EE 338 : Digital Signal Processing

Filter Design Assighnment

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Instructor: Prof. V M Gadre

Contents

1 Bandpass Filter Details

m = 43
Since m $\leq$ 75, therefore the passband will be monotonic
q(m) = 4
r(m) = 3
Passband is from $B_l(m)$ and $B_h(m)$
$B_l(m)$ = 150 + 17*4 + 13*3 = 257 kHz
$B_h(m)$ = $B_l(m)$ + 45 = 302 kHz
Transition band : 20 kHz Sampling Rate : 1200 kHz

1.1 Un-normalised discrete time filter specifications

- **Passband :** 257-302 kHz
- **Stopband :** 0-237 kHz on left side and 322-600 kHz on the right
- **Tolerance :** 0.15
- **Passband nature :** Monotonic
- **Stopband nature :** Monotonic

1.2 Normalized Digital Filter Specifications

Any frequency(Ω) upto half of the sampling rate can be represented on the normalized axis(ω) by:

$$\omega = \frac{\Omega * 2\pi}{SamplingRate} \tag{1}$$

Therefore the specifications for the normalized digital filter are:

- **Passband :** 0.4283π - 0.5033π
- **Stopband :** 0 - 0.3950π on left side and 0.5375π - π on the right
- **Passband nature :** Monotonic
- **Stopband nature :** Monotonic

1.3 Analog filter specifications using Bilinear Transformation

Bilinear transform is given as :

$$\Omega = tan\frac{\omega}{2} \tag{2}$$

Applying the transformation, we get:

ω	Ω
0	0
0.3950π	0.7149
0.4283π	0.7973
0.5033π	1.0112
0.5375π	1.1231
π	∞

Table 1: Bilinear Transformation

- **Passband :** 0.7973(Ω_{P1}) - 1.0112(Ω_{P2})
- **Transition band :** 0.7149 - 0.7973 and 1.0112 - 1.1231
- **Stopband :** 0-0.7149(Ω_{S1}) and 1.1231(Ω_{S2}) - ∞
- **Passband nature :** Monotonic
- **Stopband nature :** Monotonic

1.4 Frequency Transformation and Relevant Parameters

For bandpass filter, we need to convert the specifications into their low pass forms. This is done using the frequency transformation:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{3}$$

where the parameters Ω_0 and B are defined as follow:

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = \sqrt{0.7973 * 1.0112} = \mathbf{0.8979} \tag{4}$$

$$B = \Omega_{P2} - \Omega_{P1} = 1.0112 - 0.7973 = \mathbf{0.2139} \tag{5}$$

Applying the transformation, we get:

Ω	Ω_L
0	$-\infty$
0.7149 (Ω_{S1})	-1.9292 ($\Omega_{L_{S1}}$)
0.7973 (Ω_{P1})	-1 ($\Omega_{L_{P1}}$)
1.0112 (Ω_{P2})	1 ($\Omega_{L_{P2}}$)
1.1231 (Ω_{S2})	1.8952 ($\Omega_{L_{S2}}$)
∞	∞

Table 2: Frequency Transformation

1.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband edge :** 1
- **Stopband edge:** $\min(-\Omega_{L_{S1}}, \Omega_{L_{S2}}) = 1.8952$
- **Passband nature :** Monotonic
- **Stopband nature :** Monotonic

1.6 Analog Lowpass Transfer Function

For analog filter with monotonic passband and monotonic stopband, we use **Butterworth** approximation. Since tolerance for both band (δ) is 0.15, we define D1 and D2 as:

$$D1 = \frac{1}{1-\delta^2} - 1 = \mathbf{0.384} \tag{6}$$

$$D2 = \frac{1}{\delta^2} - 1 = \mathbf{43.444} \tag{7}$$

The order of butterworth filter is given as :

$$N = \lceil \frac{log\frac{\sqrt{D2}}{\sqrt{D1}}}{log\frac{\Omega_S}{\Omega_P}} \rceil = \lceil 3.69 \rceil = 4 \tag{8}$$

The cut-off frequency Ω_c follows the relation:

$$\frac{\Omega_P}{D1^{\frac{1}{2N}}} \le \Omega_c \le \frac{\Omega_S}{D2^{\frac{1}{2N}}} \tag{9}$$

$$1.127 \le \Omega_c \le 1.183$$

Let $\Omega_c = 1.15$
Poles of transfer function are the roots of the equation:

$$1 + (\frac{s}{j\Omega_c})^{2N} = 0 \tag{10}$$

$$1 + (\frac{s}{1.15j})^8 = 0$$

The roots of the equation are given as:

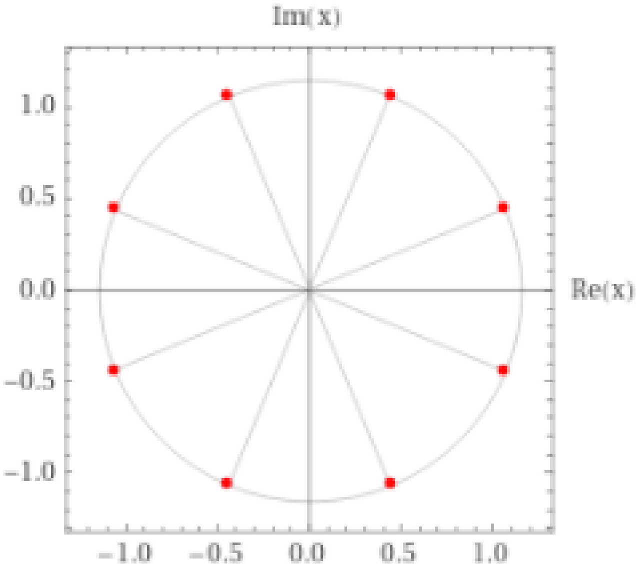


Figure 1: Poles of the transfer function

For stable system, we choose left-hand side poles.

$$p1 = -1.06246 - 0.440086j$$

$$p2 = -1.06246 + 0.440086j$$
$$p3 = -0.440086 - 1.06246j$$
$$p4 = -0.440086 + 1.06246j$$

The analog lowpass transfer function (considering DC gain 1) can be written as:

$$H_{analog,LPF}(s_L) = \frac{\Omega_c^N}{(s_L - p1)(s_L - p2)(s_L - p3)(s_L - p4)}$$

$$H_{analog,LPF}(s_L) = \frac{1.749}{s_L^4 + 3.005s_L^3 + 4.515s_L^2 + 3.974s_L + 1.749}$$

