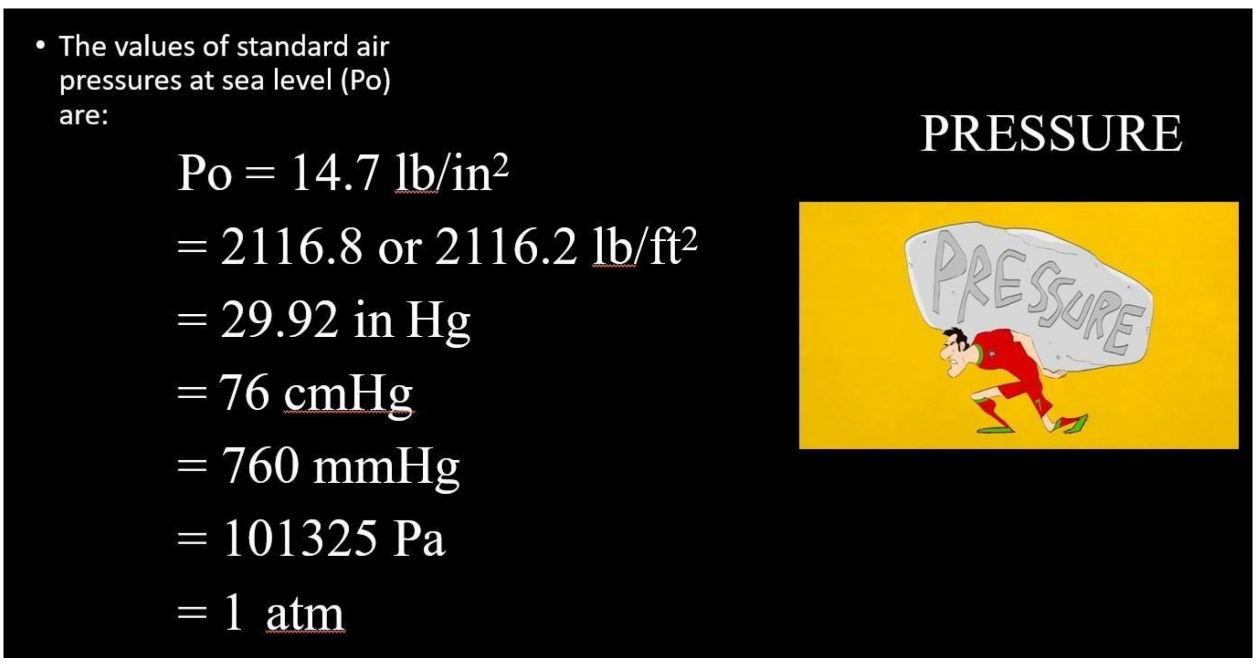
MODULE 3

PROPERTIES OF AIR

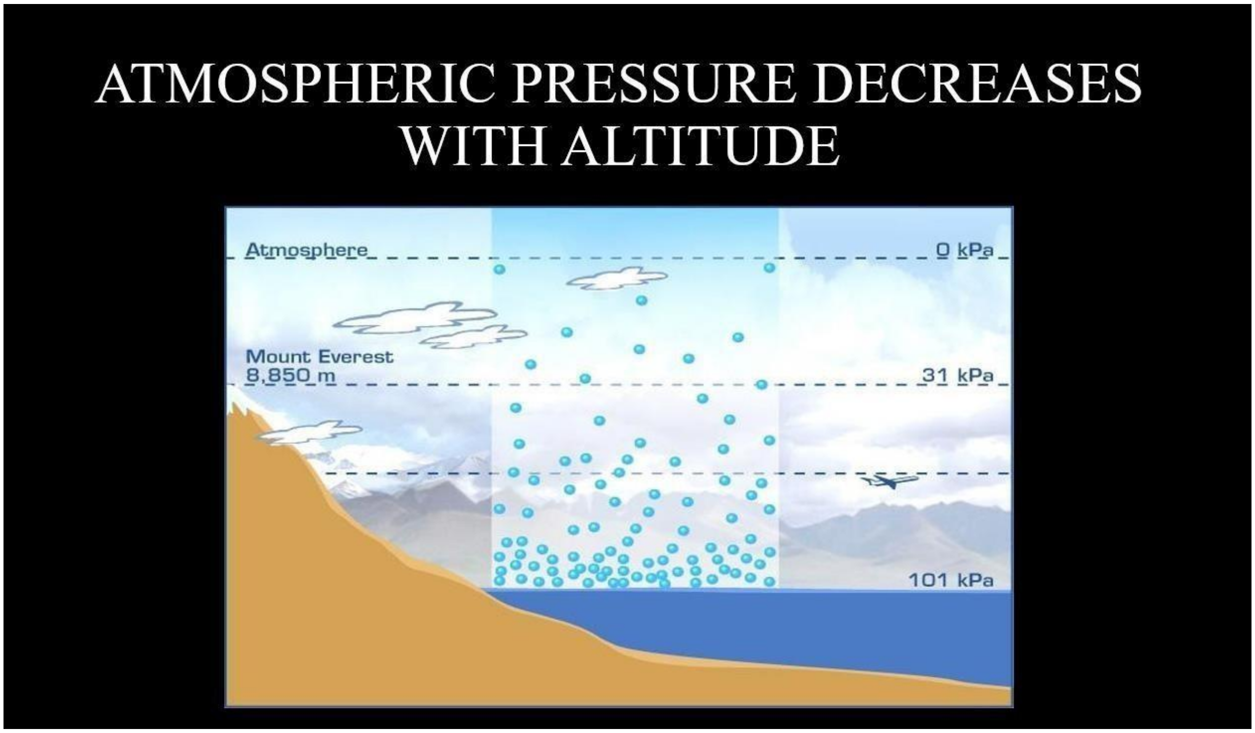
PRESSURE

Pressure P is the force applied perpendicular to the surface of an object per unit area over which that force is distributed. Various units are used to express pressure. Some of these derive from a unit of force divided by a unit of area; the SI unit of pressure, the pascal (Pa), for example, is one newton per square meter (N/m2); similarly, the pound-force per square inch (psi) is the traditional unit of pressure in the imperial and U.S. customary systems.

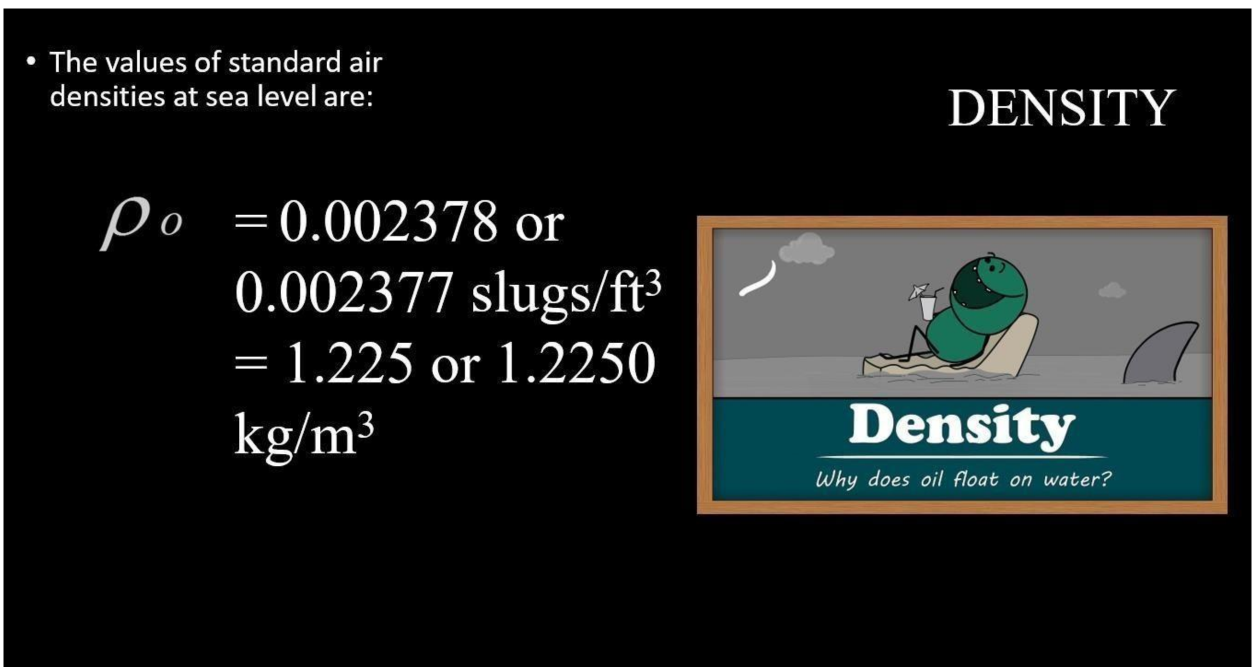


Pressure may also be expressed in terms of standard atmospheric pressure; the atmosphere (atm) is equal to this pressure, and the torr is defined as 1⁄760 of this.

