REPORT

Computation of Lid-Driven Cavity Flow Using Vorticity-Stream Function Formulation

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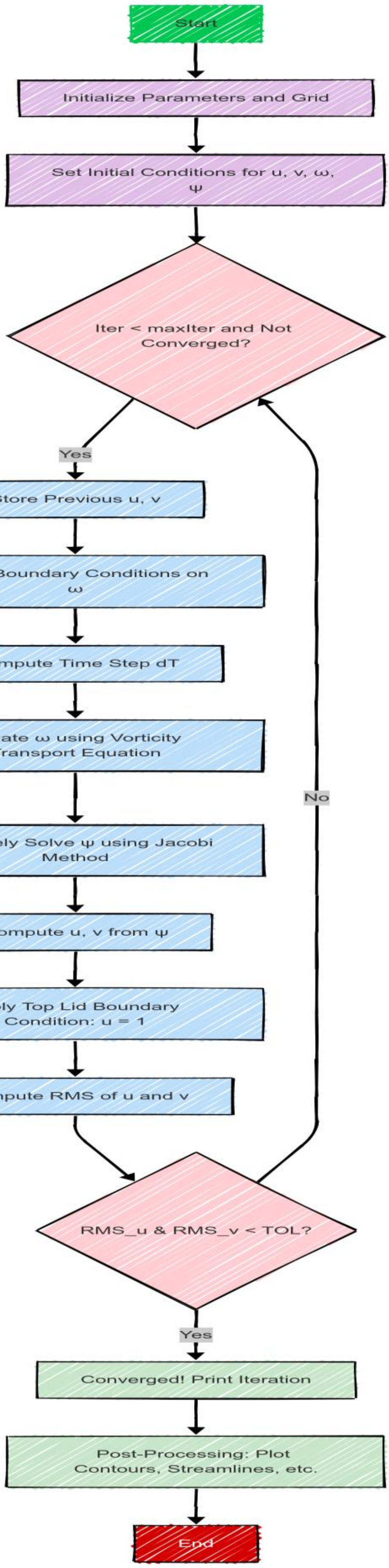
1. Flowchart


Start
Initialize Parameters and Grid
Set Initial Conditions for u, v, ω, ψ
Iter < maxIter and Not Converged?
Yes
Store Previous u, v
Apply Boundary Conditions on ω
Compute Time Step dT
Update ω using Vorticity Transport Equation
Iteratively Solve ψ using Jacobi Method
Compute u, v from ψ
Apply Top Lid Boundary Condition: u = 1
Compute RMS of u and v
No
RMS_u & RMS_v < TOL?
Yes
Converged! Print Iteration
Post-Processing: Plot Contours, Streamlines, etc.
End

2. Lid Driven Cavity Problem

The Lid-Driven Cavity Problem is a classical benchmark problem in computational fluid dynamics (CFD) used to validate numerical methods for incompressible viscous flows. It's simple to define but captures complex flow features, such as vortices and shear layers, making it a favourite for testing solvers.

Physical Setup

- Imagine a square box filled with a viscous fluid.
- The top lid moves at a constant velocity.
- All other walls are stationary and no-slip.
- There is no inflow or outflow — it is a closed domain.

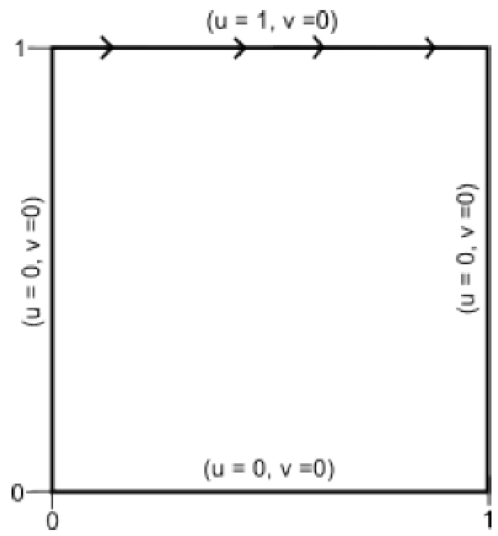


Figure 2.1 Lid Driven Cavity Setup

This simple setup induces complex flow patterns within the cavity due to viscous effects, particularly the formation of a primary vortex at the centre and secondary vortices in the corners, especially at higher Reynolds numbers.

