Parul University
Faculty of Engineering and Technology
Department of Applied Science & Humanities
Academic Year 2025-26
Subject: Quant and Reasoning (303105311)
Branch: CSE

Unit -1: Number system, LCM &. HCF simplification and approximation

1.1 Numbers

1.1.1 Types of Numbers:

a. Counting Numbers (Natural numbers)

1, 2, 3 ...

b. Whole Numbers

0, 1, 2, 3 ...

c. Integers

-3, -2, -1, 0, 1, 2, 3 ...

d. Rational Numbers

Rational numbers can be expressed as a/b where a and b are integers and $b \neq 0$
Examples: 112112, 4242, 00, −811−811 etc.

All integers, fractions and terminating or recurring decimals are rational numbers.

e. Irrational Numbers

Any number which is not a rational number is an irrational number. In other words, an irrational number is a number which cannot be expressed as a/b where a and b are integers.

For instance, numbers whose decimals do not terminate and do not repeat cannot be written as a fraction and hence they are irrational numbers.

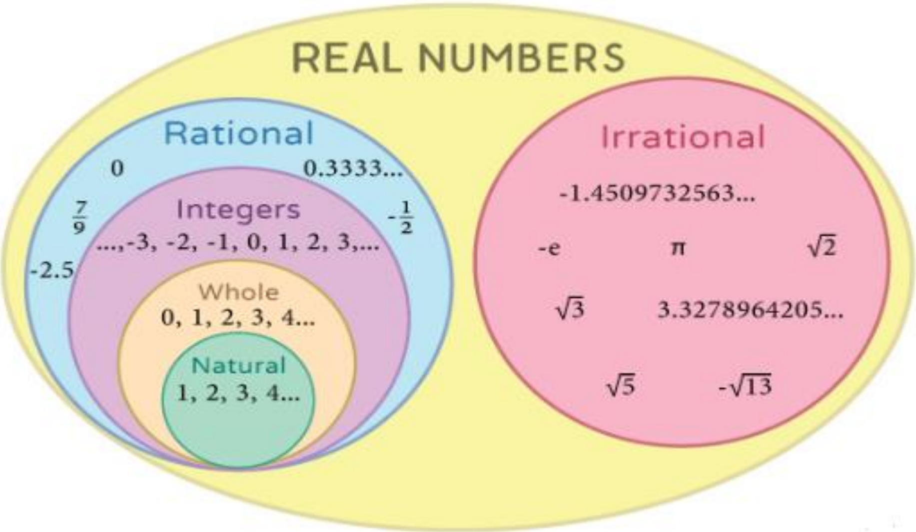
Example: π, $\sqrt{2}$, $(3+\sqrt{5})$, $4\sqrt{3}$ (meaning $4\times\sqrt{3}$), $3\sqrt{6}$ etc

Please note that the value of π = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 58209 74944 59230 78164 06286 20899 86280 34825 34211 70679...

We cannot π as a simple fraction (The fraction 22/7 = 3.14.... is just an approximate value of π)

f. Real Numbers

Real numbers include counting numbers, whole numbers, integers, rational numbers and irrational numbers.



g. Surds

The Surds are the values in square root that cannot be further simplifiednto whole numbers or integers.

Let a be any rational number and n be any positive integer such that n√a is irrational. Then n√a is a surd.

Example: √3, 6√10, 4√3 etc

Please note that numbers like √9, √81 etc are not surds because they are not irrational numbers

Every surd is an irrational number. But every irrational number is not a surd. (eg : π , e etc are not surds though they are irrational numbers.)

1.1.2. Addition, Subtraction and Multiplication Rules for Even and Odd Numbers

Addition Rules for Even and Odd Numbers
1. Sum of any number of even numbers is always even
2. Sum of even number of odd numbers is always even
3. Sum of odd number of odd numbers is always odd

Subtraction Rules for Even and Odd Numbers
1. Difference of two even numbers is always even
2. Difference of two odd numbers is always even

Multiplication Rules for Even and Odd Numbers
1. Product of even numbers is always even
2. Product of odd numbers is always odd
3. If there is at least one even number multiplied by any number of odd numbers, the product is always even

1.1.3. Divisibility

One whole number is divisible by another if the remainder we get after the division is zero.

Examples

36 is divisible by 4 because $36 \div 4 = 9$ with a remainder of 0.

36 is divisible by 6 because $36 \div 6 = 6$ with a remainder of 0.

36 is not divisible by 5 because $36 \div 5 = 7$ with a remainder of 1.

Divisibility Rules

By using divisibility rules, we can easily find whether a given number is divisible by another number without actually performing the division. This saves time especially when working with numbers. Divisibility rules of numbers 1 to 20 are provided below.