Manometric units such as the centimeter of water, millimeter of mercury, and inch of mercury are used to express pressures in terms of the height of column of a particular fluid in a manometer.

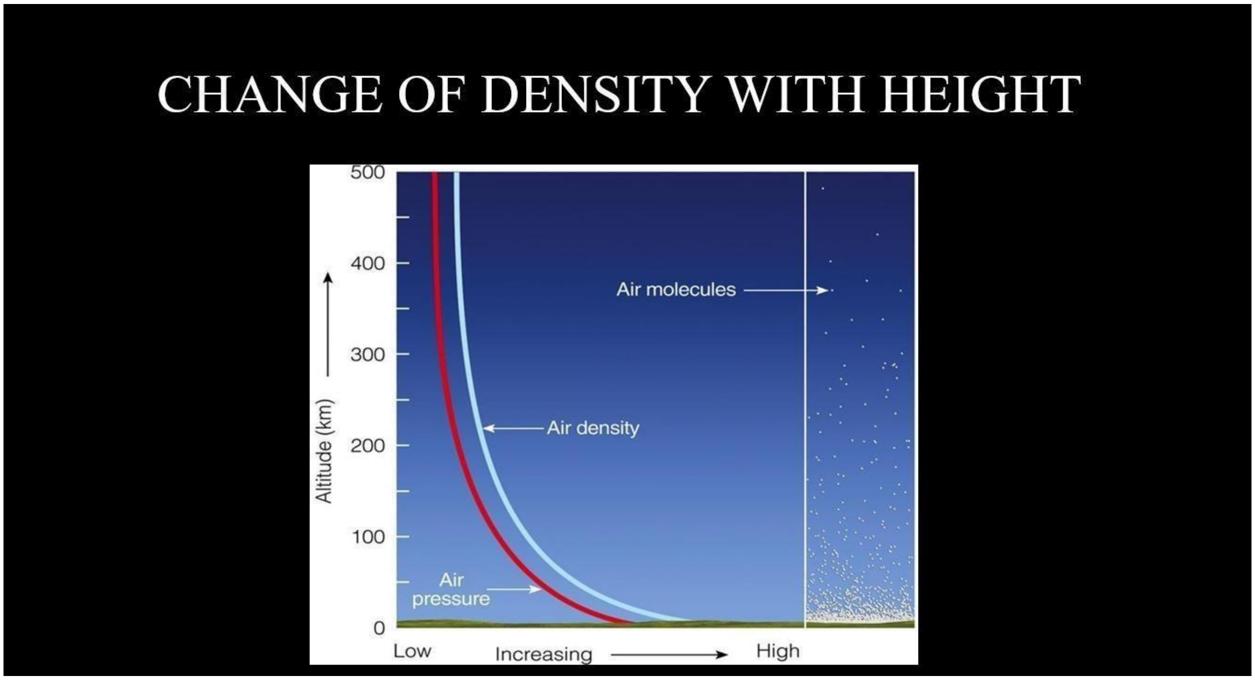


DENSITY

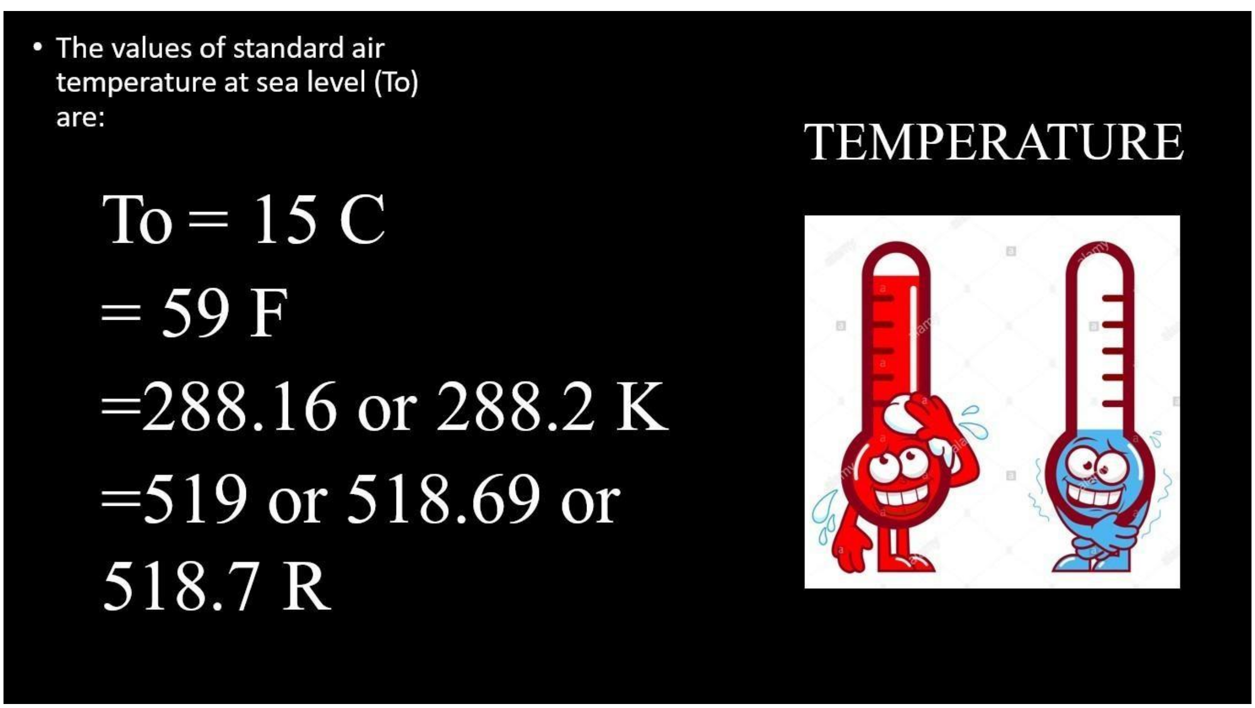


The density (more precisely, the volumetric mass density; also known as specific mass), of a substance is its mass per unit volume. The symbol most often used for density is ρ although