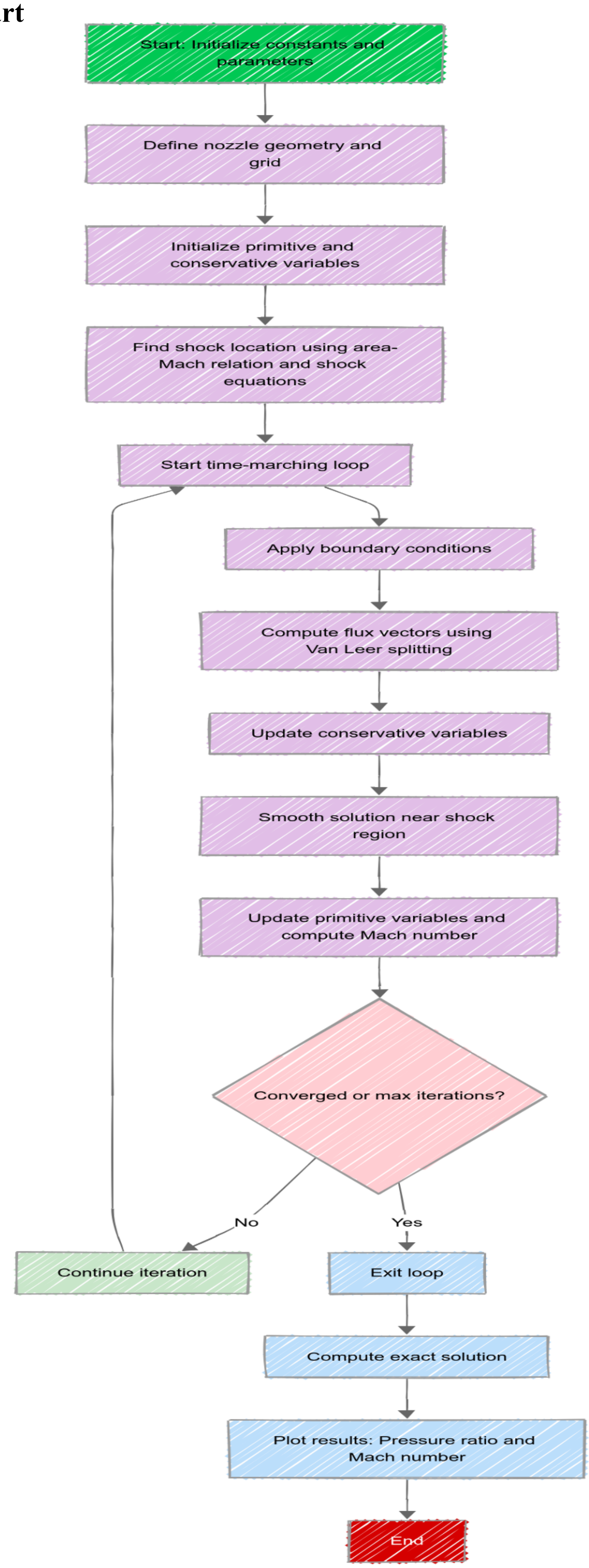
1. Flowchart


Start: Initialize constants and parameters
Define nozzle geometry and grid
Initialize primitive and conservative variables
Find shock location using area-Mach relation and shock equations
Start time-marching loop
Apply boundary conditions
Compute flux vectors using Van Leer splitting
Update conservative variables
Smooth solution near shock region
Update primitive variables and compute Mach number
Converged or max iterations?
No
Yes
Continue iteration
Exit loop
Compute exact solution
Plot results: Pressure ratio and Mach number
End

2. Convergent-Divergent Nozzle

A convergent-divergent nozzle, also known as a de Laval nozzle, is a specialized type of nozzle used in propulsion systems, particularly in rocket engines and supersonic jet engines. The CD nozzle consists of three main sections: a converging section, a throat, and a diverging section. The converging section gradually narrows from its inlet to the throat, accelerating the flow of fluid passing through it and increasing its velocity.

