Guaranteed VWAP Execution under Market Impact and Risk

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Introduction 👤

About Me

- Education, experience, and finance background

Presentation Overview

- ATQS project: Trade Execution Framework
- Automating consistency & cost-efficiency for hedge funds

Motivation

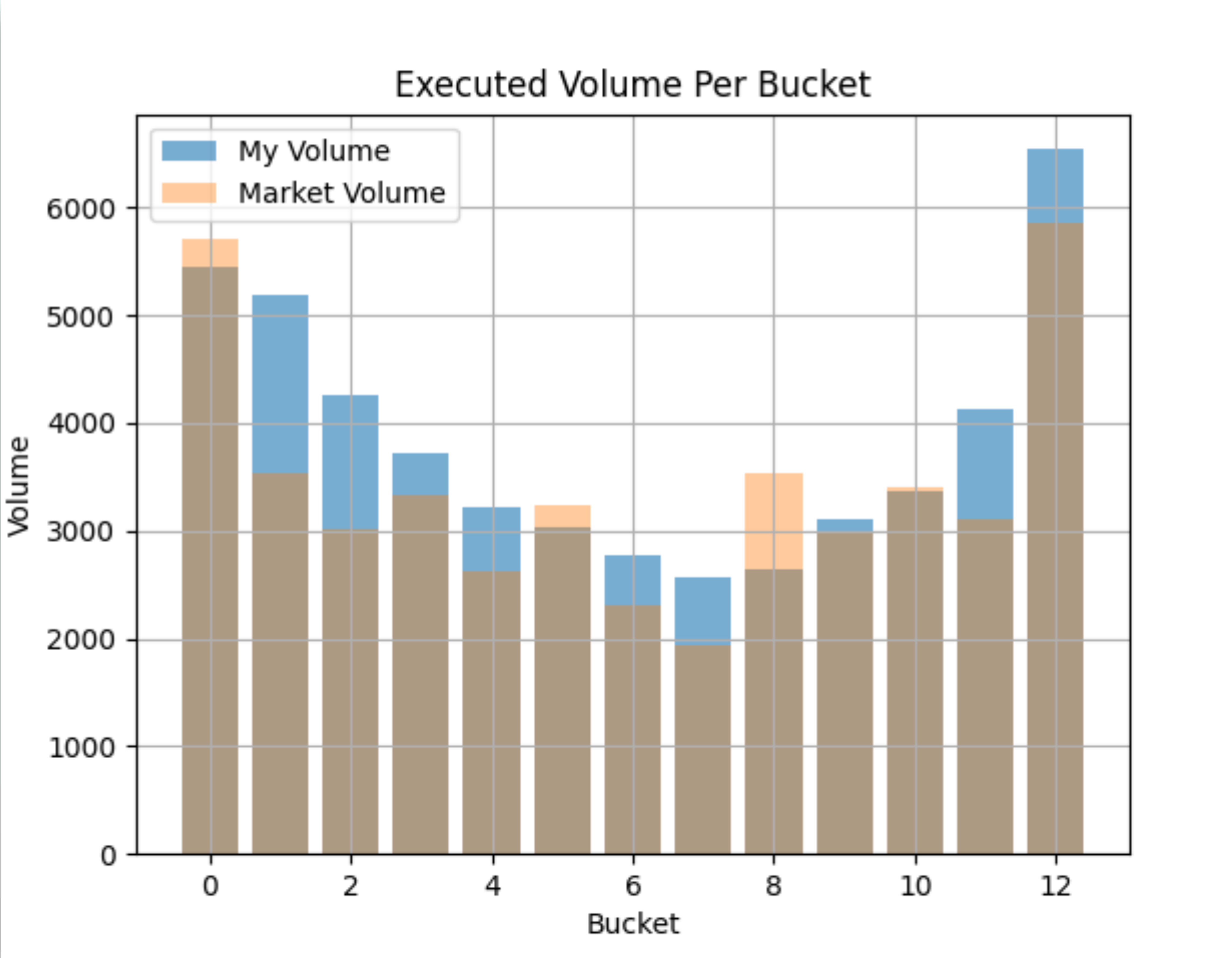
- Excited to tackle these problems

Execution as a Reliability Problem

- Guaranteed VWAP: commitment to benchmark price
- Uncertain market conditions: liquidity swings, volatility bursts, momentum shifts
- Requires adaptive control under uncertainty
  - Reliability > Prefect VWAP
- Reliability = stable cost + bounded tracking error + low variance