the Latin letter D can also be used. Mathematically, density is defined as mass divided by volume $\rho = \frac{m}{V}$ where ρ is the density, m is the mass, and V is the volume.

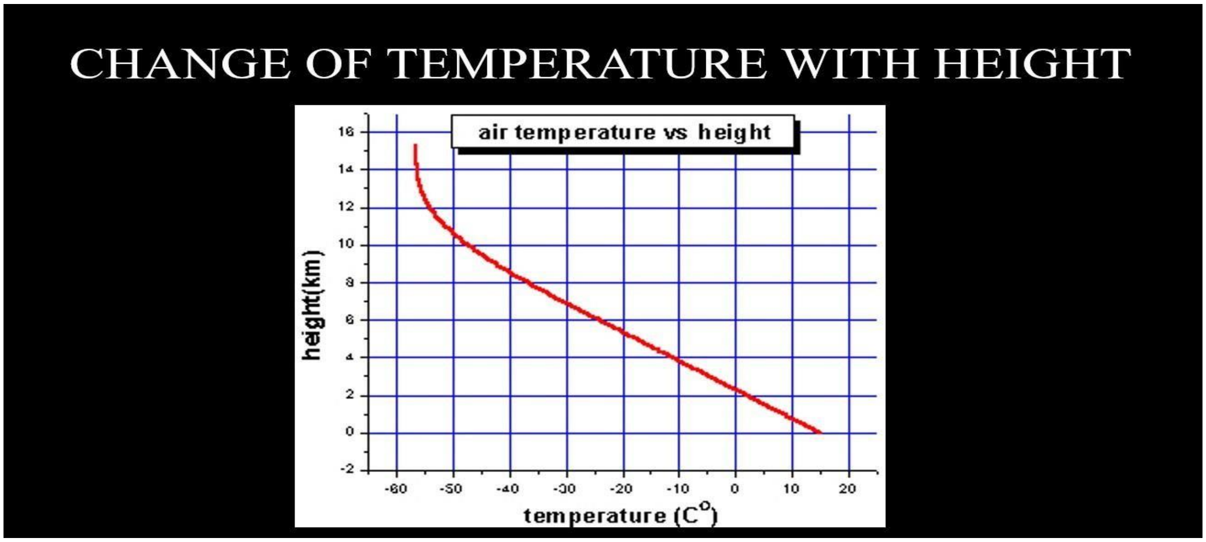
Temperature is a physical quantity that expresses hot and cold. It is the manifestation of thermal energy, present in all matter, which is the source of the occurrence of heat, a flow of energy, when a body is in contact with another that is colder.