Divisible by:	Test	Example
2	Last digit even (= 0,2,4,6 or 8)	3,489,076: Last digit = 6 6 = 3 x **2**
3	Add up all the digits in the number Repeat until sum is 2-digits 2-digits divisible by 3	16,499,205,854,376: 1+6+4+9+9+2++0+5+8+5+4+3+7 +6= 69 6+9=15 15 = 5 x **3**
4	1. Last two digits divisible by 4 2. Tens digit even and ones digit = 0, 4 or 8 Tens digit odd and ones digit = 2 or 6	358,912: 1. Last 2 digits = 12 12 = 3x**4** 2. tens digit =1, odd & ones digit = 2
5	Last digit = 5 or 0	3,783,953,495: Last digit = **5**
6	Divisible by 2 (even) **and** 3	57,342: Last digit = 2 **and** 5+7+3+4+2=21 21 = 7 x **3**
7	Double the last digit, then subtract result from the rest of the digits Repeat for larger numbers until result is a 2-digit number 2-digit # divisible by 7	357: double 7 = 14 subtract 14 from 35 = 21 21 = 3 x **7**
8	1. Hundreds digit even: last 2 digits divisible by 8 Hundreds digit odd: add 4 to last 2 digits and sum divisible by 8 2. Last 3 digits divisible by 8	986,104: 1. hundreds digit = 1, odd 04 + 4 = 8 2. Last 3 digits = 104 104 = 13 x **8**
9	Add all the digits in the number Repeat until sum is 2-digits 2-digits divisible by 9	24,343,785: 2+4+3+4+3+7+8+5=36 36 = 4 x **9**
10	Last digit is 0	34,789,013,467,593,487,540: Last digit = 0
11	1. Alternately subtract then add the digits from L to R; result divisible by 11 2. Subtract last digit from the rest	918,082: 1. 9 - 1 + 8 - 0 + 8 - 2 = 22 22 = 2x**11**. 2. 627: 62 - 7 = 55 55 = 5 x **11**
12	Divisible by 3 **and** 4	324: 3 + 2 + 4 = 9, divisible by **3** 24 divisible by **4**
13	Iteratively add 4 times the last digit to the rest	637: 63 + (7 × 4) = 91, 9 + (1 × 4) = **13**
14	Divisible by 2 **and** 7	

15	Divisible by 3 **and** 5	
16	If the thousands digit is even, take the last three digits If the thousands digit is odd, add 8 to the last three digits With the 3-digit #, multiply hundreds digit by 4, then add the last two digits.	254,176: Thousands digit = 4, so take 176 (1 × 4) + 76 = 80 80 = 5 x **16** 693,408: Thousands digit = 3, 408 + 8 = 416 (4 x 4) + 16 = 32 32 = 2 x **16**
17	Subtract 5 times the last digit from the rest.	221: 22 - (1 × 5) = **17**
18	Divisible by 2 **and** 9	
19	Add twice the last digit to the rest	437: 43 + (7 × 2) = 57 5 + (7 x 2) = **19**
20	Divisible by 10 and the tens digit is even	360: last digit = 0 and 6 is even
25	Last 2 digits = 25, 50 or 75	
50	Last 2 digits = 50 or 00	

Example 1: Check if 64 is divisible by 2.
The last digit of 64 is 4 (even).
Hence 64 is divisible by 2

Example 2: Check if 69 is divisible by 2.
The last digit of 69 is 9 (not even).
Hence 69 is not divisible by 2

Example 3: Check if 387 is divisible by 3.
3 + 8 + 7 = 18.
18 is divisible by 3.
Hence 387 is divisible by 3

Example 4: Check if 421 is divisible by 3.
4 + 2 + 1 = 7.
7 is not divisible by 3.
Hence 421 is not divisible by 3

Example 5: Check if 416 is divisible by 4.
Number formed by the last two digits = 16.
16 is divisible by 4.
Hence 416 is divisible by 4

Example 6: Check if 481 is divisible by 4.

Number formed by the last two digits = 81.
81 is not divisible by 4.
Hence 481 is not divisible by 4

Example 7: Check if 305 is divisible by 5.
Last digit is 5.
Hence 305 is divisible by 5.

Example 8: Check if 420 is divisible by 5.
Last digit is 0.
Hence 420 is divisible by 5.

Example 9: Check if 546 is divisible by 6.
546 is divisible by 2.
546 is also divisible by 3. *(Refer divisibility rule of 2 and 3)*
Hence 546 is divisible by 6

Example 10: Check if 635 is divisible by 6.
635 is not divisible by 2.
635 is also not divisible by 3.
Hence 635 is not divisible by 6

Example 11: Check if 349 is divisible by 7.
Given number = 349
34 - (9 × 2) = 34 - 18 = 16
16 is not divisible by 7.
Hence 349 is not divisible by 7

Example 12: Check if 3374 is divisible by 7.
Given number = 3374
337 - (4 × 2) = 337 - 8 = 329
32 - (9 × 2) = 32 - 18 = 14
14 is divisible by 7.
Hence 329 is also divisible by 7.
Hence 3374 is also divisible by 7.

