DTL Assignment 2

Jayesh Bapu Gadilohar
MIS : 111910048
Div : 2
Batch : S4

Mathematical Formulae

Here are some general mathematical formulae for definite and indefinite integration,differentiation,limits and summation formulae.

General mathematical formulae :

1) $(a+b)^2 = a^2+b^2+2ab$
2) $(a-b)^2 = a^2+b^2-2ab$
3) $a^2-b^2 = (a-b)(a+b)$
4) $a^3+b^3 = (a+b)(a^2-2ab+b^2)$
5) $a^3-b^3 = (a-b)(a^2+2ab+b^2)$
6) $a^2+b^2 = c^2$ ....(pythagoras' theorem)
7) $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Indefinite integration Formulae :

(a)General Integration Formulae :

1) $\int \sin x dx = -\cos x + c$
2) $\int \cos x dx = \sin x + c$
3) $\int \tan x dx = -\ln|\cos x| + c$
4) $\int \cot x dx = \ln|\sin x| + c$
5) $\int \sec x dx = \ln|\sec x + \tan x| + c$
6) $\int \csc x dx = \ln|\csc x - \cot x| + c$
7) $\int x^n dx = \frac{x^{n+1}}{n+1} + c(n \neq -1)$
8) $\int \frac{1}{x} dx = \ln|x| + c$
9) $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
10) $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$
11) $\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$
12) $\int \tan^2 x dx = \tan x - x + c$
13) $\int \cot^2 x dx = -\cot x - x + c$
14) $\int \sec^2 x dx = \tan x + c$
15) $\int \csc^2 x dx = -\cot x + c$

(b)Substitution Formulae :

1) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + c$
2) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + c$
3) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln|\frac{x-a}{x+a}| + c$
4) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln|\frac{a+x}{a-x}| + c$
5) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln x + \sqrt{x^2+a^2} + c$
6) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln x + \sqrt{x^2-a^2} + c$
7) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}(\frac{x}{a}) + c$

(c) By Part Formulae :

1) $\int uv' = uv - \int u'v dx$ (by parts)
2) $\int \ln x dx = x\ln x - x + c$
3) $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$
4) $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$
5) $\int \sec^3 x dx = \frac{1}{2}(\ln|\sec x + \tan x| + \sec x \tan x) + c$
6) $\int \csc^3 x dx = \frac{1}{2}(\ln|\csc x - \cot x| + \csc x \cot x) + c$

(d)Partial Fraction :

1) $\int \frac{px+q}{(x-a)(x-b)} dx = \frac{A}{x-a} + \frac{B}{x-b}$
2) $\int \frac{px^2+qx+r}{(x-a)(x-b)(x-c)} dx = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
3) $\int \frac{dx}{(x-a)(x-b)^2} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$
4) $\int \frac{dx}{(x-a)(x-b)} = \frac{1}{(a-b)}\int(\frac{1}{x-a} - \frac{1}{x-b})dx$

Definite Integrals :

1) if $\int F(x)dx = \phi(x)$ then
   $\int_a^b F(x)dx = \phi(b) - \phi(a)$
2) $\int_a^b F(x)dx = -\int_b^a F(x)dx$
3) $\int_a^b F(x)dx = \int_a^b F(t)dt$
4) $\int_0^a F(x)dx = \int_0^a F(a-x)dx$
5) $\int_0^{2a} F(x)dx = \int_0^a F(x)dx + \int_0^a F(2a-x)dx$
6) $\int_{-a}^a F(x)dx = 2\int_0^a F(x)dx$ (....if F(x) is even function)
7) $\int_{-a}^a F(x)dx = 0$ (....if F(x) is odd function)
8) $\int_0^a (\frac{F(x)}{F(x)+F(a-x)})dx = \frac{a}{2}$
9) $\int_a^b (\frac{F(x)}{F(x)+F(a+b-x)})dx = \frac{(b-a)}{2}$
10) $\int_0^{\pi/2} \ln(\sin x)dx = -\frac{\pi}{2}\ln 2$
11) $\int_0^{\pi/2} \ln(\cos x)dx = -\frac{\pi}{2}\ln 2$
12) $\int_0^{\pi/2} \ln(\tan x)dx = 0$
13) $\int_0^{\pi/2} \ln(\cot x)dx = 0$
14) $\int_0^{\pi/2} \ln(\sec x)dx = \frac{\pi}{2}\ln 2$
15) $\int_0^{\pi/2} \ln(\csc x)dx = \frac{\pi}{2}\ln 2$

Limits :