TEMPERATURE



Temperature is measured with a thermometer. Thermometers are calibrated in various temperature scales that historically have used various reference points and thermometric substances for definition. The most common scales are the Celsius scale (formerly called centigrade, denoted °C), the Fahrenheit scale (denoted °F), and the Kelvin scale (denoted K), the last of which is predominantly used for scientific purposes by conventions of theInternational System of Units (SI).



AIR PRESSURE CHANGES WITH ALTITUDE.

Air is all around us, but we cannot see it. Gravity from the Earth pulls air down - this is called air pressure. We don't feel this pressure because our bodies push an equal amount of pressure outward. This graph shows how air density and air pressure changes with altitude (the distance above sea level). Barometers are used to measure air pressure in millibars.

International Standard Atmosphere (ISA)

INTERNATIONAL STANDARD ATMOSPHERE SEA LEVEL CONDITIONS

	Metric Value	Imperial Value
Pressure	101325Pa	2116.2lb/ft^2
Density	1.225kg/m^3	0.002378slug/ft^3
Temperature	15^{o}C/288.2K	59^{o}F/518.69R
Speed of Sound	340.2m/s	1116.4ft/s
Viscosity	1.789x10^{-5} kg/m/s	3.737x10^{-7} slug/ft/s
Kinematic Viscosity	1.460x10^{-5} m^2/s	1.5723x10^{-4} ft^2/s
Thermal Conductivity	0.02596W/m/K	0.015 BTU/hr/ft/R
Gas Constant	287.1 J/kg/K	1715.7 ft-lbf/slug/R
Specific Heat Cp	1005 J/kg/K	6005 ft-lbf/slug/R
Specific Heat Cv	717.98 J/kg/K	4289 ft-lbf/slug/R
K=Cp/Cv	1.4	
Gravitational Acceleration	9.80665m/s^2	32.174ft/s^2

Description

Also known as the ICAO Standard Atmosphere, ISA is a standard against which to compare the actual atmosphere at any point and time.

The ISA is based the following values of pressure, density, and temperature at mean sea level eachOF which decreases with increase in height:

- Pressure of 1013.2 millibar - Pressure is taken to fall at about 1 millibar per 30 feet in the lower atmosphere (up to about 5,000 feet).
- Temperature of +15 °C - Temperature falls at a rate of 2 °C per 1,000 feet until the tropopause is reached at 36,000 feet above which the temperature is assumed to be constant at -57 °C. (The precise numbers are 1.98 °C, -56.5 °C and 36,090 feet)
- Density of 1,225 gm/m3. The real atmosphere differs from ISA in many ways. Sea level pressure varies from day to day, and there are wide extremes of temperature at all levels. Variation in pressure, vertically and horizontally, affects the operation of the pressure altimeter.

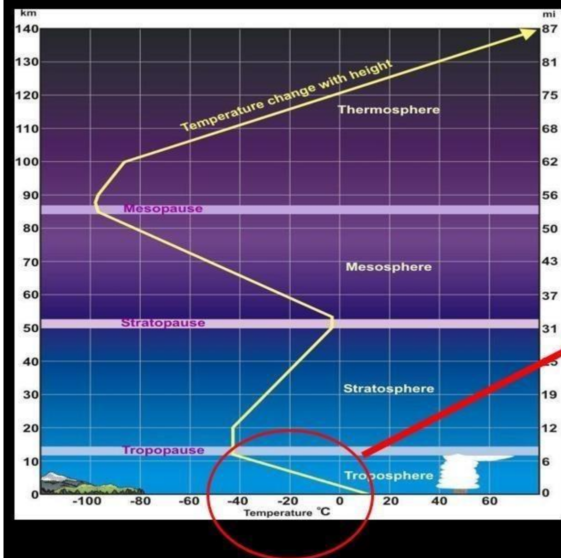
LAPSE RATE

The lapse rate is the rate at which an atmospheric variable, normally temperature in Earth's atmosphere, falls with altitude. Lapse rate arises from the word lapse, in the sense of a gradual

fall.