Example 13: Check if 7624 is divisible by 8.
The number formed by the last three digits of 7624 = 624.
624 is divisible by 8.
Hence 7624 is also divisible by 8.

Example 14: Check if 129437464 is divisible by 8.
The number formed by the last three digits of 129437464 = 464.
464 is divisible by 8.
Hence 129437464 is also divisible by 8.

Example 15: Check if 367821 is divisible by 9
3 + 6 + 7 + 8 + 2 + 1 = 27
27 is divisible by 9.
Hence 367821 is also divisible by 9.

Example 16: Check if 47128 is divisible by 9
4 + 7 + 1 + 2 + 8 = 22
22 is not divisible by 9.
Hence 47128 is not divisible by 9.

Example 17: Check if 2570 is divisible by 10.
Last digit is 0.
Hence 2570 is divisible by 10.

Example 18: Check if 5462 is divisible by 10.
Last digit is not 0.
Hence 5462 is not divisible by 10

Example 19: Check if 85136 is divisible by 11.
8 + 1 + 6 = 15
5 + 3 = 8
15 - 8 = 7
7 is not divisible by 11.
Hence 85136 is not divisible by 11.

Example 20: Check if 2737152 is divisible by 11.
2 + 3 + 1 + 2 = 8
7 + 7 + 5 = 19
19 - 8 = 11
11 is divisible by 11.
Hence 2737152 is also divisible by 11.

Example 21: Check if 720 is divisible by 12.
720 is divisible by 3.
720 is also divisible by 4. *(Refer divisibility rules of 3 and 4)*
Hence 720 is divisible by 12

1.1.4. Factors of a number

What are Factors of a Number and how to find it out?
If one number is divisible by a second number, the second number is a factor of the first number.

The lowest factor of any positive number = 1

The highest factor of any positive number = the number itself.

Example

The factors of 36 are 1, 2, 3, 4, 6, 9 12, 18, 36 because each of these numbers divides 36 with a remainder of 0

How to find out factors of a number

Write down 1 and the number itself (lowest and highest factors).

Check if the given number is divisible by 2 (Reference: Divisibility by 2 rule)

If the number is divisible by 2, write down 2 as the second lowest factor and divide the given number by 2 to get the second highest factor

Check for divisibility by 3, 4,5, and so on. till the beginning of the list reaches the end

Example 1: Find out the factors of 72

Write down 1 and the number itself (72) as lowest and highest factors.

1 . . . 72

72 is divisible by 2 (Reference: Divisibility by 2 Rule).
72 ÷ 2 = 36. Hence 2^{nd} lowest factor = 2 and 2^{nd} highest factor = 36. So we can write as

1, 2 . . . 36, 72

72 is divisible by 3 (Reference: Divisibility by 3 Rule).
72 ÷ 3 = 24 . Hence 3^{rd} lowest factor = 3 and 3^{rd} highest factor = 24. So we can write as

1, 2, 3, . . . 24, 36, 72

72 is divisible by 4 (Reference: Divisibility by 4 Rule).
72 ÷ 4 = 18. Hence 4^{th} lowest factor = 4 and 4^{th} highest factor = 18. So we can write as

1, 2, 3, 4, . . . 18, 24, 36, 72

72 is not divisible by 5 (Reference: Divisibility by 5 Rule)

72 is divisible by 6 (Reference: Divisibility by 6 Rule).
72 ÷ 6 = 12. Hence 5^{th} lowest factor = 6 and 5^{th} highest factor = 12. So we can write as

1, 2, 3, 4, 6, . . . 12, 18, 24, 36, 72

72 is not divisible by 7 (Reference: Divisibility by 7 Rule)

72 is divisible by 8 (Reference: Divisibility by 8 Rule).
72 ÷ 8 = 9. Hence 6th lowest factor = 8 and 6th highest factor = 9.

Now our list is complete and the factors of 72 are

1, 2, 3, 4, 6, 8, 9 12, 18, 24, 36, 72

Example 2: Find out the factors of 22

Write down 1 and the number itself (22) as lowest and highest factors

1 . . . 22

22 is divisible by 2 (Reference: Divisibility by 2 Rule).
22 ÷ 2 = 11. Hence 2nd lowest factor = 2 and 2nd highest factor = 11. So we can write as

1, 2 . . . 11, 22

22 is not divisible by 3 (Reference: Divisibility by 3 Rule).

22 is not divisible by 4 (Reference: Divisibility by 4 Rule).

22 is not divisible by 5 (Reference: Divisibility by 5 Rule).

22 is not divisible by 6 (Reference: Divisibility by 6 Rule).

22 is not divisible by 7 (Reference: Divisibility by 7 Rule).

22 is not divisible by 8 (Reference: Divisibility by 8 Rule).