The throat is the narrowest part of the nozzle, where the flow reaches its maximum velocity. It serves as a critical point of transition between the converging and diverging sections. The diverging section widens from the throat to the outlet, allowing the flow to expand gradually. This expansion converts the kinetic energy of the flow into pressure energy, resulting in increased pressure and a corresponding decrease in velocity.

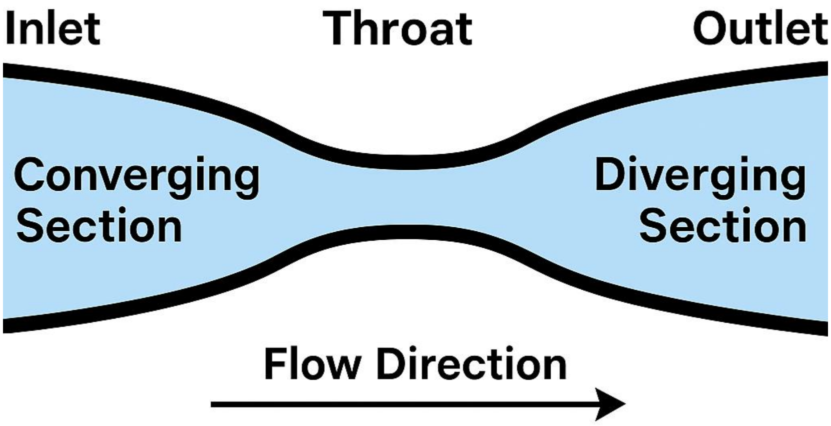


Figure 1 CD Nozzle Geometry

One of the primary advantages of CD nozzles is their ability to generate supersonic flow velocities. By carefully designing the converging and diverging sections, CD nozzles can efficiently accelerate the flow to supersonic speeds, enabling high-speed propulsion in rocket engines and supersonic aircraft.

CD nozzles are designed to achieve optimal expansion of the flow, maximizing thrust generation and efficiency. The gradual expansion in the diverging section allows for efficient conversion of kinetic energy into pressure energy, resulting in increased thrust output.

CD nozzles are versatile and can be adapted to various propulsion systems and applications, including rocket engines, supersonic jet engines, and wind tunnels. Their ability to generate supersonic flow velocities and optimize thrust output makes them suitable for a wide range of engineering tasks. CD nozzles can adapt to changing ambient pressures, making them suitable for applications where altitude variations are encountered.

Here, in the Assignment, the Nozzle Length given is 2 meters, and the governing equation of Area is given as:

$$Area = 1 + 2 * (x - 1)^2 \,, 0 \leq x \leq 2$$

$; x = Grid\ Point\ Location\ ;\ Total\ Grid\ Points = 101$

The reservoir conditions are as follows: P0 = 1.0133 × 10^5 Pa and T0 = 300 K, and the Stagnation Pressure at the exit is given as 0.585.