1) $\lim_{x\to a} \frac{x^n-a^n}{x-a} = nx^{n-1}$
2) $\lim_{x\to\infty} \frac{1}{x} = 0$
3) $\lim_{x\to 0} \frac{\sin x}{x} = 1$
4) $\lim_{x\to 0} \frac{\tan x}{x} = 1$
5) $\lim_{x\to 0} \frac{a^x-1}{x} = \log a$
6) $\lim_{x\to 0} \frac{e^x-1}{x} = \log e = 1$
7) $\lim_{x\to 0} \frac{\ln(x+1)}{x} = \log e = 1$
8) $\lim_{x\to 0} (1+x)^{1/x} = e$
9) $\lim_{x\to 0} \ln(1+x)^{1/x} = \log a$
10) $\lim_{x\to 0} \frac{a^x-1}{x} = \log a$
11) $\lim_{x\to 0} \frac{a^{mx}-1}{x} = m\log a$
12) $\lim_{x\to 0} \frac{e^{mx}-1}{x} = m$
13) $\lim_{x\to 0} \frac{\ln(1+mx)}{x} = m$
14) $\lim_{x\to 0} (1+mx)^{1/x} = e^m$

Summation Formulae :

1) $\sum_{n=1}^{k} n = \frac{k(k+1)}{2}$
2) $\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{2}$
3) $\sum_{n=1}^{k} n^3 = \frac{(k(k+1))^2}{4}$
4) $\sum_{n=1}^{k} 2n = k(k+1)$
5) $\sum_{n=1}^{k} (2n-1) = k^2$
6) $\sum_{n=1}^{k} n(n+1) = \frac{(k+1)(k+2)}{3}$
7) $\sum_{n=1}^{k} \frac{1}{n(n+1)} = \frac{k}{k+1}$

Differentiation Formulae :

The derivative of the function $F(x)$ at point $x = a$ is given by the first principle of derivative

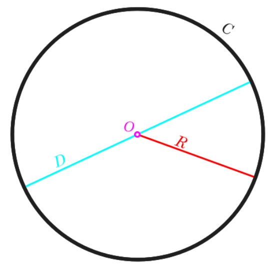
$$F(a)' = \lim_{h \to 0} \frac{F(a+h) - F(a)}{h}$$

1) $(ku)' = ku'$
2) $(u+v)' = u' + v'$
3) $(uv)' = u'v + uv'$
4) $(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$
5) $\frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx}$
6) $(x^n)' = nx^{(n-1)}$
7) $(e^x)' = e^x$
8) $(e^{ax})' = ae^{ax}$
9) $(a^x)' = a^x \ln a$
10) $(\sin x)' = \cos x$
11) $(\cos x)' = -\sin x$
12) $(\tan x)' = \sec^2 x$
13) $(\cot x)' = -\csc^2 x$
14) $(\sec x)' = \sec x \tan x$
15) $(\csc x)' = -\csc x \cot x$
16) $(\sinh x)' = \sinh x$
17) $(\cosh x)' = \cosh x$
18) $(\ln x)' = \frac{1}{x}$
19) $(\log_a x)' = \frac{\log_a e}{x}$
20) $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
21) $(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$
22) $(\tan^{-1} x)' = \frac{1}{1+x^2}$
23) $(\cot^{-1} x)' = -\frac{1}{1+x^2}$
24) $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$
25) $(\csc^{-1} x)' = -\frac{1}{x\sqrt{x^2-1}}$

CONIC SECTION

A conic section or conic is a locus of a point p in a plane such that the ratio of its distance from a fixed point to its distance from the fixed line is constant.:

Circle



Eqation of circle in different forms :

1)Standard Equation :

$$x^2 + y^2 = a^2$$

whrere a is the radius of the circle

2)Centre-Radius Form :

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h, k) are the centre and r is the radius of the circle

3)Diameter Form :

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

where $(x_1, y_1), (x_2, y_2)$ are the points on the circumference of the circle

4)General equation of the circle :

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Note :
(1)If $g^2 + f^2 - c = 0$,then the above equation represents a point circle
(2)If $g^2 + f^2 - c < 0$,then the above equation does not represents a circle.
(3)The radius of the circle is $\sqrt{g^2 + f^2 - c}$
(4)The centre of the circle is $(h, k) = (-g, -f)$

Remark :
The equation of the circle has following properties.
(1)It is the second degree equation in x and y
(2)Coefficient of x^2=Coefficient of y^2
(3)There is no term in xy

Parabola

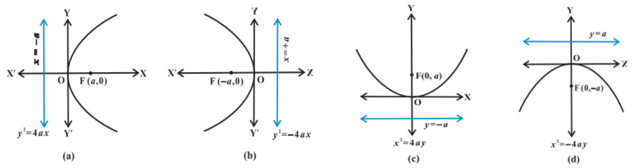


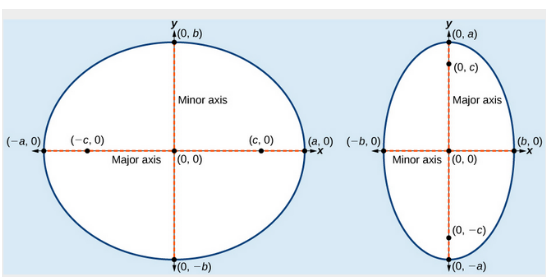
Fig. 2

Parametric Equation of Standard Parabola $y^2 = 4ax$:
Consider the equation $x = at^2, y = 2at$....(1)
Then

$$\begin{aligned} y^2 &= (2at)^2 \\ &= 4a^2t^2 \\ &= 4a(at^2) \\ &= 4ax \end{aligned}$$

$\Rightarrow$ The point $(x, y) = (at^2, 2at)$ given by equation (1) will describe the parabola.