1.7 Analog Bandpass Transfer Function

We can get analog bandpass transfer function from analog lowpass transfer function using the transformation:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs} \tag{11}$$

$$s_L = \frac{s^2 + 0.80622}{0.2139s}$$

Substituting the value, we get:

$$H_{analog,BPF}(s) = \frac{0.00366s^4}{\begin{aligned} s^8 + 0.64269s^7 + 3.43137s^6 + 1.5933s^5 + 4.23652s^4 \\ + 1.28454s^3 + 2.23030s^2 + 0.33678s + 0.42247 \end{aligned}}$$

1.8 Discrete Time Filter Transfer Function

To get the discrete time filter from analog bandpasss filter, we use the following transformation:

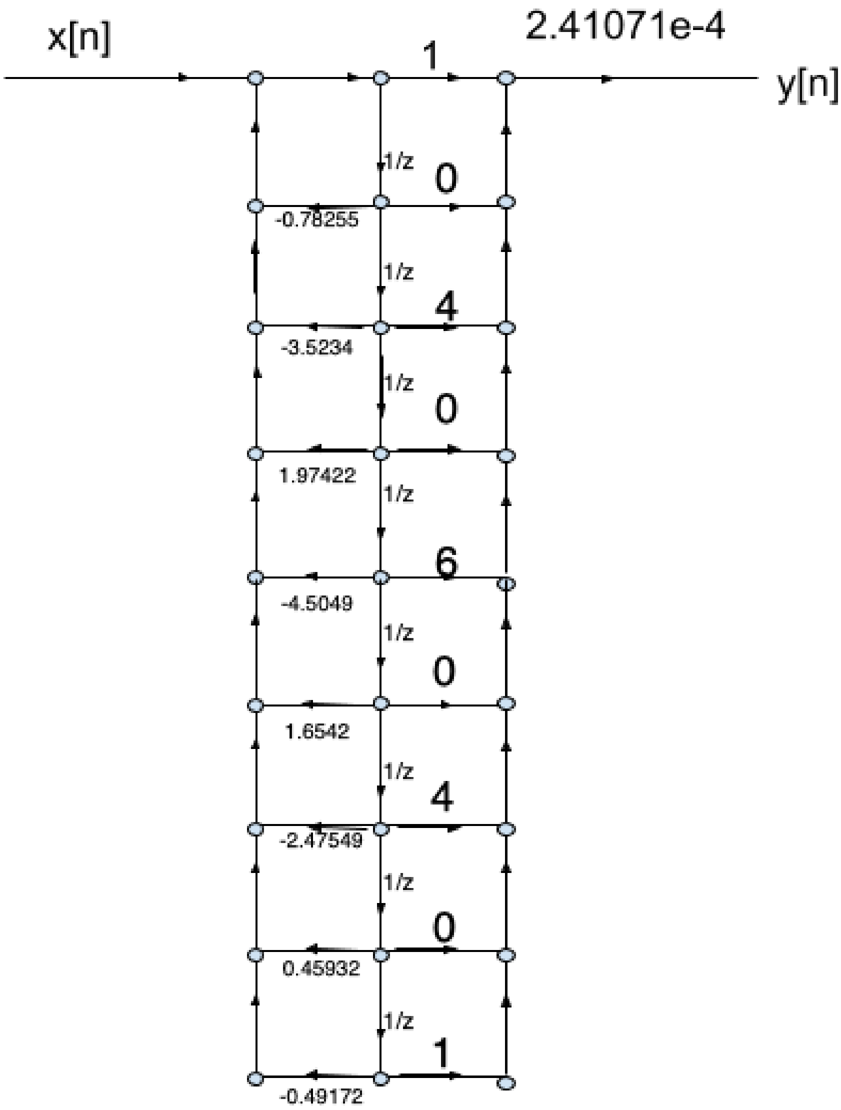
$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{12}$$

Substituting the value, we get:

$$H_{discrete,BPF}(z) = \frac{2.41071 * 10^{-4}(z^8 - 4z^6 + 6z^4 - 4z^2 + 1)}{\begin{array}{r} z^8 + 0.78255z^7 + 3.5234z^6 - 1.97442z^5 + 4.5049z^4 \\ - 1.6542z^3 + 2.47549z^2 - 0.45932z + 0.49172 \end{array}}$$

$$H_{discrete,BPF}(z) = \frac{2.41071 * 10^{-4}(1 - 4z^{-2} + 6z^{-4} - 4z^{-6} + z^{-8})}{\begin{array}{r} 1 + 0.78255z^{-1} + 3.5234z^{-2} - 1.97442z^{-3} + 4.5049z^{-4} \\ - 1.6542z^{-5} + 2.47549z^{-6} - 0.45932z^{-7} + 0.49172z{-8} \end{array}}$$

1.9 Realization Using Direct Form II



Direct Form II is obtained by treating the transfer function H(z) = B(z)/(1 - A(z)) as a cascade of 1/(1 - A(z)) followed by B(z).
Numerator coefficients have positive sign in the feedback while the denominator coefficients have a negative sign here.

1.10 FIR Filter Transfer Function using Kaiser Window

$\delta = 0.15$. Therfore the minimum stopband attenuation is given by A = -20log(0.15) = 16.4728 dB
Since A is less than 21dB, we get the shape parameter of the Kaiser window $\beta = 0$
The minimum transition width $\Delta\omega_t = 0.0333\pi$
To estimate the window length, we have the formula for the lower bound as :