22 is not divisible by 9 (Reference: Divisibility by 9 Rule).

22 is not divisible by 10 (Reference: Divisibility by 10 Rule).

Now our list is complete and the factors of 22 are

1, 2, 11, 22

Important Properties of Factors

If a number is divisible by another number, then it is also divisible by all the factors of that number.

Example: 108 is divisible by 36 because 106 ÷ 38 = 3 with remainder of 0.

The factors of 36 are 1, 2, 3, 4, 6, 9 12, 18, 36 because each of these numbers divides 36 with a remainder of 0.

Hence, 108 is also divisible by each of the numbers 1, 2, 3, 4, 6, 9, 12, 18, 36.

1.1.5. Prime Numbers and Composite Numbers

Prime Numbers

A prime number is a positive integer that is divisible by itself and 1 only. Prime numbers will have exactly two integer factors.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

Please note the following facts

Zero is not a prime number because zero is divisible by more than two numbers. Zero can be divided by 1, 2, 3 etc.
($0 \div 1 = 0$, $0 \div 2 = 0$ ...)

One is not a prime number because it does not have two factors. It is divisible by only 1

Composite Numbers

Composite numbers are numbers that have more than two factors. A composite number is divisible by at least one number other than 1 and itself.

Examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc.

Please note that zero and 1 are neither prime numbers nor composite numbers.

Every whole number is either prime or composite, with two exceptions 0 and 1 which are neither prime nor composite

1.1.6. Prime Factorization and Prime factors

Prime factor

The factors which are prime numbers are called prime factors

Prime factorization

Prime factorization of a number is the expression of the number as the product of its prime factors.

Example 1:
Prime factorization of 280 can be written as $280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$ and the prime factors of 280 are 2, 5 and 7

Example 2:
Prime factorization of 72 can be written as $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ and the prime factors of 72 are 2 and 3

How to find out prime factorization and prime factors of a number

Repeated Division Method : In order to find out the prime factorization of a number, divide the number repeatedly by the smallest prime number possible(2,3,5,7,11, ...) until the quotient is 1.

Example 1: Find out prime factorization of 280

$$\begin{array}{r|c}
2 & 280 \\ \hline
2 & 140 \\ \hline
2 & 70 \\ \hline
5 & 35 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

Hence, prime factorization of 280 can be written as
$280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$
and the prime factors of 280 are 2, 5 and 7

Example 2: Find out prime factorization of 72

$$\begin{array}{r|c}
2 & 72 \\ \hline
2 & 36 \\ \hline
2 & 18 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}$$

Hence, prime factorization of 72 can be written as $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ and the prime factors of 72 are 2 and 3

Note: Every whole number greater than 1 can be uniquely expressed as the product of its prime factors. For example, $700 = 2^2 \times 5^2 \times 7$

1.1.7. Multiples

Multiples of a whole number are the products of that number with 1, 2, 3, 4, and so on

Example: Multiples of 3 are 3, 6, 9, 12, 15, ...

If a number x divides another number y exactly with a remainder of 0, we can say that x is a factor of y and y is a multiple of x

For instance, 4 divides 36 exactly with a remainder of 0. Hence 4 is a factor of 36 and 36 is a multiple of 4

1.1.8. Comparing fractions

Type 1: Fractions with same denominators.

Example 1: Compare 3/5 and 1/5

These fractions have same denominator. So just compare the numerators. Bigger the numerator, bigger the number.

3 > 1. Hence 3/5>1/5

Example 2: Compare 2/7 and 3/7 and 8/7

These fractions have same denominator. So just compare the numerators. Bigger the numerator, bigger the number.

8 > 3 > 2. Hence 8/7>3/7>2/7

Type 2: Fractions with same numerators

Example 1: Compare 3/5 and 3/8

These fractions have same numerator. So just compare the denominators. Bigger the denominator, smaller the number.

8 > 5. Hence 3/8<3/5

Example 2: Compare 7/8 and 7/2 and 7/5

These fractions have same numerator. So just compare the denominators. Bigger the denominator, smaller the number.

8 > 5 > 2. Hence 7/8<7/5<7/2

Type 3: Fractions with different numerators and denominators

Example 1: Compare 3/5 and 4/7

To compare such fractions, find out LCM of the denominators. Here, LCM(5, 7) = 35

Now , convert each of the given fractions into an equivalent fraction with 35 (LCM) as the denominator.

The denominator of 3/5 is 5.   5 needs to be multiplied with 7 to get 35. Hence,

The denominator of 4/7 is 7.   7 needs to be multiplied with 5 to get 35. Hence,

21/35>20/35   Hence, 3/5>4/7

1.1.9. Co-prime Numbers or Relatively Prime Numbers

Two numbers are said to be co-prime (also spelled co-prime) or relatively prime if they do not have a common factor other than 1. i.e., if their HCF is 1.

