Parul University
Faculty of Engineering and Technology
Department of Applied Science & Humanities
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Subject: Quant and Reasoning (303105311)
Branch: CSE/ IT

Unit-2: Averages and Progressions

2.1 Average

$$Average = \frac{Sum\ of\ quantities}{Number\ of\ quantities}$$

Example-1: A man purchasedd 5 toys at Rs.220 each, 6 toys at Rs. 250 each and 9 toys at Rs. 300 each. Calculate the average cost of 1 toy.

Solution: Price of 5 toys = 200 ×5 = 1000

Price of 6 toys = 250×6 = 1500

Price of 9 toys = 300×9 = 2700

Total number of toys = 5 + 6 + 9 = 20

Average Price of 1 toy = $\frac{1000+1500+2700}{20}$ = 260.

Shorcut method

1. The Average of two or more groups taken together.
(a)If the number of quantities in two groups are $n1$ and $n2$ and their average is x and y, respectively, the combined average (average of all of them put together) is $\frac{n_1x+n_2y}{n_1+n_2}$.
(b) If the average of $n1$ quantities is x, and the average of n_2 quantities out of them is y, the average of the remaining group (rest of the quantities) is $\frac{n_1x-n_2y}{n_1-n_2}$.
2. If x is the average of $x_1, x_2, \ldots, x_n$, then
(a) The average of $x_1 + a, x_2 + a, \ldots, x_n + a$ is $x + a$.
(b) The average of $x_1 - a, x_2 - a, \ldots x_n - a$ is $x - a$.
(c) The average of $ax_1, ax_2, \ldots, ax_n$ is ax, provided $a \neq 0$.
(d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots, \frac{x_n}{a}$, is $\frac{x}{a}$ provided $a \neq 0$.

Example-2: The average weight of 24 students of section *A* of a class is 58 Kg, whereas the average weight of 26 students of section *B* of the same class is 60.5 Kg. Find out average weight of all the 50 students of the class.
Solution: Here, $n_1 = 24, n_2 = 26, x = 58\ and\ y = 60.5$

$$\therefore The\ average\ weight\ of\ all\ 50\ students = \frac{n_1 x + n_2 y}{n_1 + n_2}$$

$$= \frac{24 \times 58 + 26 \times 60.5}{24 + 26}$$

$$= \frac{1392 + 1573}{50}$$

$$= \frac{2965}{50} = 59.3\ Kg$$

Example-3: Average salary of all the 50 employees including 5 officers is Rs. 850. If the average salary of the officers is Rs. 2500, find the average salary of the remaining staff of the company.
Solution: Here, $n_1 = 50, n_2 = 5, x = 850\ and\ y = 2500$

$$\therefore The\ average\ salary\ of\ the\ remaining\ staff = \frac{n_1 x - n_2 y}{n_1 - n_2}$$

$$= \frac{50 \times 8500 - 5 \times 2500}{50 - 5}$$

$$= \frac{42500 - 12500}{45}$$

$$= \frac{30000}{45} = 667\ Kg$$

Example-4: The Average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Solution: The new average = 19+8 =27

Example-5: The average of *x* numbers is 5*x*. If each number is multiplied by 8, find the average of a new set of numbers.
Solution: The average of a new set of numbers = a*x* = 8×21= 168

Exercise

1. A grocer has a sale of Rs 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs, 6500 ?

A) 4991 B) 5467

C) 5987 D) 6453

Answer: A) 4991
Explanation: Total sale for 5 months = Rs. (6435 + 6927 + 6855 + 7230 + 6562) = Rs. 34009.

Required sale = Rs.[(6500 x 6) - 34009]

= Rs. (39000 - 34009)

= Rs. 4991.

2. The average of runs of a cricket player of 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ?

A) 76 B) 79

C) 85 D) 87

Answer: A) 76
Explanation: Average = total runs / no.of innings = 32

So, total = Average x no.of innings = 32 x 10 = 320.

Now increase in avg = 4runs. So, new avg = 32+4 = 36runs

Total runs = new avg x new no. of innings = 36 x 11 = 396

Runs made in the 11th inning = 396 - 320 = 76

3. A pupil's marks were wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half. The number of pupils in the class is :

A) 45 B) 40

C) 39 D) 37

Answer: B) 40

Explanation: Let there be x pupils in the class.

Total increase in marks = (x * 1/2) = x/2.

x/2 = (83 - 63) => x/2 = 20 => x = 40.

4. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person ?