The Challenge: Tracking VWAP Under Uncertainty🧩

- VWAP execution depends on accurate intraday volume prediction
- Real-world **volume curves drift** with market volatility and news
- Volume drift leads to misalignment → risk of over- or under-trading
- **Objective**: adapt schedule dynamically to minimize cost: VWAP deviation + λ × risk



Teaser: Intraday volume curve drift between forecasted and actual distributions

System Overview ⚜️

Design Focus: data-driven, adaptive, reliable, risk-aware

Static Volume Model (Bayesian Dirichlet) ⚓

- Per-stock training on **historical** intraday TAQ data
- Learns baseline day-shape from normalized volume buckets
- Blends **global + stock-specific** patterns for stability
  - α captures cumulative confidence from historical patterns
  - Stable during low-data or high-volatility days
- Acts as the static prior for real-time adaptive updates

$$\alpha_{\text{stock}} = w \cdot V_{\text{global}} + (1 - w) \cdot V_{\text{stock}}$$

$$\hat{v}_{\text{stock}} = \frac{\alpha_{\text{stock}}}{\sum \alpha_{\text{stock}}}$$

- w -> balances global vs. stock-specific evidence
- α -> represents Dirichlet parameters
- $\hat{v}$ -> the expected intraday volume profile.

Dynamic Volume Model (Bayesian Ridge) 🧮

- **Global regression** trained on pooled stocks and days
- Ridge regression is used because coefficients remain small and stable under multicollinearity
- Adapts bucket-by-bucket as trading unfolds
- Provides fast corrections to static Dirichlet baseline

$$\hat{y}t = \beta_0\, s_t + \beta_1\, c_{t-1} + \beta_2\, \mu_{t-3:t-1} + \beta_3\, \sigma_{t-3:t-1} + \varepsilon$$

- s_t -> static prior for bucket t
- c_{t-1}-> represents market volume traded till now
- μ, σ -> mean & std of recent market volumes

Modeling the Market Environment

- **Price Simulation:** Models *market* price for buckets using stochastic shocks

$$P_{t+1} = P_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

- **Market Volume Simulation:** Adds realistic liquidity noise via log-normal scaling

$$V_t^{\text{market}} = V_t^{\text{static}} \times \text{LogNormal}(0, \sigma)$$

- **Residual Execution Risk Estimation:**
  - Penalizes uncertainty from unexecuted volume
  - σ = 0.05 (assumed volatility; ideally estimated from price variance post-simulation)

$$\text{Risk}_t = \left(\frac{\text{Remaining Shares}}{\text{Total Shares}}\right)^2 \sigma^2$$

Trade Trajectory & Execution Impact

- Models temporary execution impact via a sinh-shaped trade path
- Controls trade intensity (constantly) across 30-min execution buckets
  - η measures price sensitivity. Higher η → stronger slippage
- Impact from prior bucket partly decays into the next interval
  - Non Linear Impact = $\eta \, \sigma \, v^{\beta}$
  - Algrem-Chris Parameters: λ= 1; β = 0.5

$$X_t = X_0 \frac{\sinh[\kappa(T - t)]}{\sinh(\kappa T)}$$

$$\kappa = \sqrt{\frac{\lambda \sigma^2}{\eta}} \quad \Rightarrow \quad \eta = \frac{\lambda \sigma^2}{\kappa^2}$$

$$P_t^{exec} = P_t^{market} + \eta \, \sigma \, v_t^{\beta} + 0.5 \, \eta \, \sigma \, v_{t-1}^{\beta}$$

End-to-End Execution Simulation 🎬

- Start with static volume baseline from the prior model
- From buckets 2–12, apply dynamic volume updates based on real-time forecasts
- In the final bucket (13), execute all remaining shares to close out
- In each bucket:
  - Compute execution price including temporary impact
  - Estimate residual risk from untraded volume
- In end compute: Market VWAP, Execution VWAP, Total Risk

Code: https://github.com/JayeshK10/Guaranteed-VWAP.git

A Fair Trade: The Guaranteed VWAP Strategy Holds Its Ground 💰

Metric	Per Share Value ($)
Market VWAP	157.9966
Execution VWAP	158.5643
VWAP Deviation	0.5677
VWAP Deviation %	0.359
Execution Risk	0.011214
Guaranteed Cost	0.5789