Example 1: 3, 5 are co-prime numbers (Because HCF of 3 and 5 = 1)

Example 2: 14, 15 are co-prime numbers (Because HCF of 14 and 15 = 1)

A set of numbers is said to be pairwise co-prime (or pairwise relatively prime) if every two distinct numbers in the set are co-prime

Example 3: The numbers 10, 7, 33, 13 are pairwise co-prime, because HCF of any pair of the numbers in this is 1.
HCF (10, 7) = HCF (10, 33) = HCF (10, 13) = HCF (7, 33) = HCF (7, 13) = HCF (33, 13) = 1.

Example 4: The numbers 10, 7, 33, 14 are not pairwise co-prime because HCF(10, 14) = 2 ≠ 1 and HCF(7, 14) = 7 ≠ 1.

Note:-
1. If a number is divisible by two co-prime numbers, then the number is divisible by their product also.

2. If a number is divisible by more than two pairwise co-prime numbers, then the number is divisible by their product also.

Example 5: Check if the number 14325 is divisible by 15.
3, 5 are co-prime numbers (Because HCF of 3 and 5 = 1)

14325 is divisible by 3 and 5.
$3 \times 5 = 15$
Hence 14325 is divisible by 15 also.

Example 6: Check if the number 1440 is divisible by 60.
The numbers 3, 4, 5 are pairwise co-prime because HCF of any pair of numbers in this is 1.
1440 is divisible by 3, 4 and 5.
$3 \times 4 \times 5 = 60$. Hence 1440 is also divisible by 60.

Exercise

1.
Which one of the following is not a prime number?

A. 31

B. 61

C. 71

D. 91

Explanation:
91 is divisible by 7. So, it is not a prime number.

2.
The sum of first five prime numbers is:

A. 11

B. 18

C. 26

D. 28

Explanation:

Required sum = $(2 + 3 + 5 + 7 + 11) = 28$.

Note: 1 is not a prime number.

3.
The smallest 3 digit prime number is:

A. 101

B. 103

C. 109

D. 113

Explanation:

The smallest 3-digit number is 100, which is divisible by 2.

∴ 100 is not a prime number.

101 < 11 and 101 is not divisible by any of the prime numbers 2, 3, 5, 7, 11.

∴ 101 is a prime number.

Hence 101 is the smallest 3-digit prime number.

4.
Which one of the following numbers is exactly divisible by 11?

A. 235641

B. 245642

C. 315624

D. 415624

Explanation:

(4 + 5 + 2) - (1 + 6 + 3) = 1, not divisible by 11.

(2 + 6 + 4) - (4 + 5 + 2) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 3) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 4) = 0, So, 415624 is divisible by 11.

5.
Which of the following number is divisible by 24 ?

A. 35718

B. 63810

C. 537804

D. 3125736

Explanation:

24 = 3 x8, where 3 and 8 co-prime.

Clearly, 35718 is not divisible by 8, as 718 is not divisible by 8.

Similarly, 63810 is not divisible by 8 and 537804 is not divisible by 8.

Consider option (D),

Sum of digits = (3 + 1 + 2 + 5 + 7 + 3 + 6) = 27, which is divisible by 3.

Also, 736 is divisible by 8.

$\therefore$ 3125736 is divisible by (3 x 8), *i.e.*, 24.

6.
The difference between a positive proper fraction and its reciprocal is 9/20. The fraction is:

A. $\frac{3}{5}$

B. $\frac{3}{10}$

C. $\frac{4}{5}$

D. $\frac{4}{3}$

Explanation:

Let the required fraction be x. Then $\frac{1}{x} - x = \frac{9}{20}$

$\therefore \quad \frac{1 - x^2}{x} = \frac{9}{20}$

$\Rightarrow \quad 20 - 20x^2 = 9x$

$\Rightarrow \quad 20x^2 + 9x - 20 = 0$

$\Rightarrow \quad 20x^2 + 25x - 16x - 20 = 0$

$\Rightarrow \quad 5x(4x + 5) - 4(4x + 5) = 0$

$\Rightarrow \quad (4x + 5)(5x - 4) = 0$

$x = \frac{4}{5}$

7.
On dividing a number by 5, we get 3 as remainder. What will the remainder when the square of the this number is divided by 5 ?

A. 0

B. 1

C. 2

D. 4

Explanation:
Let the number be x and on dividing x by 5, we get k as quotient and 3 as remainder.
$\therefore \quad x = 5k + 3$
$\Rightarrow \quad x^2 = (5k + 3)^2$
$= (25k^2 + 30k + 9)$
$= 5(5k^2 + 6k + 1) + 4$
$\therefore$ On dividing x^2 by 5, we get 4 as remainder.

8.
The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number?

A. 240

B. 270

C. 295

D. 360

Explanation:
Let the smaller number be x. Then larger number = (x + 1365).
$\therefore \quad x + 1365 = 6x + 15$
$\Rightarrow 5x = 1350$
$\Rightarrow x = 270$

$\therefore$ Smaller number = 270.