A) 70 kg B) 75 kg

C) 80 kg D) 85 kg

Answer: D) 85 kg

Explanation: Total weight increased = (8 x 2.5) kg = 20 kg.

Weight of new person = (65 + 20) kg = 85 kg.

5. The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers :

A) 4 B) 8

C) 12 D) 16

Answer: B) 8
Explanation: Let the numbers be x, x + 2, x + 4, x + 6 and x + 8.

Then [x + (x + 2) + (x + 4) + (x + 6) + (x + 8)] / 5 = 61.

or 5x + 20 = 305 or x = 57. So, required difference = (57 + 8) - 57 = 8

Sequence: Succession of numbers arranged in a definite order forming a definite pattern is known as sequence.

Series: If $a_1 , a_2 , a_3 , a_4 , \dots , a_n$, ... is a sequence, then the expression

$a_1 + a_2 + a_3 + a_4 + \dots + a_n + \dots$ is a series.

A series is finite or infinite according to as the number of terms in the corresponding sequence is finite or infinite.

Progressions: Those sequences whose terms follow certain patterns are called progressions. Arithmetic Progression (A.P.): A sequence is called an Arithmetic Progression if the difference between two consecutive terms is always same.

i.e., $a_{n+1} - a_n = constant = d$ for all $n \in N$

The constant difference, generally denoted by 'd' is called the common difference.

a_n is called the n^{th} or last term of an A.P.

$a_n = l = a + (n - 1)d$

(i) Three consecutive, terms in an A.P are taken as $a - d, a, a + d$.
(ii) Four consecutive terms in an A.P taken as $a - 3d, a - d, a + d, a + 3d$.

Note : If each term of an A.P. is (increased/ decreased) by K then A.M. is also (increased/ decreased) by K.

If each term of an A.P. is (multiplied/Divided) by K, then A.M is also (multiplied/Divided) by same number K.

Rule 1. Let a be the first term and d be the common difference of an A.P. Then its n^{th} term is $a + (n - 1)d$ i.e., $a_n = a + (n - 1)d$.

Rule 2. The sum S_n of n terms of an A.P. with first term is 'a' and common difference is 'd' is $S_n = \frac{n}{2}[2a + (n - 1)d] \ or \ S_n = \frac{n}{2}[a + l]$, where $l = last \ term = a + (n - 1)d$.

Geometric Progression :

A sequence of non-zero numbers is called a Geometric Progression (abbreviated as G.P.) if the ratio of a term and the term preceding to it is always same. The constant ratio is called the common ratio (r) of the G.P.

In other words, a sequence $a_1 , a_2 , a_3 , \dots a_n$ is called a Geometric Progression if

$$\frac{a_{n+1}}{a_n} = Constant \; \forall \; n \in N$$

Three numbers in G.P is taken as $a, ar, ar^2 \; or \; \frac{a}{r}, a, ar$

Geometric Series : If $a_1, a_2, a_3, a_4, \ldots, a_n$, ... is a G.P., then the expression

$a_1 + a_2 + a_3 + a_4 + \ldots + a_n + \ldots$ is called a geometric series.

Rule 3. The n^{th} term of a G.P. with first term a and common ratio r is given by

$a_n = ar^{n-1}$.

Rule 4. The sum of n terms of a G.P. with first term 'a' and common ratio 'r' is given by

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \; for \; r < 1$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right) \; for \; r > 1$$

In fact these two are exactly identical. The only thing which must be noted is that the above formulas do not hold for $r = 1$, the sum of n the sum of n terms of the G.P. is $S_n = na$. where $r = 1$.

Rule 5. The sum of an infinite G.P. with 1st term is 'a' and common ratio is $r(-1 < r < 1$ i.e., $|r| < 1)$ is given by

$$S_\infty = \frac{a}{1-r}$$

Rule 6. Three non-zero numbers a, b, c are in G.P. if $b^2 = ac$ or $= \sqrt{ac}$.

Here, b is known as the Geometric Mean of $a \; and \; c$.

Harmonic Progression: If a, b, c, d, are in H.P. then, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ will form an A.P. and then we can apply all rules of A.P.

$$n^{th} \; term \; of \; HP = \frac{1}{n^{th} \; term \; of \; AP}$$

Note: No term of HP can be zero.