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_t} \tag{13}$$

$$N \geq \frac{16.4728 - 7.95}{2.285 * 0.0333}$$

π

N comes out to be greater than 35.6.
Since it is a loose bound, we do not get get desired result at N = 36. It is seen that at N = 36 + 17 i.e. 53, all the design parameters are full-filled. Also the window is rectangular since β is 0.
The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MatLab function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters.

```
FIR_BandPass =

  Columns 1 through 7

  -0.021932557695063  -0.010410589634580   0.017976200559516   0.012259883911282  -0.011525003337975  -0.009971779059690   0.004427800208257

  Columns 8 through 14

   0.002789635227787   0.000938354208062   0.008904095029732  -0.002207700410433  -0.023531962691505  -0.002384020552433   0.038556692749331

  Columns 15 through 21

   0.013502594882402  -0.050970048997599  -0.030440448059742   0.057932583306609   0.051144337089667  -0.057419084533496  -0.072555601838608

  Columns 22 through 28

   0.048716778618285   0.091197416465821  -0.032653180701306  -0.103881786400547   0.011491444483159   0.108376937438767   0.011491444483159

  Columns 29 through 35

  -0.103881786400547  -0.032653180701306   0.091197416465821   0.048716778618285  -0.072555601838608  -0.057419084533496   0.051144337089667

  Columns 36 through 42

   0.057932583306609  -0.030440448059742  -0.050970048997599   0.013502594882402   0.038556692749331  -0.002384020552433  -0.023531962691505

  Columns 43 through 49

  -0.002207700410433   0.008904095029732   0.000938354208062   0.002789635227787   0.004427800208257  -0.009971779059690  -0.011525003337975

  Columns 50 through 53

   0.012259883911282   0.017976200559516  -0.010410589634580  -0.021932557695063
```

The z-transform can be found by using the coefficients of this finite series obtained through MatLab.

2 Bandpstop Filter Details

m = 43
Sice m ¡ 75, therefore the stopband will be equiripple
q(m) = 4
r(m) = 3
Stopband is from $B_l(m)$ and $B_h(m)$
$B_l(m)$ = 157 + 17*4 + 13*3 = 264 kHz
$B_h(m)$ = $B_l(m)$ + 55 = 319 kHz
Transition band : 20 kHz Sampling Rate : 1200 kHz

2.1 Un-normalised discrete time filter specifications

- **Stopband :** 264-319 kHz
- **Passband :** 0-244 kHz on left side and 339-600 kHz on the right
- **Tolerance :** 0.15
- **Passband nature :** Equiripple
- **Stopband nature :** Monotonic

2.2 Normalized Digital Filter Specifications

Any frequency(Ω) upto half of the sampling rate can be represented on the normalized axis(ω) by:

$$\omega = \frac{\Omega * 2\pi}{SamplingRate} \tag{14}$$

Therefore the specifications for the normalized digital filter are:

- **Stopband :** 0.4400π - 0.5317π
- **Passband :** 0 - 0.4067π on left side and 0.5650π - π on the right
- **Passband nature :** Equiripple
- **Stopband nature :** Monotonic

2.3 Analog filter specifications using Bilinear Transformation

Bilinear transform is given as :

$$\Omega = tan\frac{\omega}{2} \tag{15}$$

Applying the transformation, we get:

ω	Ω
0	0
0.4067π	0.7431
0.4400π	0.8277
0.5317π	1.1055
0.5650π	1.2292
π	∞

Table 3: Bilinear Transformation

- **Stopband :** 0.8277(Ω_{S1}) - 1.1055(Ω_{S2})
- **Transition band :** 0.7431 - 0.8277 and 1.1055 - 1.2292
- **Passband :** 0-0.7431(Ω_{P1}) and 1.2292(Ω_{P2}) - ∞
- **Passband nature :** Equiripple
- **Stopband nature :** Monotonic

2.4 Frequency Transformation and Relevant Parameters

For bandstop filter, we need to convert the specifications into their low pass forms. This is done using the frequency transformation:

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2} \tag{16}$$

where the parameters Ω_0 and B are defined as follow:

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = \sqrt{0.7431 * 1.2292} = \mathbf{0.9557} \tag{17}$$

$$B = \Omega_{P2} - \Omega_{P1} = 1.2292 - 0.7431 = \mathbf{0.4861} \tag{18}$$

Applying the transformation, we get:

Ω	Ω_L
0	0^+
0.7431 (Ω_{P1})	1 ($\Omega_{L_{P1}}$)
0.8277 (Ω_{S1})	1.7632 ($\Omega_{L_{S1}}$)
0.9557 (Ω_0^-)	∞
0.9557 (Ω_0^+)	-∞
1.1055 (Ω_{S2})	-1.7403 ($\Omega_{L_{S2}}$)
1.2292 (Ω_{P2})	-1 (Ω_{L_2})
∞	0^-

Table 4: Frequency Transformation

2.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband edge :** 1
- **Stopband edge:** $\min(\Omega_{L_{S1}}, -\Omega_{L_{S2}}) = 1.7403$
- **Passband nature :** Equiripple
- **Stopband nature :** Monotonic

2.6 Analog Lowpass Transfer Function

For analog filter with equiripple passband and monotonic stopband, we use **Chebyshev** approximation. Since tolerance for both band (δ) is 0.15, we define D1 and D2 as:

$$D1 = \frac{1}{1-\delta^2} - 1 = \mathbf{0.384} \tag{19}$$

$$D2 = \frac{1}{\delta^2} - 1 = \mathbf{43.444} \tag{20}$$

The order of chebyshev filter is given as :

$$N = \lceil \frac{cosh^{-1}\frac{\sqrt{D1}}{\sqrt{D2}}}{cosh^{-1}\frac{\Omega_S}{\Omega_P}} \rceil = \lceil 2.6519 \rceil = 3 \tag{21}$$

Poles of transfer function are the roots of the equation:

$$1 + D1.cosh^2(Ncosh^{-1}(s/j)) = 0 \tag{22}$$

$$1 + 0.3841 * cosh^2(3 * cosh^{-1}(s/j)) = 0$$

The roots of the equation are given as:

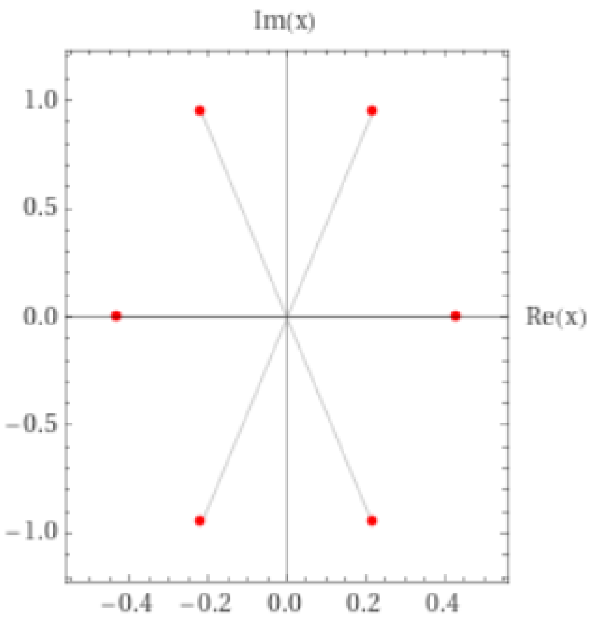


Figure 2: Poles of the transfer function

The poles of the function as obtained through MatLab are:

```
p =

                                   -0.431053737450153370897062479495 39
                                    0.431053737450153370897062479495 39
 - 0.215526868725076685448531239747 7 + 0.943056463541451645155258002361 85i
 - 0.215526868725076685448531239747 7 - 0.943056463541451645155258002361 85i
   0.215526868725076685448531239747 7 + 0.943056463541451645155258002361 85i
   0.215526868725076685448531239747 7 - 0.943056463541451645155258002361 85i
```

For stable system, we choose left-hand side poles.

$$p1 = -0.43105$$

$$p2 = -0.21552 + 0.90306j$$

$$p2 = -0.21552 - 0.90306j$$

The analog lowpass transfer function (considering DC gain 1) can be written as:

$$H_{analog,LPF}(s_L) = \frac{(-1)^N .p1.p2.p3}{(s_L - p1)(s_L - p2)(s_L - p3)}$$

It should be noted that since N is odd, therefore the DC gain is 1.

$$H_{analog,LPF}(s_L) = \frac{0.4034}{s_L^3 + 0.8621s_L^2 + 1.1216s_L + 0.4034}$$

2.7 Analog Bandpass Transfer Function

We can get analog bandpass transfer function from analog lowpass transfer function using the transformation:

$$s_L = \frac{Bs}{s^2 + \Omega_0^2} \tag{23}$$

$$s_L = \frac{0.4861s}{s^2 + 0.99142}$$

Substituting the value, we get:

$$H_{analog,BSF}(s) = \frac{s^6 + 2.7401s^4 + 2.5028s^2 + 0.762}{s^6 + 1.3517s^5 + 3.2452s^4 + 2.7541s^3 + 2.9641s^2 + 1.1277s + 0.762}$$

2.8 Discrete Time Filter Transfer Function

To get the discrete time filter from analog bandpasss filter, we use the following transformation:

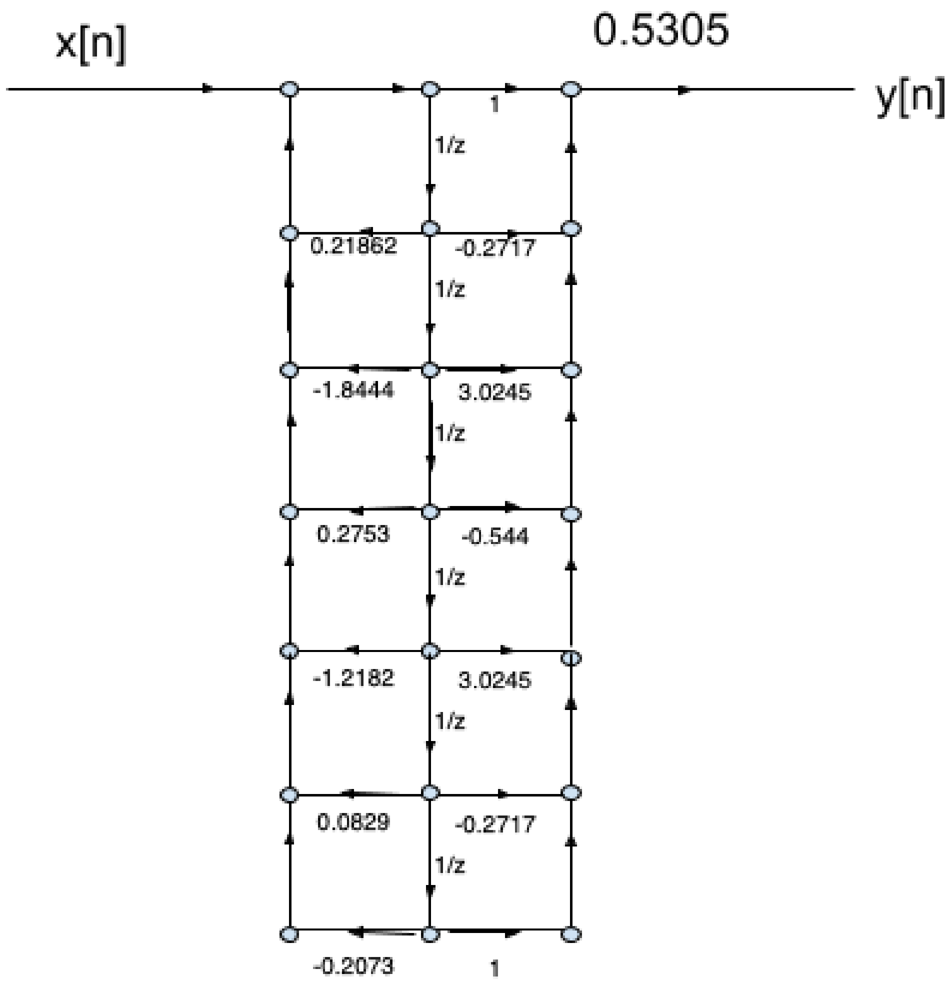
$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{24}$$

Substituting the value, we get:

$$H_{discrete,BSF}(z) = \frac{0.5305z^6 - 0.1441z^5 + 1.6045z^4 - 0.2886z^3 + 1.6045z^2 - 0.1441z + 0.5305}{z^6 - 0.21862z^5 + 1.8444z^4 - 0.2753z^3 + 1.2182z^2 - 0.0829z + 0.2073}$$

$$H_{discrete,BSF}(z) = \frac{0.5305(1 - 0.2717z^{-1} + 3.0245z^{-2} - 0.544z^{-3} + 3.0245z^{-4} - 0.2717z^{-5} + z^{-6})}{1 - 0.21862z^{-1} + 1.8444z^{-2} - 0.2753z^{-3} + 1.2182z^{-4} - 0.0829z^{-5} + 0.2073z^{-6}}$$

2.9 Realization Using Direct Form II



Direct Form II is obtained by treating the transfer function H(z) = B(z)/(1 - A(z)) as a cascade of 1/(1 - A(z)) followed by B(z).
Numerator coefficients have positive sign in the feedback while the denominator coefficients have a negative sign here.

