Chapter:3 2D and 3D Transformation

Transformation

Transformation means that dynamical some graphics into one thing else by applying rules. we will have numerous styles of transformations like translation, scaling up or down, rotation, shearing, etc. once a change takes place on a 2nd plane, it's known as 2nd transformation.

Transformations play a vital role in lighting tricks to reposition the graphics on the screen and alter their size or orientation.

Homogenous Coordinates

To perform a sequence of transformation like translation followed by rotation and scaling, we'd like to follow a sequent method method

Translate the coordinates,

Rotate the translated coordinates, and then

Scale the revolved coordinates to complete the composite transformation.

To shorten this method, we've got to use 3×3 transformation matrix rather than 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we've got to feature an additional dummy coordinate W.

In this manner, we will represent the purpose by three numbers rather than a pair of numbers, that is termed homogeneous arrangement. during this system, we will represent all the transformation equations in matrix operation. Any Cartesian point P[*X*,*Y*], X,Y can be converted to homogenous coordinates by P' (X_h, Y_h, h).

Transformation Types:

1) **Translation**
2) **Scaling**
3) **Reflection**
4) **Rotation**
5) **Shear / Skewing**

Translation

A translation moves an object(Shape) to a different position on the screen. You can translate a point in 2D by adding translation coordinate (t_x, t_y) to the original coordinate *X*,*Y* , to get the new coordinate X′,Y′.

If the P(x,y)=P(6,6) and new point after Transation is

P'(x',y') = P'(12,12) then what are the Translation values $t_{x=6}$ and $t_{y=6}$?

P' = P + T where T=t_x, t_y

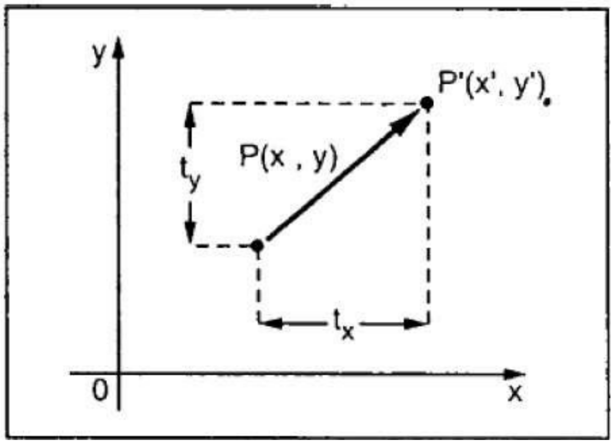


Figure 1 : Translating point p to p'

From the above figure, you can write that −

$X' = X + t_x$

$Y' = Y + t_y$

The pair (t_x, t_y) is called the **translation vector or shift vector** or **Translation Distance**. The above equations can also be represented using the **column vectors.**

***P* = [*X*]/[*Y*]**

P' = [*X′*]/[*Y′*]

T = [t_x]/[t_y]

We can write it as −

P' = P + T

1) What are the coordinates for A, B, C? What are the coordinates A', B'. C'? Are the new coordinates same or different? If different then what is the translation vector T value (t_x,t_y) ie t_x=6 and t_y=0
2) A(-4,5) B(-1,1) C(-4,-1)
3) A'(2,5) B'(5,1) C'(2,-1)

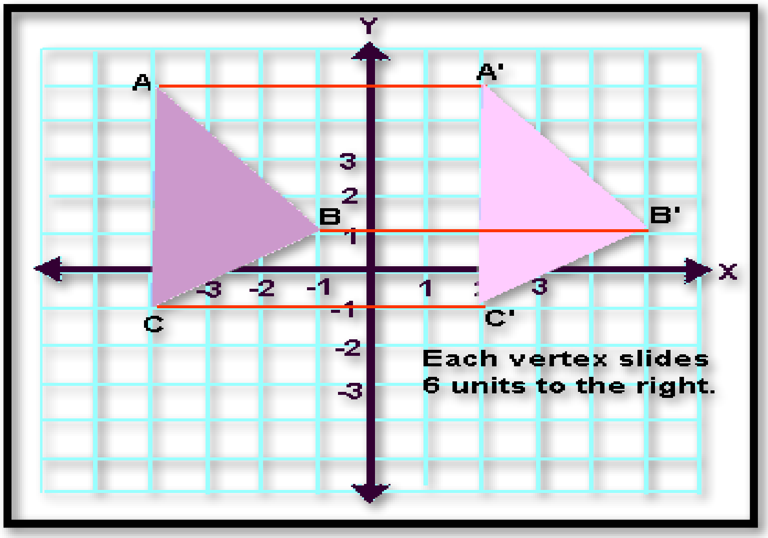


Figure 2 : Translating pont ABC to A'B'C'

Q2. Find the Translation vectors and all the New and Old Vertex values

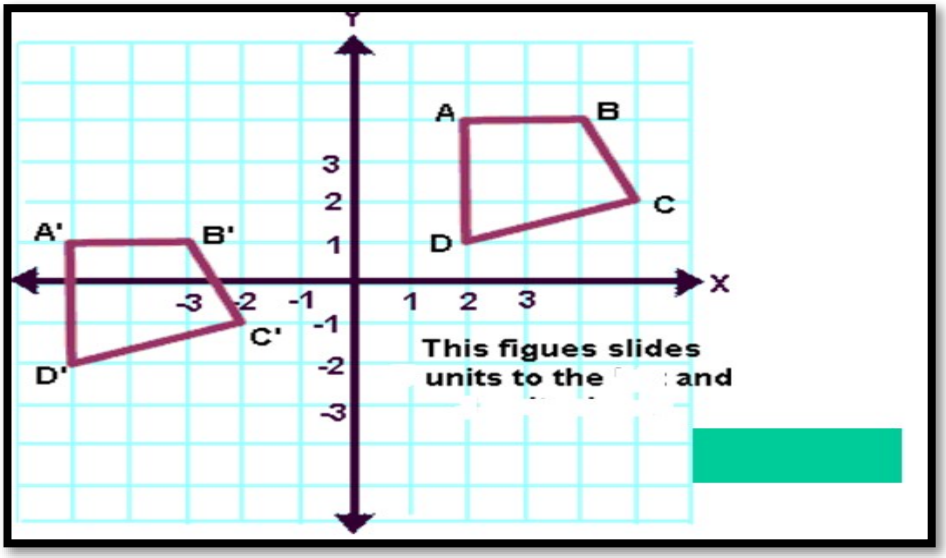


Figure 3 : Translating points ABCD to A'B'C'D'

A(2,4) B(4,4) C(5,2) and D(2,1)

A'(-5,1) B'(-3,1) C'(-2,-1) and D'(-5,-2)

(tx,ty)=(-7,-3)

Y'=y+ty

Ty=y'-y

Rotation

In rotation, we rotate the object at particular angle θ theta from its origin. From the following figure, we can see that the point P -X,Y is located at angle φ from the horizontal X coordinate with distance r from the origin.

Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P' - X′,Y′.

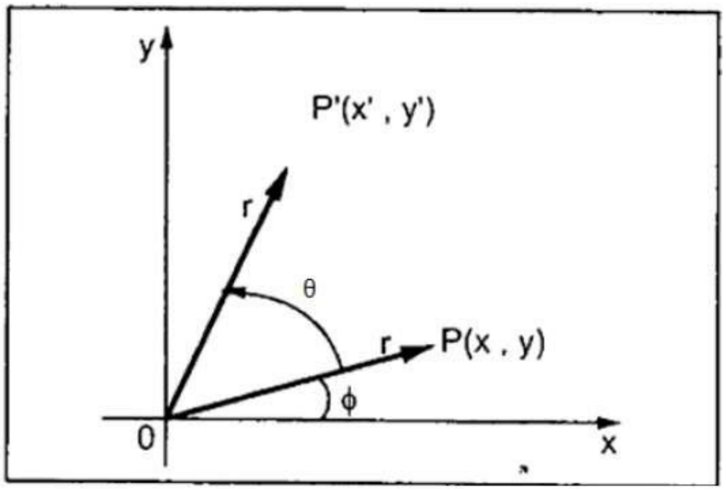


Figure 4 : Rotating point P to P'

Using standard trigonometric the original coordinate of point P*X*,*Y*X,Y can be represented as X=rcosϕ......(1)

Y=rsinϕ......(2)

Same way we can represent the point P' -′X′,Y′ as −

$x' = r\cos(\phi+\theta)$

$y' = r\sin(\phi+\theta)$

Sum and Difference of Angles

$$\sin(A+B) = \sin(A)\cdot\cos(B) + \cos(A)\cdot\sin(B)$$
$$\sin(A-B) = \sin(A)\cdot\cos(B) - \cos(A)\cdot\sin(B)$$

$$\cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)$$
$$\cos(A-B) = \cos(A)\cdot\cos(B) + \sin(A)\cdot\sin(B)$$

$x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$.......(3)

$y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta$.......(4)

Substituting equation 1 & 2 in 3 & 4 respectively, we will get

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

Representing the above equation in matrix form,

$$[X'Y'] = [XY]\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} OR$$

P' = P . R

Where R is the rotation matrix

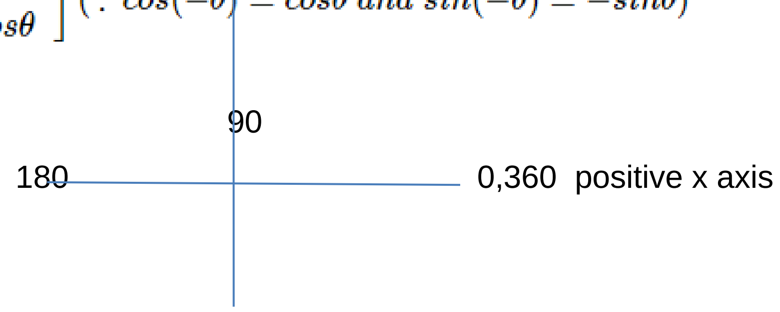
$$R = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

The rotation angle can be positive and negative.

For positive rotation angle, we can use the above rotation matrix. However, for negative angle rotation, the matrix will change as shown below −

$$R = \begin{bmatrix} cos(-\theta) & sin(-\theta) \\ -sin(-\theta) & cos(-\theta) \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} (\because cos(-\theta) = cos\theta \; and \; sin(-\theta) = -sin\theta)$$



-y axis

270

Rules Table

Trigonometry Table

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Figure 5 : Trigonometry Table

Example1:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

(x,y)=(4,4)

Rotation Angle(θ) = 30 degree

Find new coordinates(x’,y’) after applying the rotation.

$x' = x\cos\theta - y\sin\theta$

x’=4cos30-4sin 30

x’=4 $\frac{\sqrt{3}}{2}$ -4(1/2)

x’=2 $\sqrt{3}$ -2

x’=2(1.73-1)

x’=1.46

$y' = x\sin\theta + y\cos\theta$

y’=4sin30+4cos30

$=4(1/2)+4\frac{\sqrt{3}}{2}$

$=2+2\sqrt{3}$

=2(1+1.73)

Y’=5.46

Conclusion: After applying rotation at angle 30 degree the new coordinates found are (1.46,5.46)

2) Rotation

This rotation is achieved by using the following rotation equations-

- X’ = X* cosθ – Y * sinθ
- Y’ = X * sinθ + Y*cosθ

<u>Example</u>

Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

We rotate a polygon by rotating each vertex of it with the same rotation angle.

Given-

- Old corner coordinates of the triangle = A (0, 0), B(1, 0), C(1, 1)
- Rotation angle = θ = 90º

For Coordinates A(0, 0)

Let the new coordinates of corner A after rotation = (x’,y’).

Applying the rotation equations, we have-

X’

= x*cosθ – Y* sinθ

= 0 * cos90º – 0 * sin90º

= 0

Y’

= x* sinθ +y* cosθ

= 0*sin90º + 0 * cos90º

= 0

Thus, New coordinates of corner A after rotation = (0, 0).

For Coordinates B(1, 0)

Let the new coordinates of corner B after rotation = (x’,y’).

X’

$= x^*\cos\theta - y^*\sin\theta$

$= 1 \times \cos 90^o - 0 \times \sin 90^o$

$= 0$

Y’

$= x^*\sin\theta + y^*\cos\theta$

$= 1 \times \sin 90^o + 0 \times \cos 90^o$

$= 1 + 0$

$= 1$

Thus, New coordinates of corner B after rotation = (0, 1).

<u>For Coordinates C(1, 1)</u>

Let the new coordinates of corner C after rotation = (X_{new}, Y_{new}).