9.
If the number 517*324 is completely divisible by 3, then the smallest whole number in the place of * will be:

A. 0

B. 1

C. 2

D. None of these

Explanation:
Sum of digits = (5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x), which must be divisible by 3.
$\therefore \quad x = 2$.

10.
On dividing a number by 357, we get 39 as remainder. On dividing the same number 17, what will be the remainder ?

A. 0

B. 3

C. 5

D. 11

Explanation:
Let x be the number and y be the quotient. Then,
$x = 357 \times y + 39$
$= (17 \times 21 \times y) + (17 \times 2) + 5$
$= 17 \times (21y + 2) + 5)$

$\therefore$ Required remainder = 5.

1.2 LCM and HCF

1.2.1 LCM and HCF

Least common multiple is a number which is multiple of two or more than two numbers.

For example: The common multiples of 3 and 4 are 12, 24 and so on. Therefore, l.c.m.is smallest positive number that is multiple of both. Here, l.c.m. is 12.

Highestcommon factors are those integral values of number that can divide that number.

For example: The common Factors of 24 and 36 are 3 and 4. Therefore, h.c.f .is highest positive number which is multiplication of all common factors that is 3*4=12.

Some important LCM and HCF tricks:

1) Product of two numbers = Their HCF × Their LCM
2) HCF of given numbers always divides their LCM
3) HCF = HCF of Numerators/LCM of Denominators
4) LCM = LCM of Numerators/HCF of Denominators
5) If d is the HCF of two positive integer a and b, then there exist unique integer such that d=am+bn
6) If p is prime and a, b are any integer then P/ab, This implies P/a or P/b
7) HCF of a given number always divides its LCM

Some Properties of HCF, LCM and Remainder

1) Largest number which divides x, y, z to leave same remainder is HCF (x - y, y - z, z – x)
2) Largest number which divides x, y, z to leave remainder a,b,c is HCF (x-a, y -b, z – c)
3) Least number when divided by x, y, z and leaves a remainder R in eachcase = LCM (x, y, z) + R
4) Least number when divided by x, y, z leaves a remainders a, b, c and if x-a = y-b = z-c = k, is LCM(x,y,z)-k.

Example 1: Least number which when divided by 35, 45, 55 and leaves remainder18,28,38 is?
Solution: i) In this case we will evaluate LCM
ii) Here the difference between every divisor and remainder is same i.e. 17.
Therefore, required number = LCM of (35, 45, 55)-17 = (3465-17)= 3448.

Example 2: Least number which when divided by 5, 6, 7, 8 and leaves remainder 3, butwhen divided by 9, leaves no remainder?
Solution:
l.c.m. of 5, 6, 7, 8 =840
Required number = 840 k + 3
Least value of k for which (840 k + 3) is divided by 9
is 2Therefore, required number = 840 × 2 + 3
= 1683

Example 3: Greater number of 4 digits which is divisible by each one of 12, 18, 21 and 28is?
Solution: LCM of 12, 18, 21, 28 = 254
Therefore, required number must be divisible by
254Greatest four digit number = 9999
On dividing 9999 by 252, remainder = 171
Therefore, 9999-171 = 9828

Exercise:

1. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
   a) 4
   b) 7
   c) 9
   d) 13

   Answer: (a)

2. The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is:

a) 276

b) 299

c) 322

d) 345

Answer: (c)

3. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is:

a) 4

b) 5

c) 6

d) 8

Answer: (a)

4. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

a) 101

b) 107

c) 111

d) 185

Answer: (c)

5. The G.C.D. of 1.08, 0.36 and 0.9 is:

a) 0.03

b) 0.9

c) 0.18

d) 0.108

Answer: (c)

6. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

a) 1

b) 2

c) 3

d) 4

Answer: (b)

7. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:

a) 74

b) 94

c) 184

d) 364
Answer: (d)

8. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

a) 4
b) 7
c) 9
d) 13

Answer: (a)
Explanation:

Required number = H.C.F. of (91 - 43), (183 - 91) and (183 - 43)

= H.C.F. of 48, 92 and 140 = 4.

9. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

a) 4
b) 10
c) 15
d) 16

Answer: (d)
Explanation:

L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together $\frac{30}{2} + 1 = 16$ times.

10. The G.C.D. of 1.08, 0.36 and 0.9 is:

a) 0.03
b) 0.9
c) 0.18
d) 0.108

Answer: (c)
Explanation: Given numbers are 1.08, 0.36 and 0.90. H.C.F. of 108, 36 and 90 is 18,

$\therefore$ H.C.F. of given numbers = 0.18.

1.3 Simplification and Approximation

1.3.1 BODMAS RULE

Mathematical expressions with multiple operators need to be solved from left to right in the order of BODMAS.