Despite its geometric simplicity, the problem captures key features of fluid motion such as boundary layers, shear-driven circulation, and vortex formation, making it an ideal test case for validating numerical solvers, studying turbulence onset, and examining various discretization schemes. Depending on the Reynolds number, the flow can be steady or unsteady, laminar, or turbulent, and symmetric or asymmetric. The governing equations for the lid-driven cavity flow are the two-dimensional incompressible Navier–Stokes equations, which are typically solved using finite difference, finite volume, or finite element methods, either in primitive variable form (velocity and pressure) or using stream function-vorticity formulations.

3. Solution Approach

➔Vorticity Transport equation

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \vartheta\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

; $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ -----Vorticity

; $u = \frac{\partial \psi}{\partial x}$; v = $-\frac{\partial \psi}{\partial y}$ ----- Velocity Components

; $\vartheta = Kinematic\ Viscosity$

The discretization of the convection part of this formula is done using the CIR scheme (Engaging it in a second-order upwind scheme) while the diffusion part is treated central differencing scheme.

➔Poisson Equation

$$\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) = -\omega$$

➔Boundary Conditions

@ Top or Lid

U = 1 m/s

V = 0 m/s

$$\omega_{i,J} = \frac{-2(\psi_{i,J-1} - \psi_{i,J-1})}{\Delta y^2} - \frac{2u_{i,J}}{\Delta y}$$

where I × J grid is used; i = 1, 2, . . ., I and j = 1, 2, . . ., J.

@ Bottom

$$\omega_{1,J} = \frac{-2(\psi_{i,2} - \psi_{i,1})}{\Delta y^2}$$

@ Left

$$\omega_{1,J} = \frac{-2(\psi_{2,j} - \psi_{1,}j)}{\Delta y^2}$$

@ Right

$$\omega_{i,J} = \frac{-2(\psi_{I-1,j} - \psi_{I,j})}{\Delta y^2}$$

No slip condition: u = 0m/s, v = 0m/s

➔Time step determination

Convective time step

$$\Delta t_c = \frac{\Delta x \Delta y}{|u_{max}|\Delta y + |v_{max}|\Delta x}$$

Diffusion time step

$$\Delta t_d = \left(\frac{\sigma_d}{2\vartheta}\right)\frac{\Delta x^2 \Delta y^2}{\Delta x^2 + \Delta y^2}$$