X’

$= x^*\cos\theta - y^*\sin\theta$

$= 1 \times \cos 90^o - 1 \times \sin 90^o$

$= 0 - 1$

= -1

Y'

= x*sinθ + y*cosθ

= 1 x sin90º + 1 x cos90º

= 1 + 0

= 1

Thus, New coordinates of corner C after rotation = (-1, 1).

Thus, New coordinates of the triangle after rotation = A (0, 0), B(0, 1), C(-1, 1).

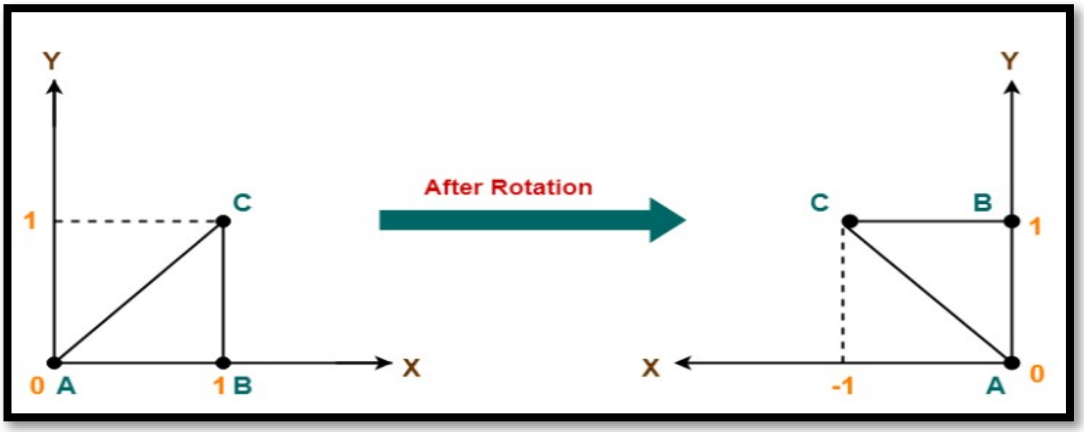


Figure 6 : Rotation applied to point ABC

3) Scaling

To change the size of an object, scaling transformation is used. In the scaling process, you either expand or compress the dimensions of the object. Scaling can be achieved by multiplying the original coordinates of the object with the scaling factor to get the desired result.

Let us assume that the original coordinates are X,Y, the scaling factors are (S_X, S_Y), and the produced coordinates are X′,Y′. This can be mathematically represented as shown below −

$X' = X \cdot S_X$ and

$Y' = Y \cdot S_Y$

The scaling factor S_X, S_Y scales the object in X and Y direction respectively. The above equations can also be represented in matrix form as below −

(X′Y′)=(XY)[Sx00Sy]

OR

P’ = P . S

Where S is the scaling matrix. The scaling process is shown in the following figure.

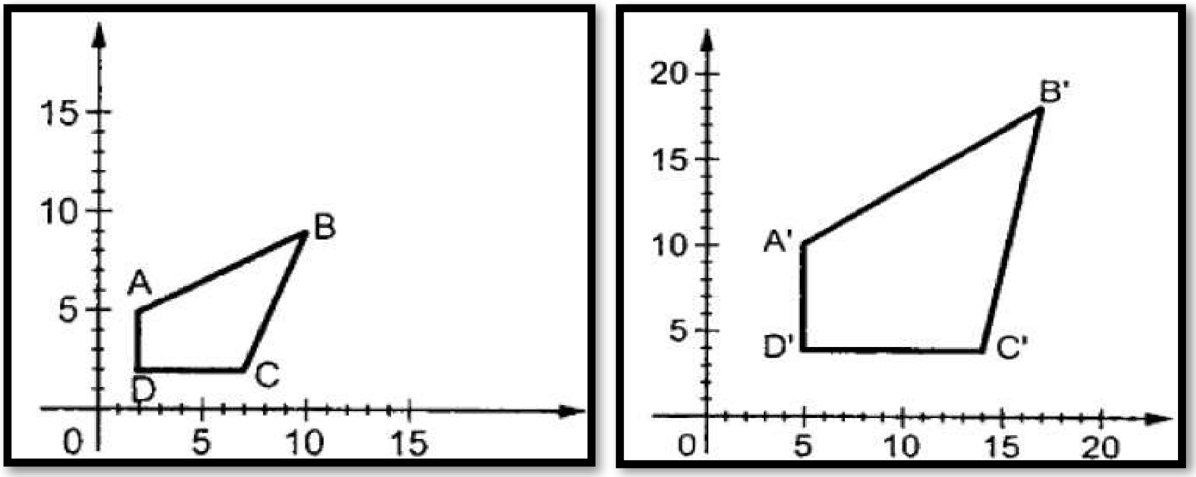


Figure 7: Scaling Transformation

If we provide values less than 1 to the scaling factor S, then we can reduce the size of the object.

If we provide values greater than 1, then we can increase the size of the object.

Example

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

- Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Scaling factor along X axis = Sx=2
- Scaling factor along Y axis = Sy=3

For Coordinates A(0, 3):x=0 and y=3

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X' = X*S_x = 0 \times 2 = 0$
- $Y' = Y* S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = A'(0, 9).

For Coordinates B(3, 3) : B'(6,9)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0): C'(6,0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0) :D'(0,0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