2.10 FIR Filter Transfer Function using Kaiser Window

$\delta = 0.15$. Therfore the minimum stopband attenuation is given by A = -20log(0.15) = 16.4728 dB
Since A is less than 21dB, we get the shape parameter of the Kaiser window $\beta = 0$
The minimum transition width $\Delta\omega_t = 0.0333\pi$
To estimate the window length, we have the formula for the lower bound as :

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_t} \tag{25}$$

$$N \geq \frac{16.4728 - 7.95}{2.285 * 0.0333}$$

π

N comes out to be greater than 35.6.
Since it is a loose bound, we do not get get desired result at N = 36. It is seen that at N = 36 + 14 i.e. 50, all the design parameters are full-filled. Also the window is rectangular since β is 0.
The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MatLab function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters.

```
FIR_BandStop =

  Columns 1 through 9

    0.0378   -0.0201   -0.0124   -0.0107    0.0394   -0.0178    0.0009   -0.0184    0.0231

  Columns 10 through 18

   -0.0209    0.0345   -0.0193   -0.0059   -0.0395    0.0810   -0.0104   -0.0308   -0.0797

  Columns 19 through 27

    0.1312   -0.0050   -0.0203   -0.1591    0.2210   -0.1310    0.5464    0.5464   -0.1310

  Columns 28 through 36

    0.2210   -0.1591   -0.0203   -0.0050    0.1312   -0.0797   -0.0308   -0.0104    0.0810

  Columns 37 through 45

   -0.0395   -0.0059   -0.0193    0.0345   -0.0209    0.0231   -0.0184    0.0009   -0.0178

  Columns 46 through 50

    0.0394   -0.0107   -0.0124   -0.0201    0.0378
```

The z-transform can be found by using the coefficients of this finite series obtained through MatLab.

3 MatLab Plots

3.1 BandPassFilter

3.1.1 IIR Filter

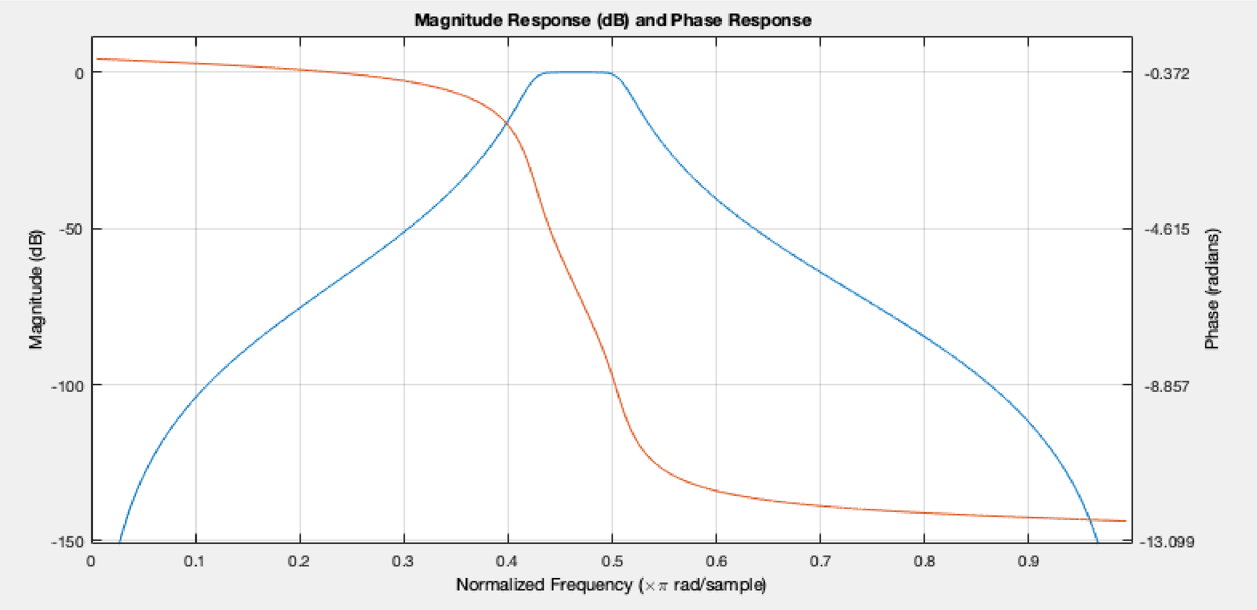


Figure 3: Normalized Magnitude response of the Butterworth Filter

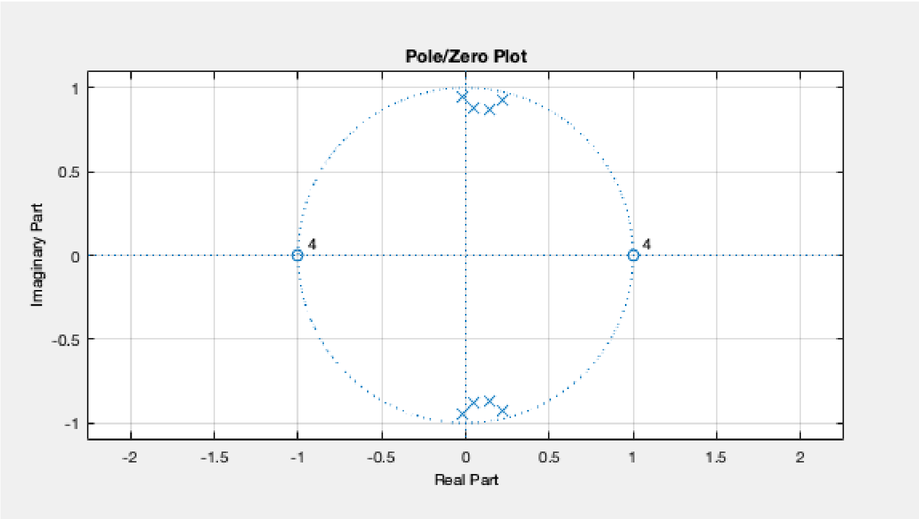


Figure 4: Poles of the transfer function

We can see that all the poles lie within the unit circle, therefore,the system is stable.