It corresponds to the vertical component of the spatial gradient of temperature. Although this concept is most often applied to the Earth's troposphere, it can be extended to any gravitationally supported parcel of gas

Variations of Pressure, Temperature, and Density in the Gradient Layers (Troposphere, 0-11 km)



LAPSE RATE

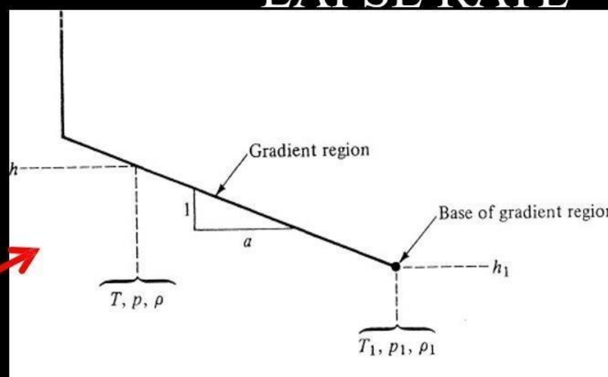


Figure 3.6 Gradient layer.

LAPSE RATE

$$\frac{T - T_1}{h - h_1} = \frac{dT}{dh} = a$$

$$a = \frac{dT}{dh}$$

Temperature Lapse Rate for the Gradient Layers

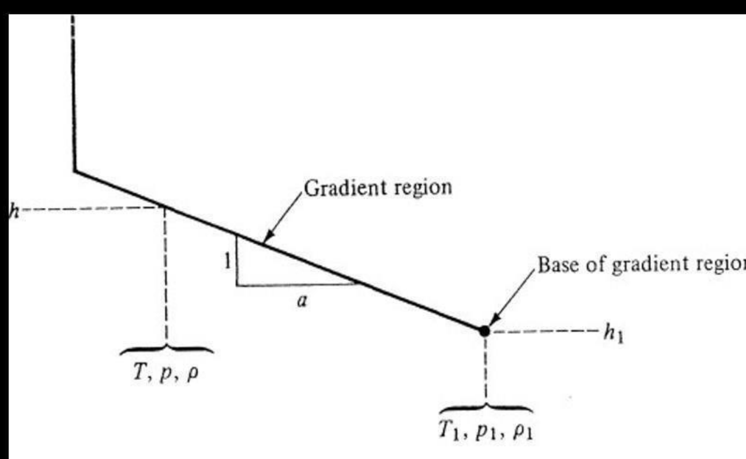


Figure 3.6 Gradient layer.

LAPSE RATE

a = -0.0065 or -0.00651 K/m
-6.5 or -6.51 K/km
-0.003566 R/ft

π

PRESSURE VARIATION WITH ALTITUDE (TROPOSPHERE 0-11KM)

- Consider an Equation

$$dp = -\rho g_0 dh$$

Dividing by the equation of state

$$\frac{dp}{p} = \frac{-\rho g_0 dh}{\rho RT} = -\frac{g_o}{RT} dh$$

BUT $dh = \frac{1}{a} dT$

Substituting the results....

$$\frac{dp}{p} = -\frac{g_0}{aR}\frac{dT}{T} \longrightarrow \int_{P_1}^{P}\frac{dp}{p} = -\frac{g_0}{aR}\int_{T_1}^{T}\frac{dT}{T}$$

$$ln\frac{P}{P_1} = -\frac{g_o}{aR} ln\frac{T}{T_1}$$

The General Equation for Pressure Variation with altitude $\longleftarrow$

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-g_o/(aR)}$$

SUBSTITUTING FOR METRIC VALUES (SAME RESULTS FOR ENGLISH VALUES)…

$$\frac{-g}{aR} = \frac{-9.81\ m/s^2}{-0.0065\ K/m(287.08\ N.m/kg.K)}$$

$$\boxed{= 5.26}$$

Therefore our equation will be.....

$$\frac{P}{P_1} = \left[\frac{T}{T_1}\right]^{5.26}$$

- Where:
  - P = pressure at any altitude up to 11 km
  - Po = standard pressure at sea level
  - T = temperature at any altitude up to 11km
  - To = standard temperature at sea level

$$\frac{P}{P_o} = \left[\frac{T}{T_o}\right]^{5.26}$$

DENSITY VARIATION WITH ALTITUDE (TROPOSPHERE 0-11KM)

FROM THE EQUATION OF STATE, $\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1}$

HENCE THE EQUATION BECOMES.....