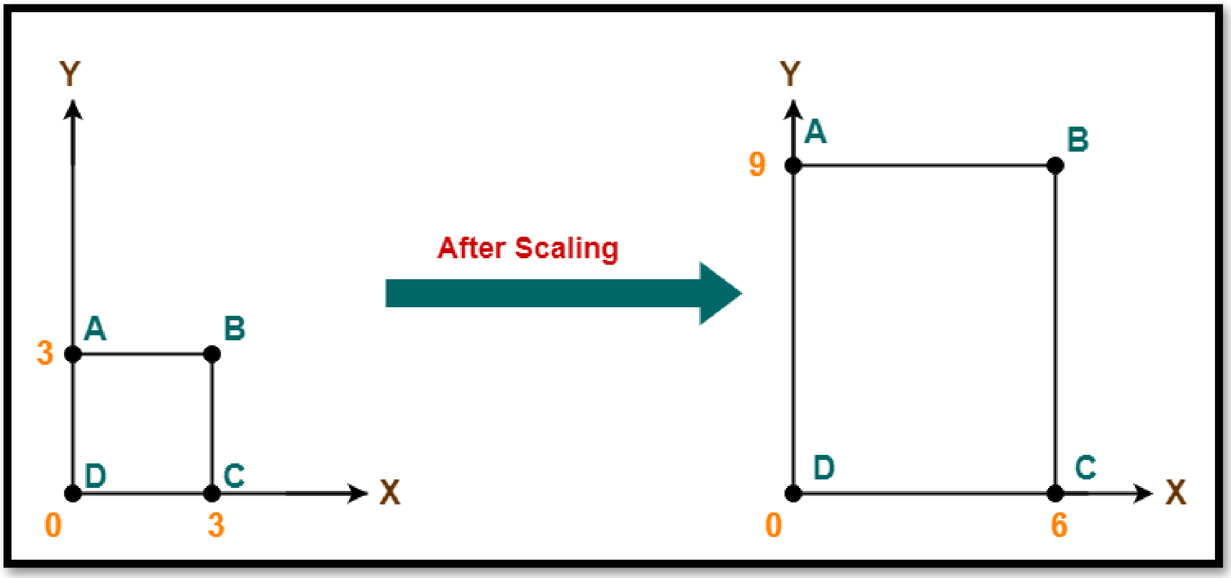


Figure 8: Scaling Transformation

Reflection

Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180°. In reflection transformation, the size of the object does not change.

The following figures show reflections with respect to X and Y axes, and about the origin respectively.

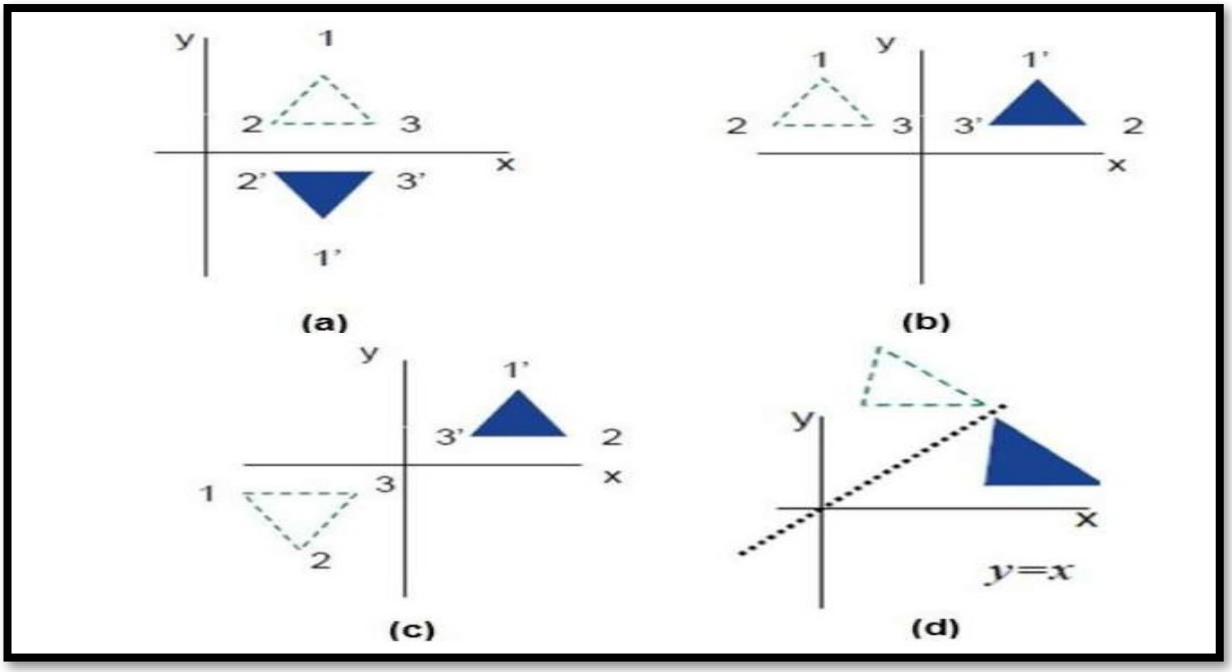


Figure 9: Reflection Transformation

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations- (X’ and Y’ are the new coordinates)

- X’ = X
- Y’ = -Y

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)

Figure 10 : Reflection along X axis

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

Figure 11 : Reflection along X axis(3*3 matrix)

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations-

- $X_{new} = -X_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

Figure 12 : Reflection along Y axis

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)

Figure 13 : Reflection along Y axis(3*3 matrix)

Example

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).

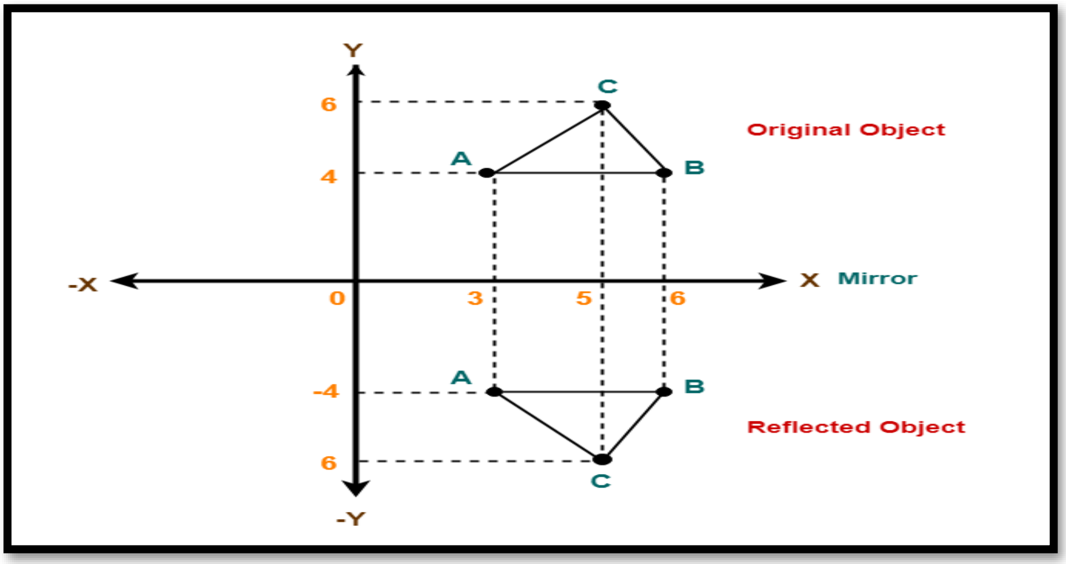


Figure 14 : Reflection Transformation

Example:

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the Y axis

<u>For Coordinates A(3, 4)</u>

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -3$

- $Y_{new} = Y_{old} = 4$

Thus, New coordinates of corner A after reflection = (-3, 4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -6$
- $Y_{new} = Y_{old} = 4$

Thus, New coordinates of corner B after reflection = (-6, 4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -5$
- $Y_{new} = Y_{old} = 6$

Thus, New coordinates of corner C after reflection = (-5, 6).