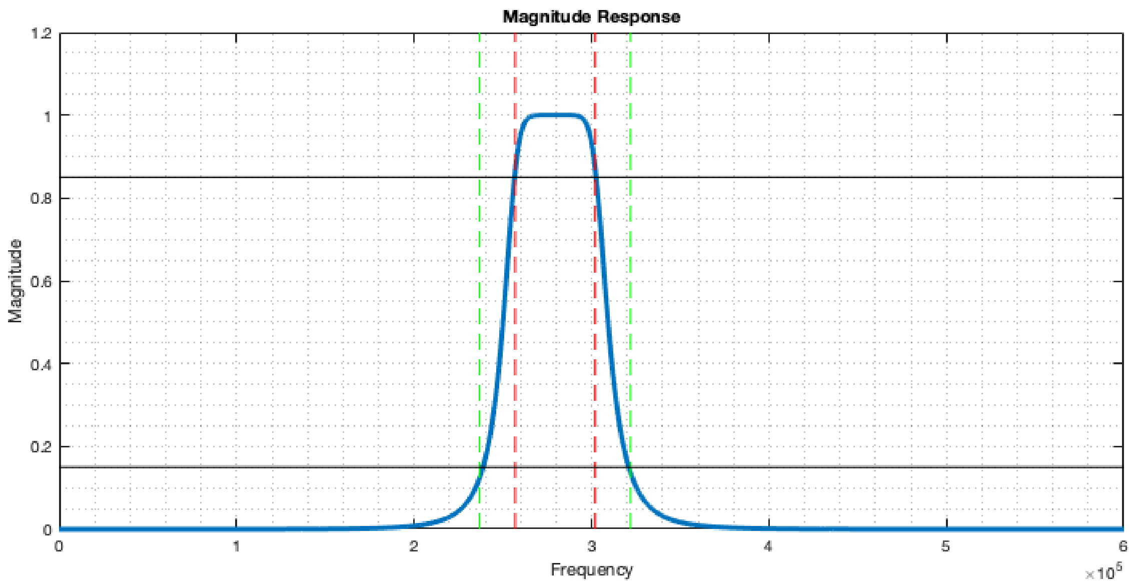


Figure 5: Un-normalized Magnitude response of the Butterworth Filter

The red margins are the bandpass edges and the green margins are bandstop edges. The black horizontal lines are the tolerance limit. As we can see, the red lines intersect the response curve above the tolerance limit and the green margins intersect it below the lower tolerance limit. Also the passband is monotonic. Thus the filter parameters are satisfied in this design.