Table: VWAP Execution Summary Metrics for Apple Inc. (500,000-Share Order)

order_size	Guaranteed Cost ($)	Market Cost ($)	Execution Cost ($)	profit/loss ($)
1,000	572.72	157,449.90	158,011.30	11.32
10,000	4,935.23	1,575,187.00	1,580,010.00	112.23
50,000	13,955.84	7,886,470.00	7,899,865.00	560.84
100,000	11,917.36	15,788,900.00	15,799,700.00	1,117.36
250,000	55,245.42	39,499,150.00	39,551,600.00	2,795.42
500,000	289,446.52	78,998,300.00	79,282,150.00	5,596.52
1,000,000	1,085,091.04	157,996,600.00	159,070,400.00	11,291.04

Table: Results for Apple Inc., showing costs and realized P/L across different order sizes

Cost Sensitivity to Order Size – AAPL 🔍

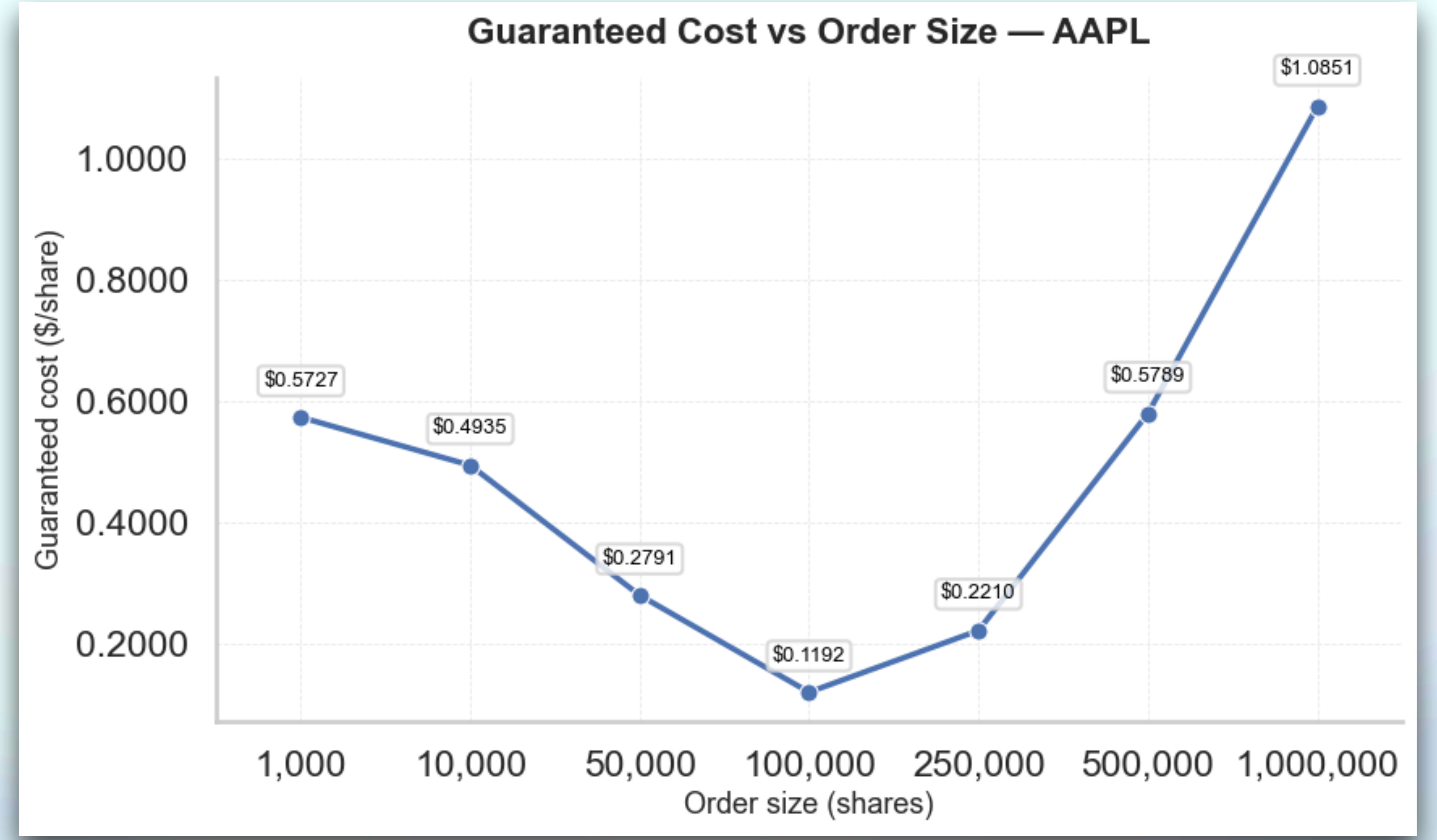


Figure: Guaranteed Cost per Share increases non-linearly with order size, reflecting higher execution difficulty and slippage for large trades