Thus, New coordinates of the triangle after reflection = A (-3, 4), B(-6, 4), C(-5, 6).

Shear

A transformation that slants the shape of an object is called the shear transformation. There are two shear transformations X-Shear and Y-Shear. One shifts X coordinates values and other shifts Y coordinate values. However; in both the cases only one coordinate changes its coordinates and other preserves its values. Shearing is also termed as Skewing.

X-Shear

The X-Shear preserves the Y coordinate and changes are made to X coordinates, which causes the vertical lines to tilt right or left as shown in below figure.

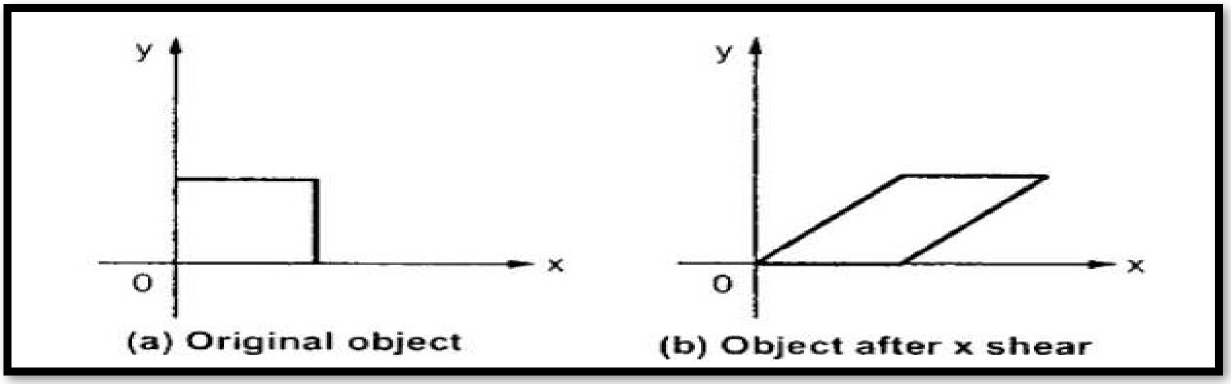


Figure 15 : x-shearTransformation

Shearing in X Axis-

$X_{new} = X_{old} + Sh_x \times Y_{old}$

$Y_{new} = Y_{old}$

The transformation matrix for X-Shear can be represented as –

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

Figure 16 : Shearing Matrix across X axis

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

(Homogeneous Coordinates Representation)

Figure 17 : Shearing 3*3 Matrix across X axis

Y-Shear

The Y-Shear preserves the X coordinates and changes the Y coordinates which causes the horizontal lines to transform into lines which slopes up or down as shown in the following figure.

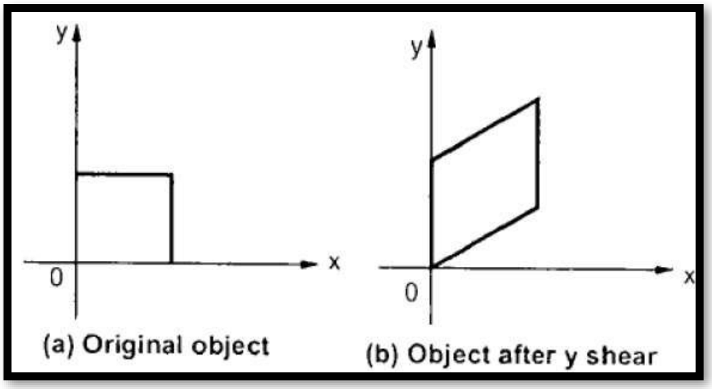


Figure 18 : ShearTransformation

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$

The Y-Shear can be represented in matrix from as −

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

Figure 19 : Shearing 2*2 Matrix across Y axis

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

Figure 20 : Shearing 3*3 Matrix across Y axis

Example:

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Given-

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

<u>**Shearing in X Axis-**</u>

<u>For Coordinates A(1, 1)</u>

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

<u>For Coordinates B(0, 0)</u>

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

<u>For Coordinates C(1, 0)</u>

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$

- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \text{ x } X_{old} = 1 + 2 \text{ x } 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \text{ x } X_{old} = 0 + 2 \text{ x } 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).

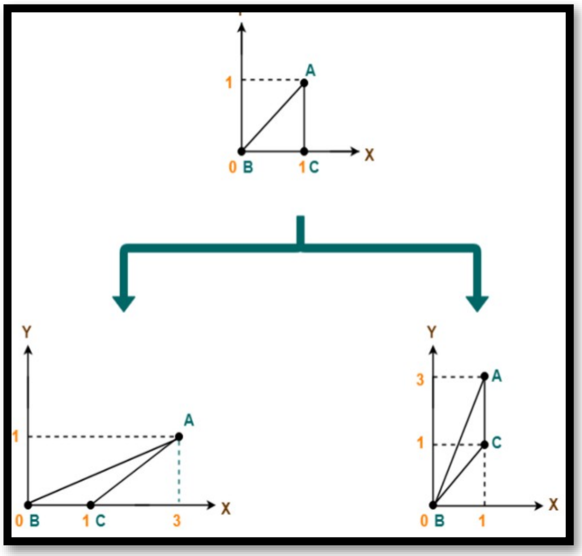


Figure 21 : Shear Transformation

Composite Transformation

If a transformation of the plane T1 is followed by a second plane transformation T2, then the result itself may be represented by a single transformation T which is the composition of T1 and T2 taken in that order. This is written as T = T1·T2.

Composite transformation can be achieved by concatenation of transformation matrices to obtain a combined transformation matrix.

A combined matrix −

[T][X] = [X] [T1] [T2] [T3] [T4] …. [Tn]

Where [Ti] is any combination of

- Translation
- Scaling
- Shearing
- Rotation
- Reflection

The change in the order of transformation would lead to different results, as in general matrix multiplication is not cumulative, that is [A] . [B] ≠ [B] . [A] and the order of multiplication.

The basic purpose of composing transformations is to gain efficiency by applying a single composed transformation to a point, rather than applying a series of transformation, one after another.