$$\frac{\rho T}{\rho_1 T_1} = \left(\frac{T}{T_1}\right)^{-g_o/(aR)}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[g_o/(aR)]-1}$$

$$\boxed{\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-([g_o/(aR)]+1)}}$$

The General Equation for Density Variation with Altitude

$$\left[\frac{-g}{aR}\right]-1=\left[\frac{-9.81\ m/s^2}{-0.0065\ K/m(287.08\ N.m/kg.K)}\right]-1$$

$$\boxed{=4.26}$$

Therefore our equation will be.....

$$\frac{\rho}{\rho_1}=\left[\frac{T}{T_1}\right]^{4.26}$$

$$\frac{\rho}{\rho_o}=\left[\frac{T}{T_o}\right]^{4.26}$$

Where:

ρ = density at any altitude up to 11 km

ρ_o = standard density at sea level

T = temperature at any altitude up to 11 km

T_o = standard temperature at sea level

TEMPERATURE VARIATION WITH ALTITUDE (TROPOSPHERE 0-11KM)

$$a = \frac{dt}{dh}$$

SOLVING FOR dt:

$$dt = adh$$

$$\int_{To}^{T} dt = a\int_{0}^{h} dh$$

$$[\ T\ ]_{To}^{T} = a\ [\ h\ ]_{0}^{h}$$

$$T - To = a\ [\ h - 0\ ]$$

$$T - To = ah$$

$$\boxed{T = To + ah}$$

Temperature variation with altitude formula

Where:

T = temperature at any altitude up to 11 km (troposphere)
To = 288.2 K or 519 R
h = height from sea level up to 11 km
a = lapse rate

EXAMPLE:

Calculate the pressure and density at 5km.

Temperature where: $T_0 = 288.2$ K

$T = T_0 + a\,h$ $a = -0.0065$

$T = (288.2 \text{ K}) + (-0.0065)(5\text{km})$

$T = 288.1675$ K

Pressure where: $P_0 = 101325$ Pa

$P = P_0 \left(\frac{T}{To}\right)^{5.26}$

$P = 101325\ Pa \left(\frac{288.1675\ K}{288.2\ K}\right)^{5.26}$

$P = 101264.912$ Pa

Density where: $\rho_0 = 1.225$ kg/m^3

$\rho = \rho_0 \left(\frac{T}{To}\right)^{4.26}$

$\rho = 1.225 \text{ kg/m}^3 \left(\frac{288.1675\ K}{288.2\ K}\right)^{4.26}$

$\rho = 1.224411624$ kg/m^3

Variations of Pressure, Temperature, and Density in the Isothermal Layers (Stratosphere, 11-32 km)

PRESSURE VARIATION WITH ALTITUDE (STRATOSPHERE, 11-32km)

CONSIDER THE HYDROSTATIC EQUATION

$$\frac{dp}{dh} = -\rho g \qquad dp = -\rho g dh$$

Divide this by the equation of state,

$$\frac{dp}{P} = \frac{-\rho g dh}{\rho RT} \rightarrow \frac{dp}{P} = \frac{-g dh}{RT}$$

PRESSURE VARIATION WITH ALTITUDE (STRATOSPHERE, 11-32km)

$$\int_{P_1}^{P} \frac{dp}{P} = \frac{-g}{RT} \int_{h_1}^{h} dh$$

$$e^{ln \frac{P}{P_1}} = e^{\frac{-g}{RT}(h - h_1)}$$

INTEGRATING, WE HAVE …

$$ln \frac{P}{P_1} = \frac{-g}{RT}(h - h_1)$$

$$\boxed{\frac{P}{P_1} = e^{-\left(\frac{g}{RT}\right)(h - h_1)}}$$

PRESSURE VARIATION WITH ALTITUDE (STRATOSPHERE, 11-32km)

Where:

P = pressure at any altitude above 11 km
P_1 = pressure at 11 km
g = gravitational constant,
(9,81 m/s², 32.2 ft/s²)
R = gas constant, for air
(287.08 J/kg.K, 53.342 ft.lbf/lbm.R)
T = constant temperature at stratosphere 216.5 K, 390.15 R
h = the given altitude above 11,000 m h_1 = 11,000 m

$$\frac{P}{P_1} = e^{-\left(\frac{g}{RT}\right)(h-h_1)}$$

DENSITY VARIATION WITH ALTITUDE (STRATOSPHERE, 11-32km)

$$\frac{P}{P_1} = e^{-\left(\frac{g}{RT}\right)(h-h_1)}$$

$$\frac{\rho}{\rho_1} = e^{-\left(\frac{g}{RT}\right)(h-h_1)}$$

From the equation of state:

$$\frac{P}{P_1} = \frac{\rho RT}{\rho_1 RT_1} = \frac{\rho \cancel{T}}{\rho_1 \cancel{T_1}} = \frac{\rho}{\rho_1}$$