Across Stocks and Sizes 📈

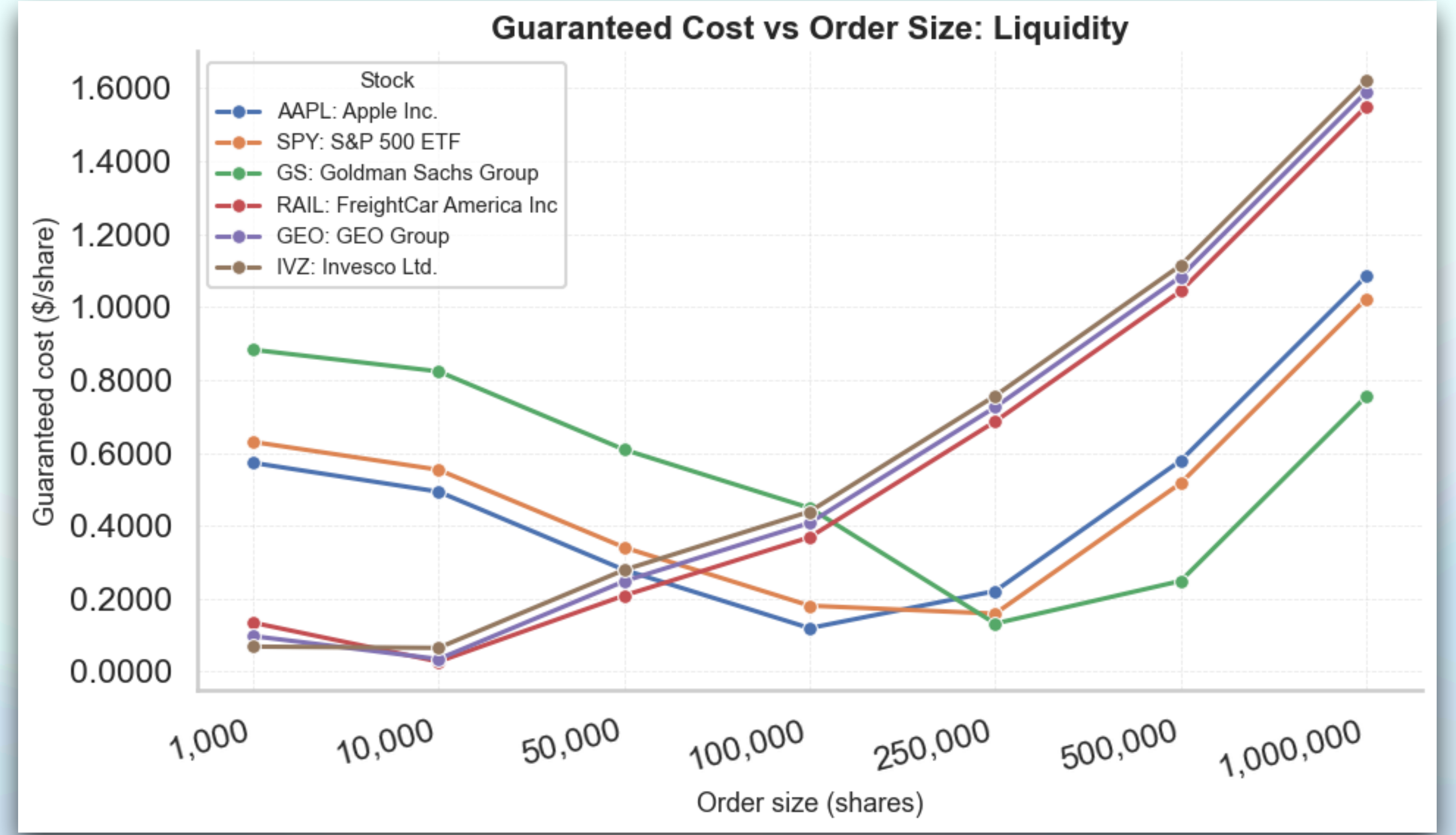


Figure: Guaranteed VWAP cost per share increases predictably with order size across assets, with higher costs for less liquid stocks

Where This Strategy Can Break ⛓️‍💥

- Forecast drift from regime breaks (news, shocks, halts)
- Assumption breakdowns
  - Volatility (σ)
  - Impact Model
  - Transaction Cost
- Liquidity Crises & Tail Events

How This Work Aligns With Financial Firms 🤝

- Data-Driven & Automated
- Reliability-First Design
- Engineering Thinking: modular
- Execution Insight
- Scalable Architecture

Q&A

Any Questions?