3.1.2 FIR Filter

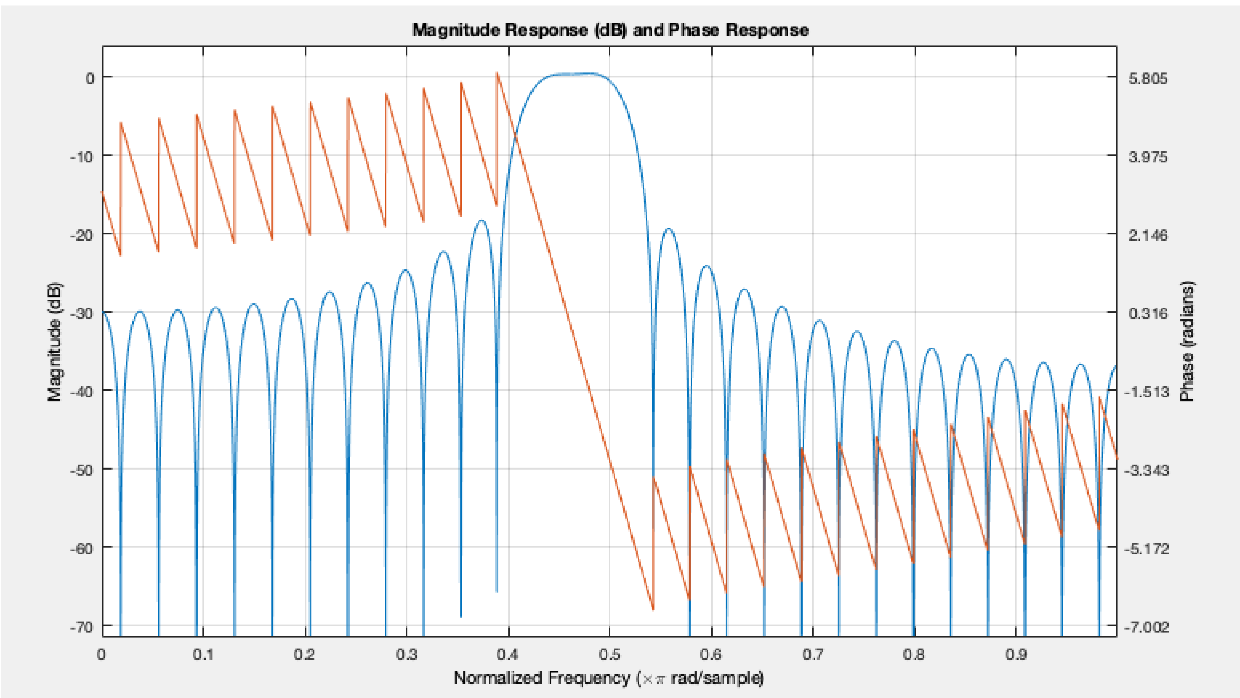


Figure 6: Normalized Magnitude response of the FIR Filter for bandpass filter

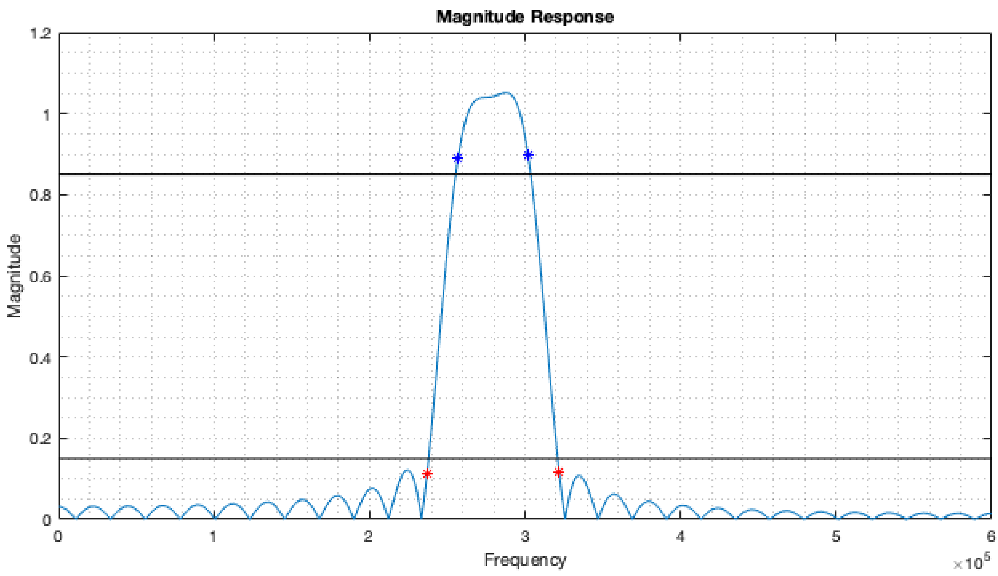


Figure 7: Un-normalized Magnitude response of the FIR Filter for bandpass filter

Blue dots are the passband edges and red dots are stopband edges. Horizontal lines are tolerance limits. As we can see, the filter design is according to specifications.

3.2 BandStopFilter

3.2.1 IIR Filter

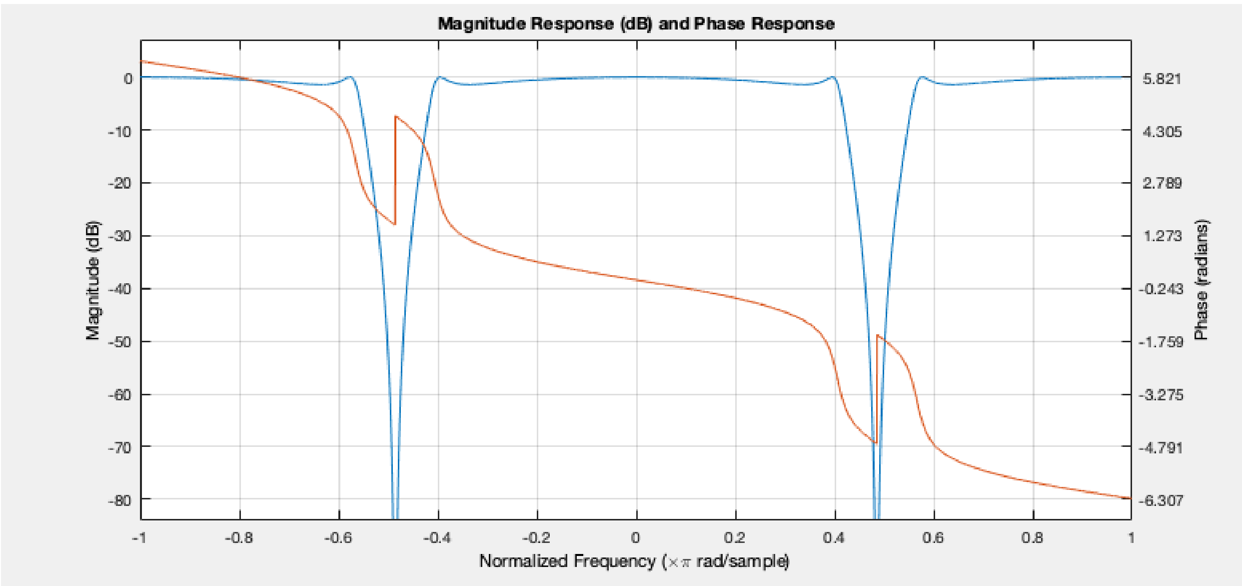


Figure 8: Normalized Magnitude response of the Chebyshev Filter

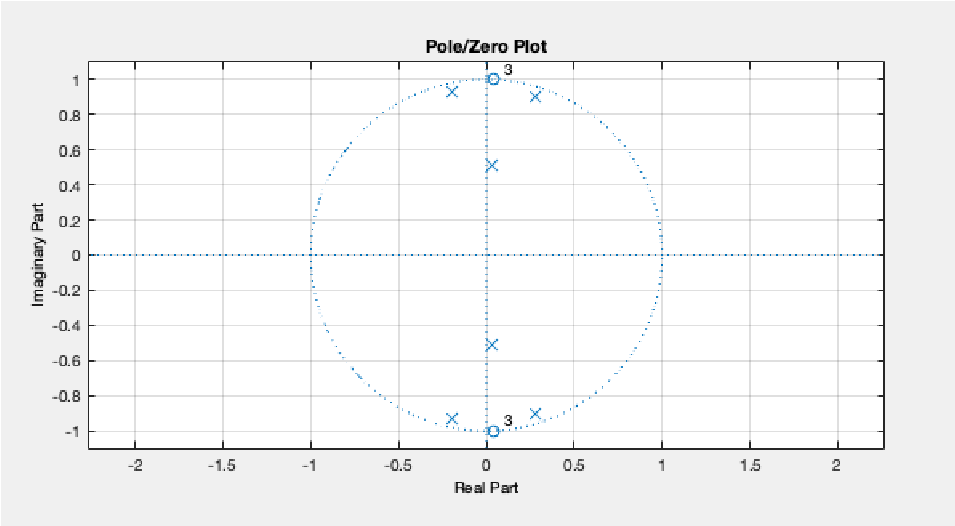


Figure 9: Poles of the transfer function

We can see that all the poles lie within the unit circle, therefore,the system is stable.