Some Special Sequences:

1) The sum of first n natural numbers,

$$\sum n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

2) The sum of squares of first n natural numbers

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3) The sum of cubes of first n natural numbers

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Exercise:

1. Show that the series 1,3,5,7,9, ...in Arithmetic progression
2. Find the $\mathrm{n^{th}}$ term and 19^{th} term of the sequence $5, 2, -1, -4, \ldots$
3. Find the sum of the series $.5 + .51 + .52 + \cdots$ to 100 terms.
4. Find the sum of 20 terms of an A. P. whose first term is 3 and last term is 57.
5. Find the sum of 8 terms and n terms of the sequence $9, -3, 1, -\frac{1}{3}, \ldots$
6. Find the 100th of the squence$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$

Exercise

1. Determine 25th term of an A.P. whose 9th term is -6 and common difference is 5/4.

(a) 16 (b) 18 (c) 12 (d) 14

Answer: (d) 14

Explanation: $d = \frac{5}{4}, a_9 = -6$

$a_n = a + (n-1)d \Rightarrow a_9 = a + 8d \Rightarrow -6 = a + 8\left(\frac{5}{4}\right)$

$\Rightarrow a = -16$

The 25^{th} term $a_{25} = a + 24d$

$\Rightarrow a_{25} = -16 + 24\left(\frac{5}{4}\right) = 14$

2. Which term of the A.P. 5, 13, 21, ... is 181?

(a) 21^{st} (b) 22^{nd} (c) 23^{rd} (d) 24^{th}

Answer: (c) 23rd

Explanation: Here, first term $a = 5$

Common difference $d = 8$

Let, 181 be the nth, i.e., $a_n = 181$

$\therefore$ 181 = 5 + (n − 1)8 or, 176 = (n − 1)8

$\therefore$ n − 1 = 176 ÷ 8 = 22

$\therefore$ n = 23

Hence, 181 is the 23rd term.

3. Determine *k* so that 2/3, *k* and 5k/8 are the three consecutive terms of an A.P.

(a) 16/ 33 (b) 14/33 (c) 12/33 (d) 18/33

Answer: (a) 16/ 33

Explanation: $\frac{2}{3}, k, \frac{5}{8}k$ be 3 terms of A.P.

Common difference between two consecutive terms is equal.

$$k - \frac{2}{3} = \frac{5}{8}k - k \Rightarrow 2k - \frac{5}{8}k = \frac{2}{3} \Rightarrow k = \frac{16}{33}$$

4. If the 12th term of an A.P. is -13 and the sum of the first four terms is 24, then what is the sum of the first 10 terms?

(a) 0 (b) 2 (c) 1 (d) 4

Answer: (a) 0

Explanation: $a_n = a + (n-1)d \Rightarrow a_{12} = a + 11d$

$\Rightarrow -13 = a + 11d$____(1)

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_4 = \frac{4}{2}[2a + 3d]$$

$\Rightarrow 12 = 2a + 3d$______(2)

Solving ___(1) and ___(2)

$a = 9, d = -2$

$$S_{10} = \frac{10}{2}[2a + 9d] = 0$$

5. How many terms are there in an A.P. whose first and fifth terms are -14 and 2, respectively and the sum of terms is 40?

(a) 15 (b) 5 (c) 10 (d) 20

Answer: (c)

Explanation: Here, $a = -14$

Let, *d* be the common difference

$a_5 = 2 \Rightarrow a + 4d = 2 \Rightarrow -14 + 4d = 2$

$\therefore d = 4$

Let 40 be the sum of *n* terms of this A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 40 = \frac{n}{2}[2(-14) + (n-1)4]$$

$\Rightarrow (n + 2)(n - 10) = 0$

$\therefore$ n = 10 or, -2. But $n \neq$ -2.

Hence, the required number of terms are 10.

6. Find the seventh term of the series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

(a) $\frac{1}{120}$ (b) $\frac{1}{128}$ (c) $\frac{1}{144}$ (d) $\frac{1}{121}$

Answer: (b)

7. How many terms are there in the G.P. 1, 2, 4, 8,…, 4096?

(a) 14 (b) 13 (c) 12 (d) 15

Answer: (b)

8. Find the sum of the series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \infty$.

(a) 1 (b) 12 (c) 15 (d) 13

Answer: (a)

9. What term of progression 18, -12, 8,…is $\frac{512}{729}$?

(a) 15 (b) 18 (c) 9 (d) 12

Answer: (c)

10. The 3rd term of G.P. is the square of the first term. If the 2nd term is 8, determine the 6th term.

(a) 136 (b) 132 (c) 128 (d) 124

Answer: (c)