For example, to rotate an object about an arbitrary point (X_p, Y_p), we have to carry out three steps

- **Translate point (X_p, Y_p) to the origin.**
- **Rotate it about the origin.**
- **Finally, translate the centre of rotation back where it belonged.**

Three Dimensional Transformations

The geometric transformations play a vital role in generating images of three Dimensional objects with the help of these transformations. The location of objects relative to others can be easily expressed. Sometimes viewpoint changes rapidly, or sometimes objects move in relation to each other. For this number of transformation can be carried out repeatedly.

For 2D transformation → use 2 X 2 matrix (2 row n 2columns)

For 3D transformation → use 3 X 3 matrix for linear algebraic equations/ use 4 X 4 matrix

Translation

It is the movement of an object from one position to another position.
Translation is done using translation vectors.
There are three vectors in 3D instead of two. These vectors are in x, y, and z directions.
Translation in the x-direction is represented using T_x.
The translation is y-direction is represented using T_y.

The translation in the z- direction is represented using T_z.

If P is a point having co-ordinates in three directions (x, y, z) is translated, then after translation its coordinates will be (x^1 y^1 z^1) after translation. T_x T_y T_z are translation vectors in x, y, and z directions respectively.

$x^1=x+ T_x$

$y^1=y+T_y$

$z^1=z+ T_z$

Three-dimensional transformations are performed by transforming each vertex of the object. If an object has five corners, then the translation will be accomplished by translating all five points to new locations. Following figure 1 shows the translation of point figure 2 shows the translation of the cube.

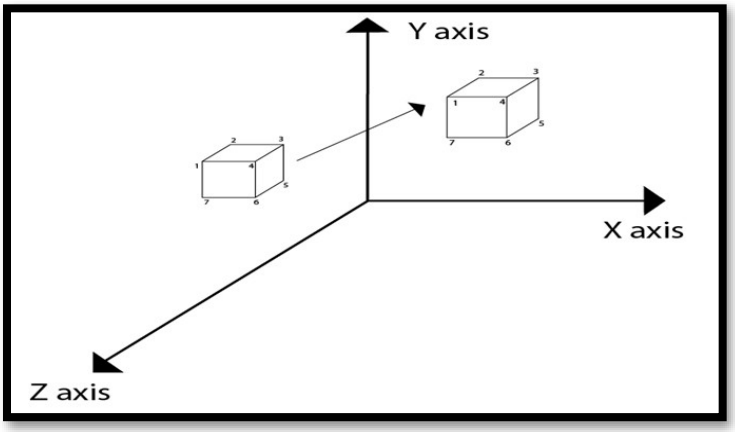


Figure 22 : Translation Transformation across 3D plane

Matrix for translation

Matrix for translation

$$\begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{Bmatrix} \text{ or } \begin{Bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

Matrix representation of point translation

Point shown in fig is (x, y, z). It become (x^1, y^1, z^1) after translation. T_x T_y T_z are translation vector.

$$\begin{pmatrix} x^1 \\ y^1 \\ z^1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Figure 23: Matrix representation of Translation

Example: A point has coordinates in the x, y, z direction i.e., (5, 6, 7). The translation is done in the x-direction by 3 coordinate and y direction. Three coordinates and in the z-direction by two coordinates. Shift the object. Find coordinates of the new position.

Solution: Co-ordinate of the point are (5, 6, 7) in 3D plane x=5, y=6 and z=7

Translation vector in x direction =Tx = 3

Translation vector in y direction = Ty =3

Translation vector in z direction = Tz=2

$x^1 = x + T_x$

$y^1 = y + T_y$

$z^1 = z + T_z$

New coordinates of point (8,9,9)

Translation matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{pmatrix}$$

Multiply co-ordinates of point with translation matrix

$$\begin{bmatrix} X^1 \\ Y^1 \\ Z^1 \\ 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

1st Matrix : 4 X 4

2nd Matrix : 1 X 4 then After doing matrix multiplication the Resulting matrix [8 9 9 1]

Rule for Matrix Multiplication: No of Rows of 1st Matrix = No of columns of 2nd Matrix

To multiply an **m×n** matrix by an **n×p** matrix, the **n**s must be the same, and the result is an **m×p** matrix.

m x n **X** n x p ⇒ m x p

Matrix is Represented as : Rows X Columns

x becomes $x^1=8$

y becomes $y^1=9$

z becomes $z^1=9$

Example 2:

Given a 3D object with coordinate points A(0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0). Apply the translation with the distance 1 towards X axis, 1 towards Y axis and 2 towards Z axis and obtain the new coordinates of the object.

Given-

- Old coordinates of the object = A (0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0)
- Translation vector = $(T_x, T_y, T_z) = (1, 1, 2)$

<u>For Coordinates A(0, 3, 1)</u>

Let the new coordinates of A = $(X_{new}, Y_{new}, Z_{new})$.

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$
- $Z_{new} = Z_{old} + T_z = 1 + 2 = 3$

Thus, New coordinates of A = (1, 4, 3).

<u>For Coordinates B(3, 3, 2)</u>

Let the new coordinates of B = $(X_{new}, Y_{new}, Z_{new})$.

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$
- $Z_{new} = Z_{old} + T_z = 2 + 2 = 4$

Thus, New coordinates of B = (4, 4, 4).

<u>For Coordinates C(3, 0, 0)</u>

Let the new coordinates of C = $(X_{new}, Y_{new}, Z_{new})$.

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$
- $Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of C = (4, 1, 2).

<u>For Coordinates D(0, 0, 0)</u>

Let the new coordinates of D = $(X_{new}, Y_{new}, Z_{new})$.

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$

- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$
- $Z_{new} = Z_{old} + T_z = 0 + 2 = 2$

Thus, New coordinates of D = (1, 1, 2).

New coordinates of the object = A (1, 4, 3), B(4, 4, 4), C(4, 1, 2), D(1, 1, 2).

Scaling

Scaling is used to change the size of an object. The size can be increased or decreased. The scaling three factors are required S_x S_y and S_z.