3. Flow Through CD Nozzle

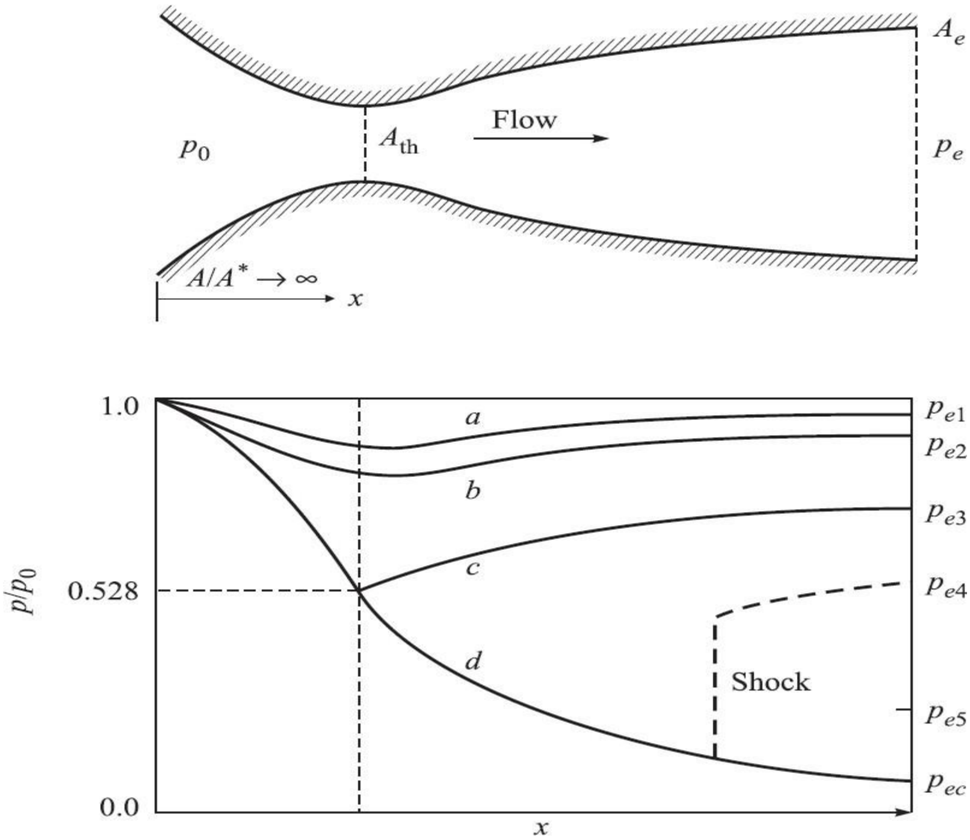


Figure 2 Pressure Variation Across CD Nozzle

For a particular total pressure P_0, when the back pressure is reduced below the upstream P_0, the pressure decreases in the nozzle as shown by the curve "a" and "b" and the flow remains subsonic throughout.

If the back pressure is reduced further as shown by curve "c," sonic flow will be attained at the throat section for a certain value of P_e^3. This condition at the throat is called a choked condition as the mass flow rate through the nozzle attains a maximum value. In other words, the mass flow cannot be increased by any further decrease in P_e^3 for a constant P_0 and T_0.

If the back pressure P_e decreases further, say to P_e^4 , then the flow cannot attain isentropic conditions since the throat is already choked. Rather, the pressure decreases further from the throat until a normal shock is formed in the divergent portion of the CD nozzle as shown by the curve "d."

In a converging-diverging (CD) nozzle, the Mach number is dependent on the nozzle geometry and flow conditions.

In the converging part, subsonic flow accelerates with decreasing area to Mach 1 at the throat if choked flow exists. Beyond the throat, in the diverging section, the flow expands and accelerates to supersonic velocities with increasing area.

The Mach number behaviour is a function of backpressure: if too elevated, the flow is subsonic; if expanded correctly, the flow is subsonic-supersonic.

4. Results obtained

The image below shows the variation of P/P0 vs Nozzle Length, the black dotted curve shows the