Brackets	→	(	or	)	←	Parenthesis
Order	→	$\sqrt{\ }$	or	x^2	←	Exponents
Division	→	÷	or	x	←	Multiplication
Multiplication	→	x	or	÷	←	Division
Addition	→	+	or	+	←	Addition
Subtraction	→	-	or	-	←	Subtraction

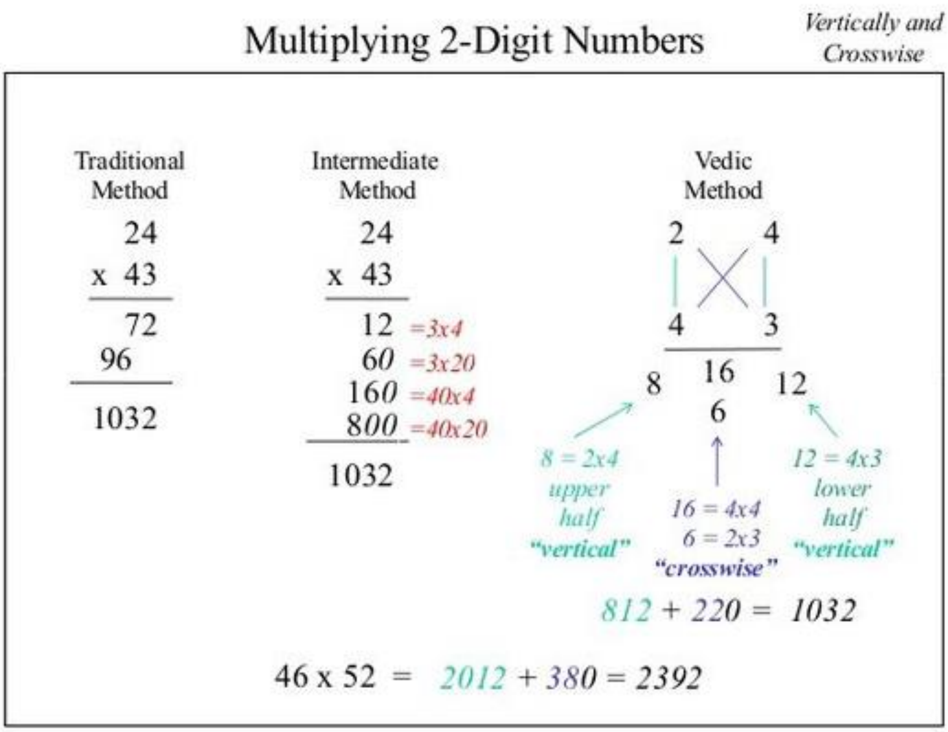
1.3.2 Modulus of a Real Number:

Modulus of a real number a is defined as

$$|a| = \begin{cases} a, \text{ if } a > 0 \\ -a, \text{ if } a < 0 \end{cases}$$

Thus, |5| = 5 and |-5| = -(-5) = 5.

1.3.3 Multiplicaion of two digit numbers



Exercise:

1. 21x11 Ans: 231

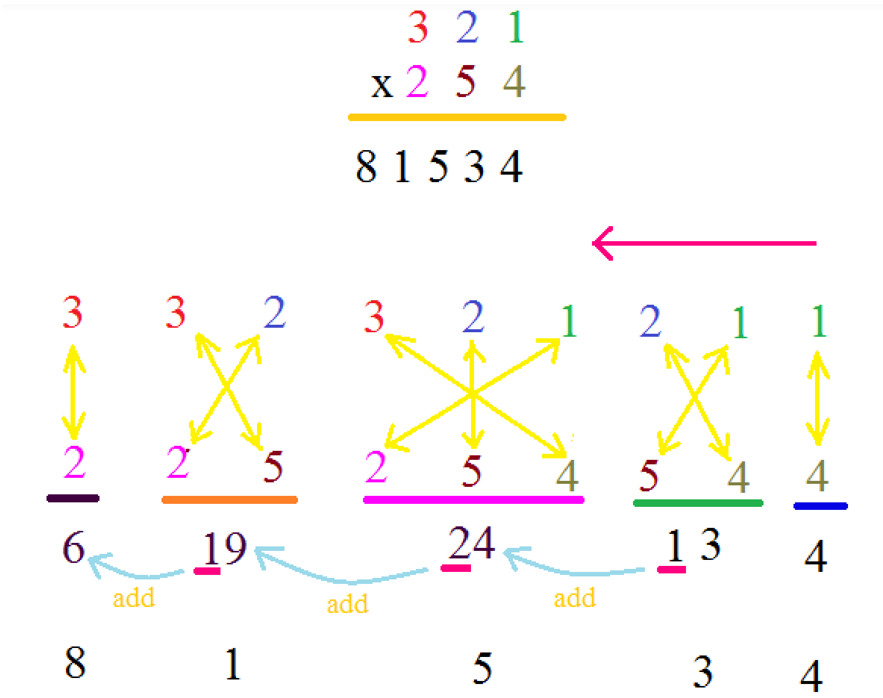
2. 43x22 Ans: 946

3.46x12 Ans: 552

4.23x76 Ans: 1748

5. 64x45 Ans: 2880

1.3.3 Multiplicaion of three digit numbers



Exercise:

1. 212x111 Ans: 23532

2. 413x212 Ans: 87556

3.146x112 Ans: 16352

4.231x176 Ans: 40656

5. 164x145 Ans: 23780

Exercise

1.
(800 ÷ 64) x (1296 ÷ 36) = ?