S_x=Scaling factor in x- direction
S_y=Scaling factor in y-direction
S_z=Scaling factor in z-direction

Matrix for Scaling

$$\left\{\begin{array}{cccc} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{array}\right\}$$

This scaling is achieved by using the following scaling equations-

- $X_{new} = X_{old} \times S_x$
- $Y_{new} = Y_{old} \times S_y$
- $Z_{new} = Z_{old} \times S_z$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix

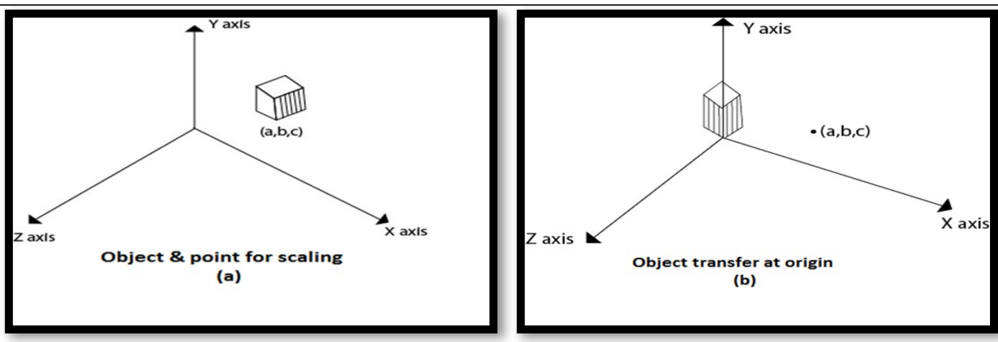
Figure 24: 3D Scaling Matrix

Scaling of the object relative to a fixed point

Following are steps performed when scaling of objects with fixed point (a, b, c). It can be represented as below:

1. Translate fixed point to the origin
2. Scale the object relative to the origin
3. Translate object back to its original position.

Note: If all scaling factors S_x=S_y=S_z.Then scaling is called as uniform. If scaling is done with different scaling vectors, it is called a differential scaling.



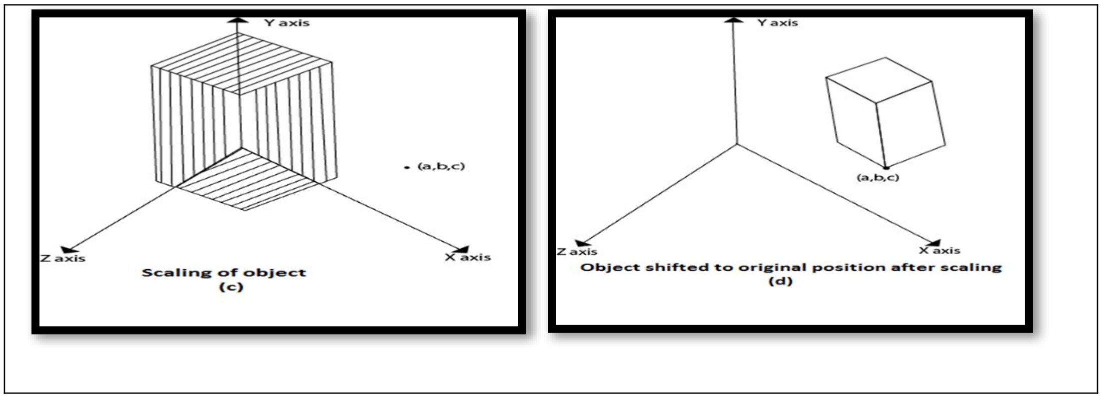


Figure 25: 3D Scaling Transformation

<u>Example:</u>

Given a 3D object with coordinate points A(0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0). Apply the scaling parameter 2 towards X axis, 3 towards Y axis and 3 towards Z axis and obtain the new coordinates of the object.

- Old coordinates of the object = A (0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0)
- Scaling factor along X axis = 2
- Scaling factor along Y axis = 3
- Scaling factor along Z axis = 3

<u>For Coordinates A(0, 3, 3)</u>

Let the new coordinates of A after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$
- $Z_{new} = Z_{old} \times S_z = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9, 9).

For Coordinates B(3, 3, 6)

Let the new coordinates of B after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$
- $Z_{new} = Z_{old} \times S_z = 6 \times 3 = 18$

Thus, New coordinates of corner B after scaling = (6, 9, 18).

For Coordinates C(3, 0, 1)

Let the new coordinates of C after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$
- $Z_{new} = Z_{old} \times S_z = 1 \times 3 = 3$

Thus, New coordinates of corner C after scaling = (6, 0, 3).

<u>For Coordinates D(0, 0, 0)</u>

Let the new coordinates of D after scaling = (X_{new}, Y_{new}, Z_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$
- $Z_{new} = Z_{old} \times S_z = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0, 0).

3D Rotation Transformation

2D Rotation: Represented over x and y axis.

- Rotation performed across the origin
- Angle used for representation of object = ϕ
- Angle of Rotation = θ
- Power coordinate representation

x'= xcosθ – ysinθ

y' =xsinθ + ycosθ

3D Rotation

Always Rotation is applied to corresponding axis, that axis new coordinate value will

Be same as the old coordinate value

Rotation is applied across X axis : X'=X , Y? and Z?

Rotation is applied across Y axis :Y'=Y , X? and Z?

Rotation is applied across Z axis :Z'=Z, X? and Y?

Inverse Transformation 4x4 Matrix

1	**0**	**0**	**0**
0	**1**	**0**	**0**
0	**0**	**1**	**0**
0	**0**	**0**	**1**

Consider a point object O has to be rotated from one angle to another in a 3D plane.

Let-

- Initial coordinates of the object O = (X, Y, Z)
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X', Y', Z')

In 3 dimensions, there are 3 possible types of rotation-

- X-axis Rotation

- Y-axis Rotation
- Z-axis Rotation

For X-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- X’ = X
- Y’ = Ycosθ – Zsinθ
- Z’ = Ysinθ + Zcosθ

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix
(For X-Axis Rotation)

Figure 26: 3D Rotation Matrix

3D-Rotation Matrix applied across X axis

1	0	0	0
0	cosθ	-sinθ	0
0	sinθ	cosθ	0
0	0	0	1

O’ = R * O

For Y-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- $X' = Z\sin\theta + X\cos\theta$
- $Y' = Y$
- $Z' = Y\cos\theta - X\sin\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Y-Axis Rotation)

Figure 27: 3D Rotation Matrix for Y-axis

3D - Rotation Transformation matrix applied across Y axis

cosθ	**0**	**sinθ**	**0**
0	**1**	**0**	**0**
-sinθ	**0**	**cosθ**	**0**
0	**0**	**0**	**1**

<u>For Z-Axis Rotation-</u>

This rotation is achieved by using the following rotation equations-

- X' = Xcosθ – YSinθ
- Y' = Xsinθ + Ycosθ
- Z' = Z

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

Figure 28 : 3D Rotation Matrix for Z-axis

3D Rotation Transformation Z axis

cos**θ**	-sin**θ**	0	0
sin**θ**	cos**θ**	0	0
0	0	1	0
0	0	0	1

Example:

Given a homogeneous point P(1, 2, 3). Apply rotation **θ** =90 degree towards X, Y and Z axis and find out the new coordinate points.