Ellipse



Types of Standard Ellipse :
(1)Horizontal Ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$$

(2)Vertical Ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$$

General equation of ellipse :

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Where $(x, y) = (h, k)$ is the center of the Ellipse.

Hyperbola

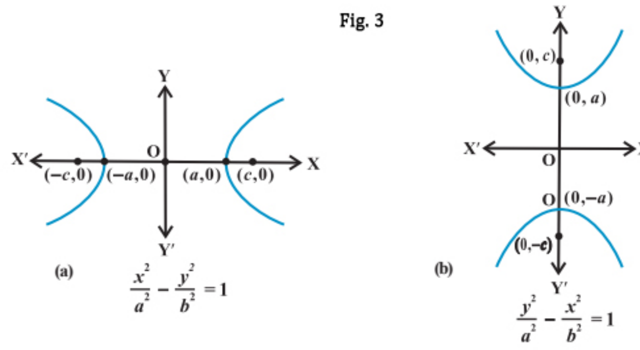


Fig. 3

Types of Standard Hyperbola :
(1)Horizontal Transverse Axis Hyperbola :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(2)Vertical Transverse axis Hyperbola :

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

General equation of ellipse :
The equation of the Ellipse having centre $(x, y) = (h, k)$ is given by

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

TABLE I

PARABOLA SYMMETRIC ABOUT Y-AXIS

Parabola	$y^2 = 4ax$	$y^2 = -4ax$
Focus	$(a, 0)$	$(-a, 0)$
Equation Directrix	$x + a = 0$	$x - a = 0$
Length of Latus rectum	$4a$	$4a$
Co-ordinators of end pts of L.R	$(a, 2a), (a, -2a)$	$(-a, 2a), (-a, -2a)$
Axis of symmetry	X-axis	X-axis
Equation of axis	$y = 0$	$y = 0$
Tangent at vertex	Y-axis	Y-axis
Focal distance of P(x_1, y_1)	$\|x_1 + 1\|$	$\|a - x_1\|$

TABLE II

PARABOLA SYMMETRIC ABOUT X-AXIS

Parabola	$x^2 = 4by$	$x^2 = -4by$
Focus	(o, b)	$(0, -b)$
Equation of Directrix	$y + a = 0$	$y - a = 0$
Length of Latus rectum	$4b$	$4b$
Co-Ordinates of end points of L.R	$(2b, b), (-2b, b)$	$(2b, -b), (-2b, -b)$
Axis of symmetry	Y-axis	Y-axis
Equation of axis	$x = 0$	$x = 0$
Tangent at vertex	X-axis	X-axis
Focal distance of point P(x_1, y_1)	$\|y_1 + 1\|$	$\|a - y_1\|$

TABLE III

STANDARD HORIZONTAL AND VERTICAL ELLIPSE

Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$
Centre	$(0, 0),$	$(0, 0)$
Vertex	$(a, 0), (-a, 0)$	$(0, b), (0, -b)$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Relation between a^2 and b^2	$b^2 = a^2(1 - e^2)$	$a^2 = b^2(1 - e^2)$
Eccentricity	$e = \frac{\sqrt{a^2 - b^2}}{a}$	$e = \frac{\sqrt{a^2 - b^2}}{a}$
CO-ordinates of Focci	$(ae, 0), (-ae, 0)$	$(0, be), (0, -be)$
Equation of Directrics	$x \pm \frac{a}{e} = 0$	$x \pm \frac{b}{e} = 0$
Distance between Focci	$2ae$	$2be$
Distance between Directrics	$\frac{2a}{e}$	$\frac{2b}{e}$

TABLE IV

TRANSVERSE AND VERTICAL AXIS HYPERBOLA

Ellipse	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Centre	$(0, 0),$	$(0, 0)$
Vertex	$(a, 0), (-a, 0)$	$(0, b), (0, -b)$
Length of Transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Relation between a^2 and b^2	$b^2 = a^2(e^2 - 1)$	$a^2 = b^2(e^2 - 1)$
Eccentricity	$e = \frac{\sqrt{a^2 + b^2}}{a}$	$e = \frac{\sqrt{a^2 + b^2}}{a}$
CO-ordinates of Focci	$(ae, 0), (-ae, 0)$	$(0, be), (0, -be)$
Equation of directrics	$x \pm \frac{a}{e} = 0$	$x \pm \frac{b}{e} = 0$
Distance between focci	$2ae$	$2be$
Distance between directrics	$\frac{2a}{e}$	$\frac{2b}{e}$