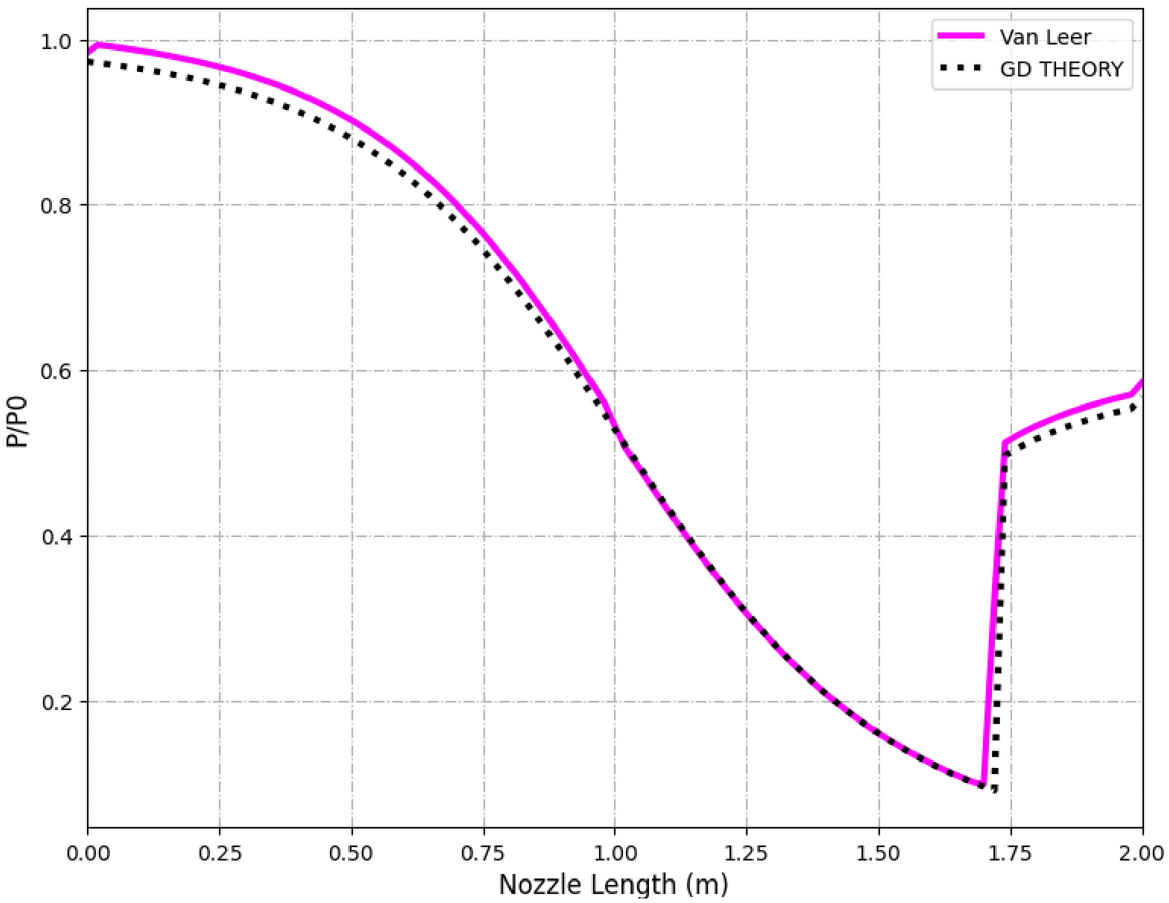


Figure 3 Pressure Distribution across Nozzle Length

exact pressure variation across the nozzle using the Gas Dynamics approach, which is explained in the later section, while the curve having green colour shows how the pressure is varying over nozzle length for given physical as well as thermodynamic constraints applying Van Leer Flux splitting over the Euler equation.

On one thought by seeing the given exit pressure ratio one may conclude that the flow through Nozzle is fully subsonic as it is greater than 0.528 but that approach is quite wrong as because of given physical and thermodynamic constraints there may be possibility of Normal Shock inside the Nozzle to meet the conditions prescribed at Nozzle Exit.

The solution obtained by Van Leer Flux splitting over the Euler equation is showing almost proper results, as seen in the figure shown above, as it shows a similar trend as the theoretical approach.

The location where the Normal Shock appears is 1.72 m, where the Nozzle area is around 2.0368 m^2. After the shock formation, the flow becomes subsonic and pressure starts to drop.

The image below shows the variation of Mach Number across the Nozzle Length, the dotted purple