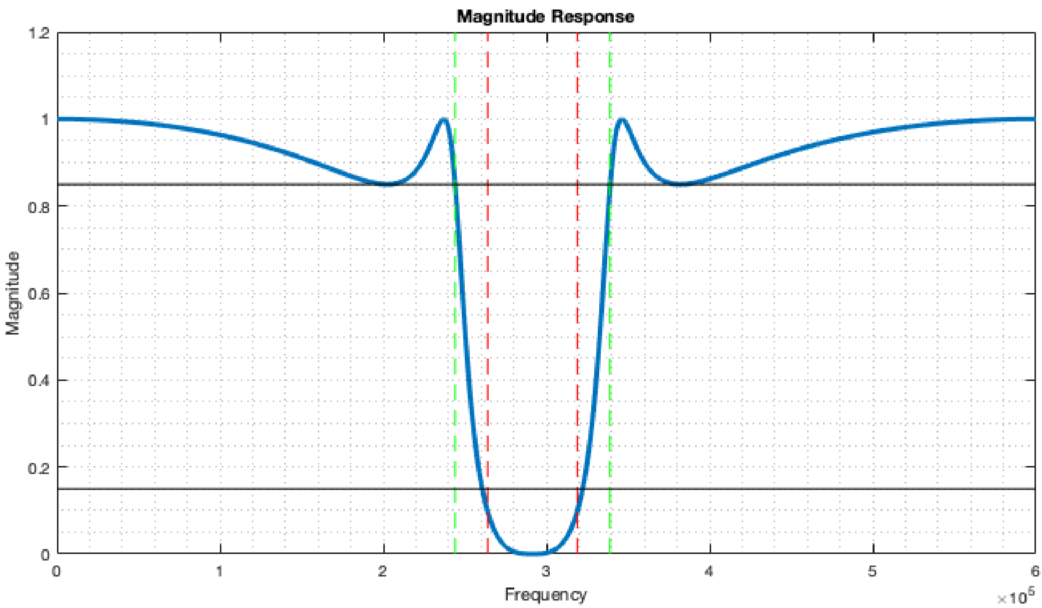


Figure 10: Un-normalized Magnitude response of the Chebyshev Filter

The red margins are the stopband edges and the green margins are passband edges. The black horizontal lines are the tolerance limit. As we can see, the red lines intersect the response curve below the tolerance limit and the green margins intersect it above the upper tolerance limit. Also the passband is eqiripple. Thus the filter parameters are satisfied in this design.

3.2.2 FIR Filter

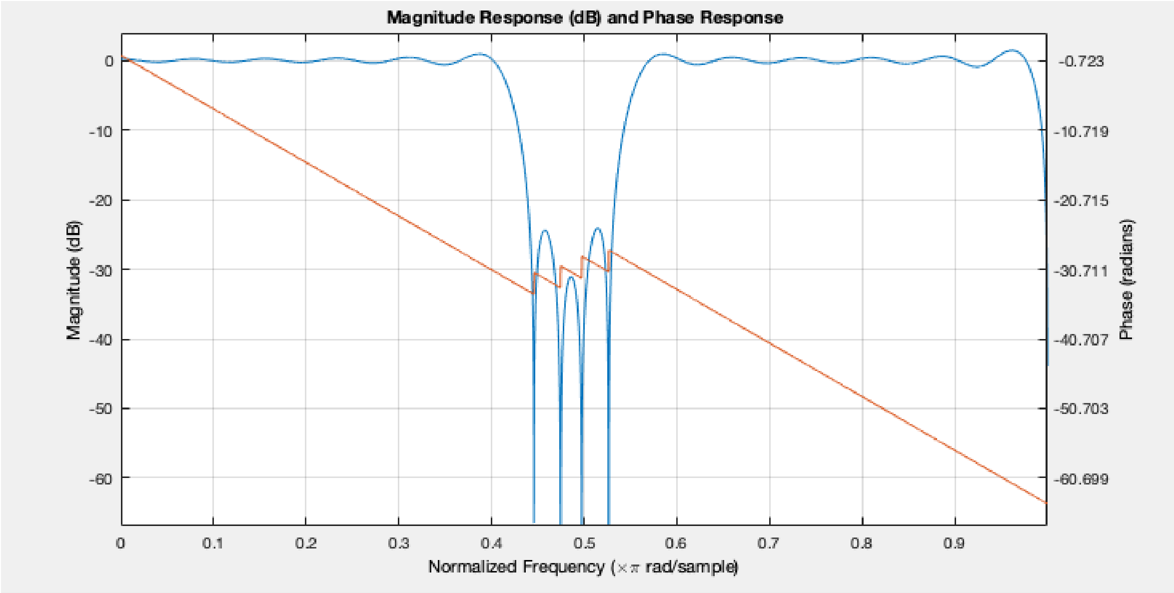


Figure 11: Normalized Magnitude response of the FIR Filter for bandstop filter

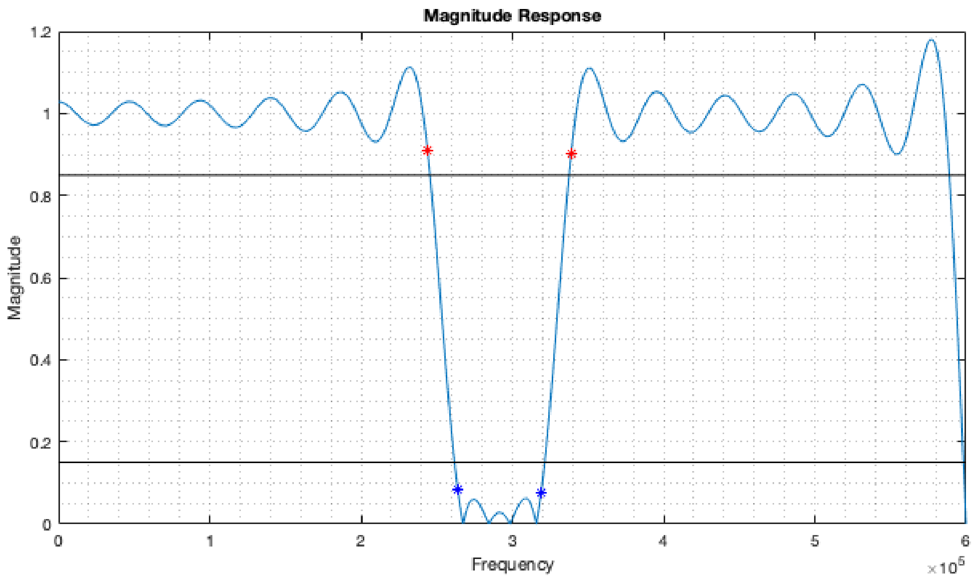


Figure 12: Un-normalized Magnitude response of the FIR Filter for bandstop filter

Blue dots are the stopband edges and red dots are passband edges. Horizontal lines are tolerance limits. As we can see, the filter design is according to specifications.

4 Peer Review