A. 420

B. 460

C. 500

D. 540

E. None of these

: Option E
Explanation:

Given Exp. = $\frac{800}{64}$ x $\frac{1296}{36}$ = 450

2.
107 x 107 + 93 x 93 = ?

A. 19578

B. 19418

C. 20098

D. 21908

E. None of these

: Option C
Explanation:

107 x 107 + 93 x 93 = $(107)^2 + (93)^2$

= $(100 + 7)^2 + (100 - 7)^2$

= $2 \times [(100)^2 + 7^2]$ [Ref: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

= 20098

3.
(?) + 3699 + 1985 - 2047 = 31111

A. 34748

B. 27474

C. 30154

D. 27574

E. None of these

: Option B
Explanation:
$x + 3699 + 1985 - 2047 = 31111$
$\Rightarrow \quad x + 3699 + 1985 = 31111 + 2047$
$\Rightarrow \quad x + 5684 = 33158$
$\Rightarrow \quad x = 33158 - 5684 = 27474.$

4.
$$\frac{753 \text{ x } 753 + 247 \text{ x } 247 - 753 \text{ x } 247}{753 \text{ x } 753 \text{ x } 753 + 247 \text{ x } 247 \text{ x } 247} = ?$$

A. $\frac{1}{1000}$

B. $\frac{1}{506}$

C. $\frac{253}{500}$

D. None of these

: Option A
Explanation:
$$\text{Given Exp.} = \frac{(a^2 + b^2 - ab)}{(a^3 + b^3)} = \frac{1}{(a + b)} = \frac{1}{(753 + 247)} = \frac{1}{1000}$$

5.
107 x 107 + 93 x 93 = ?

A. 19578

B. 19418

C. 20098

D. 21908

E. None of these

: Option C
Explanation:
$107 \text{ x } 107 + 93 \text{ x } 93 = (107)^2 + (93)^2$

$= (100 + 7)^2 + (100 - 7)^2$

$= 2 \times [(100)^2 + 7^2]$ [Ref: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 20098$

6.

$$\frac{(489 + 375)^2 - (489 - 375)^2}{(489 \times 375)} = ?$$

A. 144

B. 864

C. 2

D. 4

E. None of these

: Option D

Explanation:

Given Exp. $= \dfrac{(a + b)^2 - (a - b)^2}{ab} = \dfrac{4ab}{ab} = 4$

7.

The sum of the two numbers is 12 and their product is 35. What is the sum of the reciprocals of these numbers ?

A. $\dfrac{12}{35}$

B. $\dfrac{1}{35}$

C. $\dfrac{35}{8}$

D. $\dfrac{7}{32}$

: Option A

Explanation:

Let the numbers be a and b. Then, $a + b = 12$ and $ab = 35$.

$\therefore \dfrac{a + b}{ab} = \dfrac{12}{35} \Rightarrow \left(\dfrac{1}{b} + \dfrac{1}{a}\right) = \dfrac{12}{35}$

$\therefore$ Sum of reciprocals of given numbers = <u>12/35</u>

8.
If a and b are odd numbers, then which of the following is even ?

A. $a + b$

B. $a + b + 1$

C. ab

D. $ab + 2$

E. None of these

: Option A
Explanation:
The sum of two odd number is even. So, a + b is even.

9.
1904 x 1904 = ?

A. 3654316

B. 3632646

C. 3625216

D. 3623436

E. None of these

: Option C
Explanation:

$$
\begin{aligned}
1904 \times 1904 &= (1904)^2 \\
&= (1900 + 4)^2 \\
&= (1900)^2 + (4)^2 + (2 \times 1900 \times 4) \\
&= 3610000 + 16 + 15200. \\
&= 3625216.
\end{aligned}
$$

10.
The difference of the squares of two consecutive odd integers is divisible by which of the following integers ?

A. 3

B. 6

C. 7

D. 8

: Option D
Explanation:
Let the two consecutive odd integers be $(2n + 1)$ and $(2n + 3)$. Then,
$(2n + 3)^2 - (2n + 1)^2 = (2n + 3 + 2n + 1)(2n + 3 - 2n - 1)$
$= (4n + 4) \times 2$
$= 8(n + 1)$, which is divisible by 8.