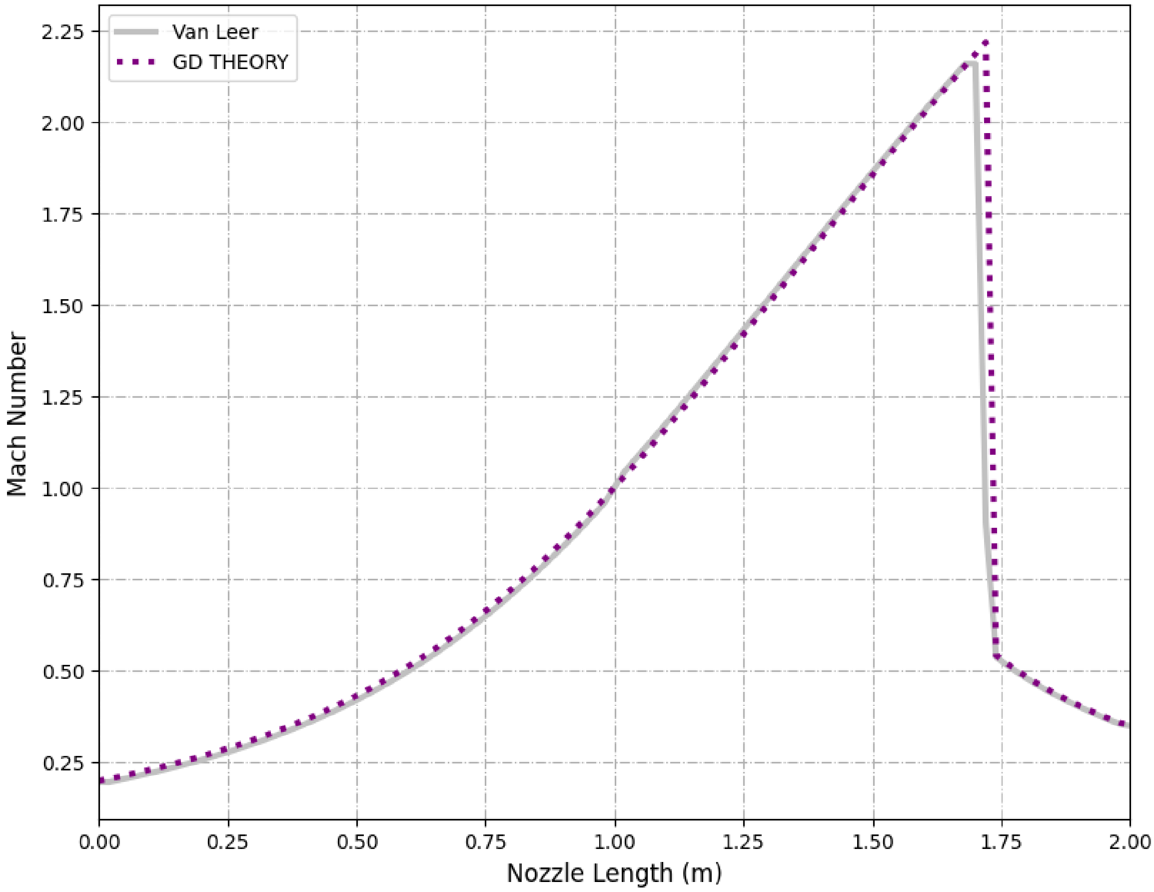


Figure 4 Mach Number Distribution across Nozzle Length

the curve shows the Mach Number variation across the nozzle using the theoretical approach, while the curve having purple colour shows how the Mach Number is varying over nozzle length for given physical as well as thermodynamic constraints, applying Van Leer Flux splitting over the Euler equation.

The flow through the Nozzle is choked as the Mach Number at the nozzle throat is 1, but as the downstream pressure is not low enough to have a fully developed flow and the shock forms, and it makes the flow subsonic at the exit.

5. Cross verification using Gas Dynamics Theory

The one way to confirm the results obtained from Van Leer is to opt for an iterative approach, which involves the following steps:

➔Guess a shock location x_s.

➔Compute the area ratio A_s / A_t

➔Determine the upstream Mach number M1 using isentropic relations.

➔Compute the downstream Mach number M2 using normal shock relations.

➔Use M2 to find the pressure ratio pe/p0 at the nozzle exit.

➔Compare with 0.585. If incorrect, adjust x_s and repeat.

The Throat occurs at the minimum area A_t

$$\frac{dA}{dx} = 4(x-1) = 0$$

Thus, at x_t =1m ➔ A_t = 1

Which makes the area ratio as:

$$\frac{A_s}{A_t} = 1 + 2(x_s - 1)^2$$

Now one has to assume the value of x_s which will give some value of this area ratio using the Mach area relation and opt for the Supersonic result of it, so we do have one value of M, Using this Mach number, one can calculate the downstream Mach Number using the Normal Shock Relation, then after you can calculate the Pe/P0.

One can compare this Pe/P0 in question if it is same as the given in question, then your assumed value is correct, or else assume a new value and follow the same procedure.

For this question around $x_s = 1.7$, The value matches with the obtained one.

6. Acknowledgement

I want to thank Prof. Mandal for his continuous guidance and support, which helped me to understand and perform this project.

7. References

➔MODERN COMPRESSIBLE FLOW: ISBN 978-1-260-57082-3