Time Step is

$$\Delta t = minimum(\Delta t_c, \Delta t_d)$$

4. Results

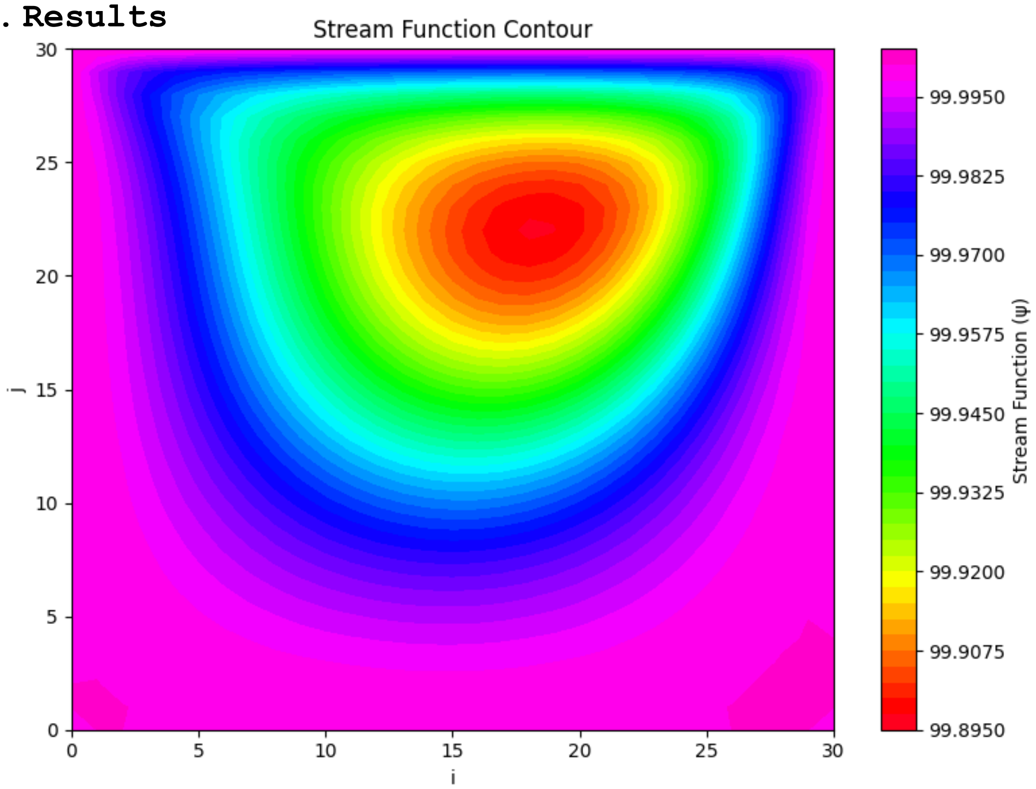


Figure 3.1 Stream Function Contour

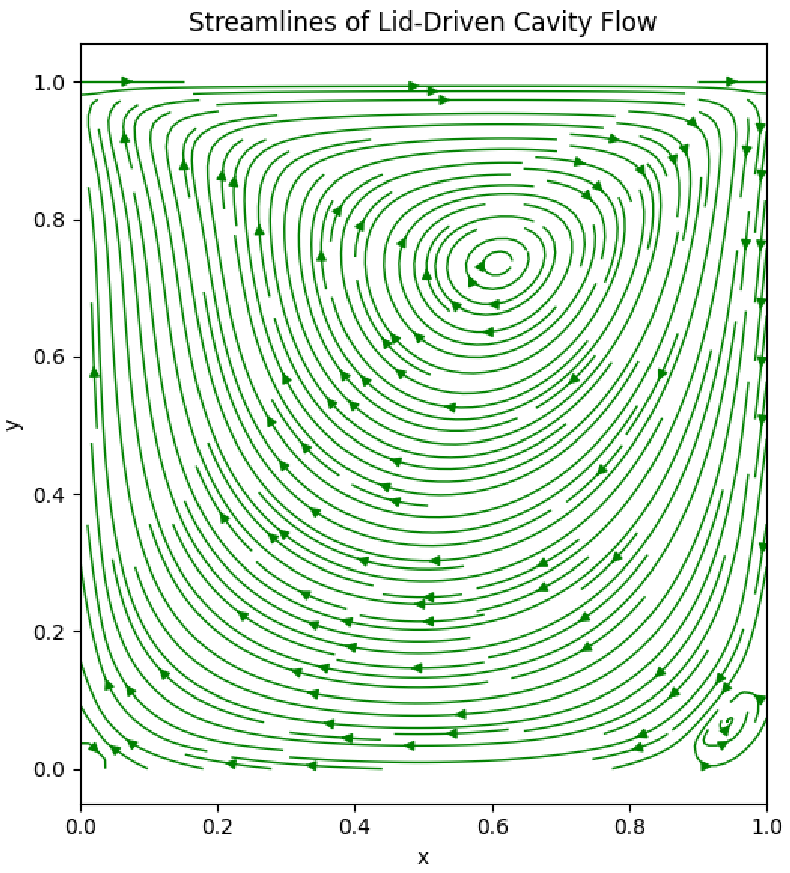


Figure 3.2 Streamlines Contour

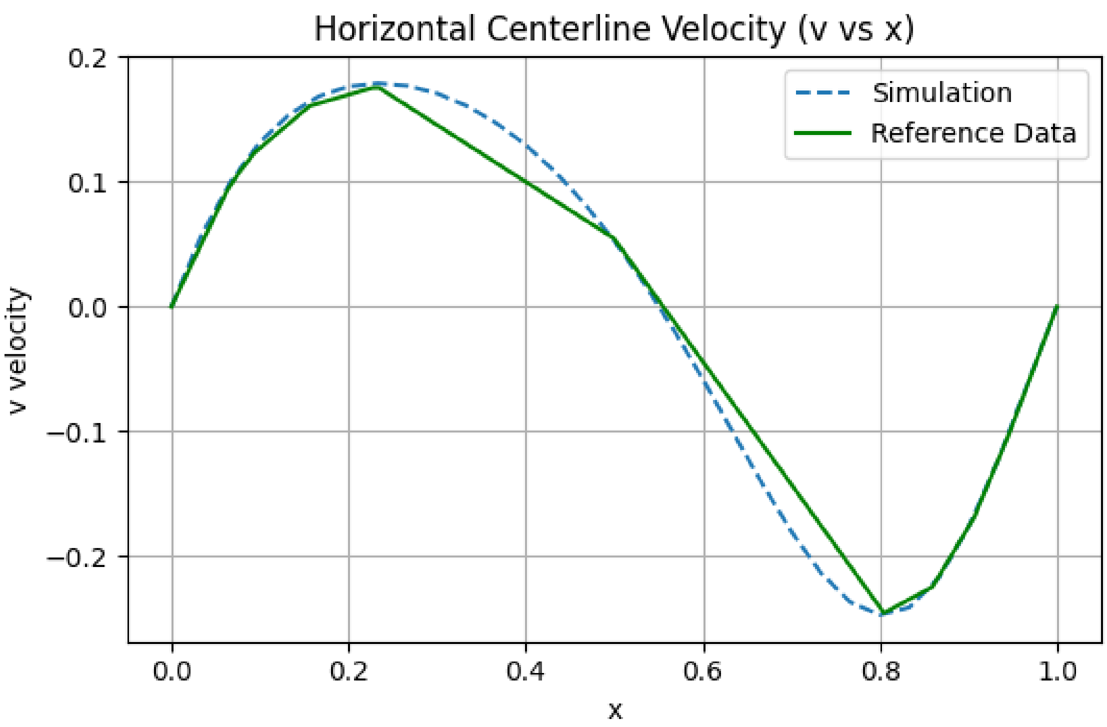


Figure 3.3 Horizontal Centerline Velocity Comparison

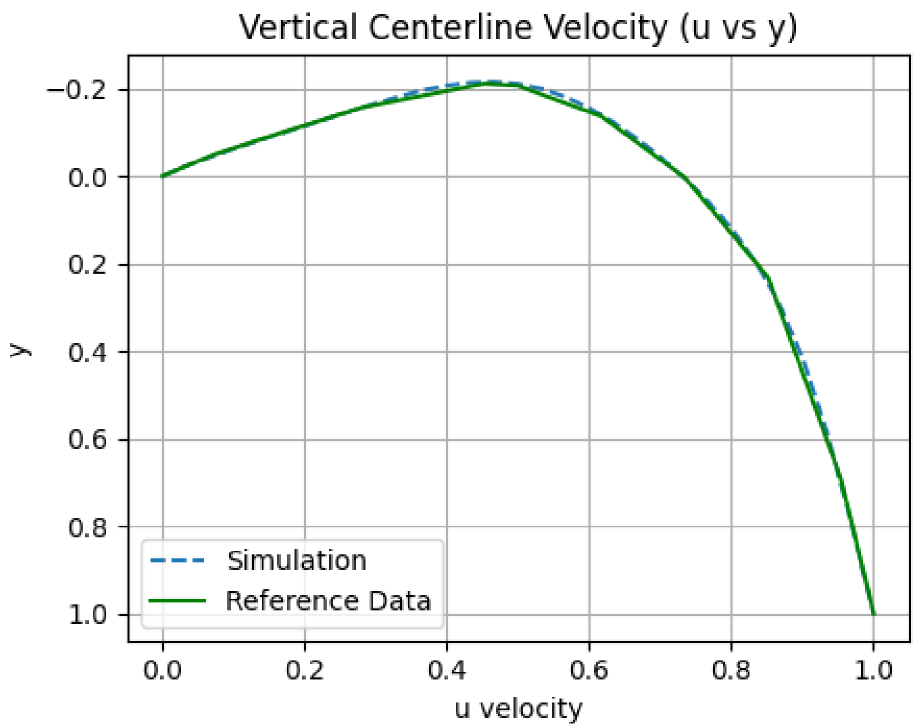


Figure 3.4 Vertical Centerline Velocity Comparison

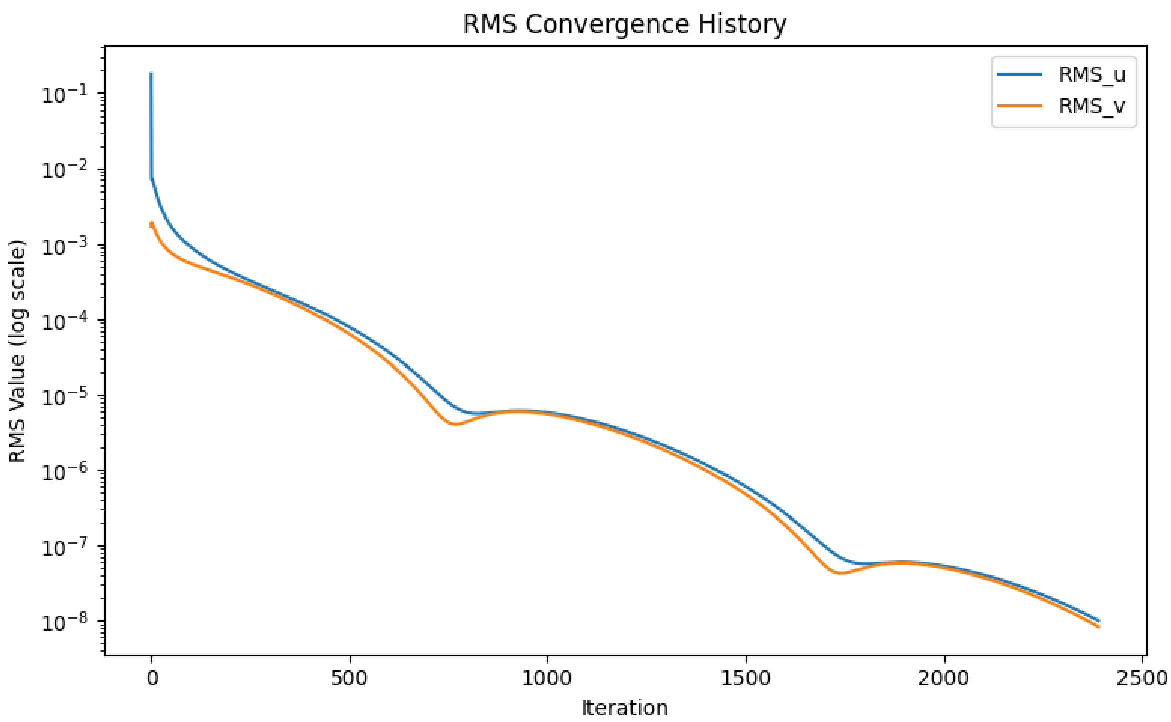


Figure 3.5 RMS convergence

The stream function contour plot (Figure 3.1) for the lid-driven cavity at a Reynolds number of 100 shows a steady, laminar, incompressible flow driven by the top-moving lid. A single dominant primary vortex forms inside the cavity, with its centre shifted to the right, indicating mild asymmetry often seen at low Reynolds numbers. The smooth, closed contours represent recirculating flow, which is strongest near the vortex core and weakens toward the walls.

The streamline plot (Figure 3.2) of the lid-driven cavity flow at Reynolds number 100 illustrates a smooth, steady, and laminar recirculating flow pattern. A prominent primary vortex is visible, with its centre shifted to the right of the cavity's geometric centre, consistent with the top wall driving the flow. The streamlines are closed and densely packed near the vortex core, indicating strong rotational motion. Notably, small secondary vortices are also observed in the bottom-right and bottom-left corners, typical at this Reynolds number and signifying subtle flow separation near the stationary walls.

In the case of horizontal and vertical velocity at the centerline (Figures 3.3 and 3.4), the simulation follows the pattern of the given reference, but the values are somewhat deviated.

5. Acknowledgement

I want to thank Prof. Mandal for his continuous guidance and support, which helped me to understand and perform this project.