DENSITY VARIATION WITH ALTITUDE (STRATOSPHERE, 11-32km)

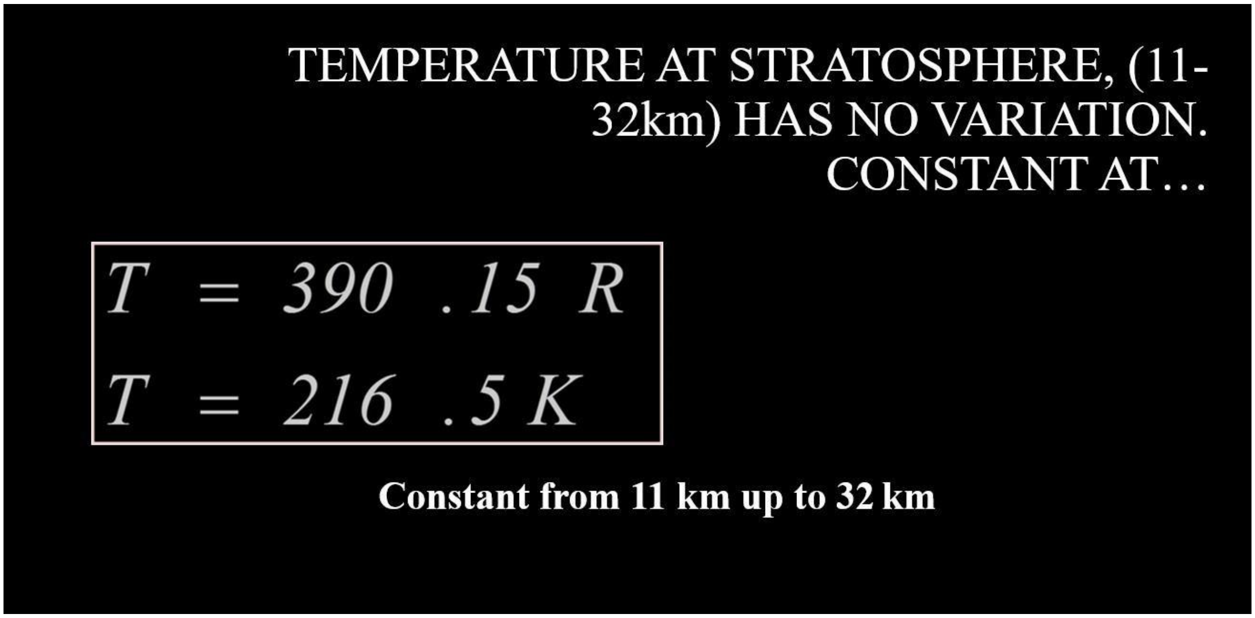
$$\frac{\rho}{\rho_1} = e^{-\left(\frac{g}{RT}\right)(h-h_1)}$$

Where:

ρ = density at any altitude above 11 km

ρ_1 = density at 11 km

g = gravitational constant,
(9,81 m/s², 32.2 ft/s²)
R = gas constant, for air
(287.08 J/kg.K, 53.342 ft.lbf/lbm.R)
T = constant temperature at stratosphere
216.5 K, 390.15 R
h = the given altitude above 11,ooo m
h_1 = 11,000 m



Example

Calculate the pressure and density at altitude 15000km.

Temperature (constant)

T = 216.5 K

Pressure where: P_{11} = 22502.7116 Pa

$P = P_{11}\, e^{(-g/RT)(h-11000)}$

$P = 22502.7116\ \text{Pa}\ e^{(-9.8/(287)216.5)(15000-11000)}$

P = 11974. 22627 Pa

Density where: ρ_{11} = 0.3622 kg/m^3

$\rho = \rho_{11}\, e^{(-g/RT)(h-11000)}$

$\rho = 0.3622\ \text{kg/m}^3\ e^{(-9.8/(287)216.5)(15000-11000)}$

ρ = 0.192735206 k