Solution-

Given-

- Old coordinates = (X, Y, Z) = (1, 2, 3)
- Rotation angle = θ = 90^o

For X-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have- (x=1,y=2,z=3)

- X' = X = 1
- Y' = Ycosθ – Zsinθ = 2cos90 – 3sin90 = 2x0 – 3x1 =-3
- Z' = Ysinθ + Zcosθ =2sin90 +3cos90 = 2x1 + 3x 0 =2
- Thus, New coordinates after rotation = (1, -3, 2)

For Y-Axis Rotation-

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have- (X,Y,Z)= (1,2,3)

- $X' = Z\sin\theta + X\cos\theta = 3\sin 90 + 1\cos 90 = 3 \times 1 + 1 \times 0 = 3$
- $Y' = Y = 2$
- $Z' = Y\cos\theta - X\sin\theta = 2 \times 0 - 1 \times 1 = -1$

Thus, New coordinates after rotation = (3, 2, -1).

<u>For Z-Axis Rotation-</u>

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new}).

Applying the rotation equations, we have-

- $X' = X\cos\theta - Y\sin\theta = 0 - 2 = -2$
- $Y' = X\sin\theta + Y\cos\theta = 1 + 0 = 1$
- $Z' = Z = 3$

Thus, New coordinates after rotation = (-2, 1, 3).

<u>3D Shearing in 3D Graphics</u>

1. Shearing in X direction
2. Shearing in Y direction
3. Shearing in Z direction

Consider a point object O has to be sheared in a 3D plane.

Let-

- Initial coordinates of the object O = (X, Y, Z)
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y

- Shearing parameter towards Z direction = Sh_z
- New coordinates of the object O after shearing = (X', Y', Z')

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- X' = X
- $Y' = Y + Sh_y * X$
- $Z' = Z + Sh_z * X$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix
(In X axis)

Figure 29: 3D Shearing Matrix for X-axis

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X' = X + Sh_x * Y$
- Y' = Y

- $Z' = Z + Sh_z * Y$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Y axis)

Figure 30: 3D Shearing Matrix for Y- axis

Shearing in Z Axis-

Shearing in Z axis is achieved by using the following shearing equations-

- $X' = X + Sh_x \times Z$
- $Y' = Y + Sh_y \times Z$
- $Z' = Z$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Z axis)

Figure 31: 3D Shearing Matrix for Z-axis

<u>Problem-01:</u>

Given a 3D triangle with points A(0, 0, 0), B(1, 1, 2) and C(1, 1, 3). Apply shear parameter 2 on X axis, 2 on Y axis and 3 on Z axis and find out the new coordinates of the object.

A(0,0,0) : A(x=0,y=0,z=0)

Shx=2

Shy=2

Shz=3

<u>Solution-</u>

Given-

- Old corner coordinates of the triangle = A (0, 0, 0), B(1, 1, 2), C(1, 1, 3)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

- Shearing parameter towards Y direction (Sh_z) = 3

Shearing in X Axis-

For Coordinates A(0, 0, 0)=A(x=0,y=0,z=0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- X'= X = 0
- Y' = Y + Sh_y x X = 0 + 2 x 0 = 0
- Z`= Z + Sh_z x X = 0 + 3 x 0 = 0

Thus, New coordinates of corner A after shearing = A'(0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- X' = X = 1
- Y' = Y + Sh_y * X = 1 + 2 x 1 = 3
- Z' = Z + Sh_z * X = 2 + 3 x 1 = 5

Thus, New coordinates of corner B after shearing = (1, 3, 5).

For Coordinates C(1, 1, 3)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- X' = X = 1
- Y' = Y + Sh_y * X = 1 + 2 x 1 = 3
- Z' = Z + Sh_z * X = 3 + 3 x 1 = 6

Thus, New coordinates of corner C after shearing = (1, 3, 6).

Thus, New coordinates of the triangle after shearing in X axis = A' (0, 0, 0), B'(1, 3, 5), C'(1, 3, 6).

Shearing in Y Axis-

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- Y' = Y = 0
- $Z_{new} = Z_{old} + Sh_z \times Y_{old} = 0 + 3 \times 0 = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

<u>For Coordinates B(1, 1, 2)</u>

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X' = X + Sh_x \times Y = 1 + 2 \times 1 = 3$
- Y' = Y = 1
- $Z' = Z + Sh_z \times Y = 2 + 3 \times 1 = 5$

Thus, New coordinates of corner B after shearing = (3, 1, 5).

<u>For Coordinates C(1, 1, 3)</u>

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X' = X + Sh_x \times Y = 1 + 2 \times 1 = 3$
- $Y' = Y = 1$
- $Z' = Z + Sh_z \times Y = 3 + 3 \times 1 = 6$

Thus, New coordinates of corner C after shearing = (3, 1, 6).

Thus, New coordinates of the triangle after shearing in Y axis = A (0, 0, 0), B(3, 1, 5), C(3, 1, 6).

Shearing in Z Axis-

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 0 + 2 \times 0 = 0$
- $Z_{new} = Z_{old} = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 1 + 2 \times 2 = 5$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 1 + 2 \times 2 = 5$
- $Z_{new} = Z_{old} = 2$

Thus, New coordinates of corner B after shearing = (5, 5, 2).

For Coordinates C(1, 1, 3)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 1 + 2 \times 3 = 7$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 1 + 2 \times 3 = 7$
- $Z_{new} = Z_{old} = 3$

Thus, New coordinates of corner C after shearing = (7, 7, 3)

Thus, New coordinates of the triangle after shearing in Z axis = A (0, 0, 0), B(5, 5, 2), C(7, 7, 3).

3D Reflection in Computer Graphics-

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 3D plane.

Let-

- Initial coordinates of the object O = (X, Y, Z)
- New coordinates of the reflected object O after reflection = (X’, Y’,Z’)

In 3 dimensions, there are 3 possible types of reflection-

- Reflection relative to XY plane : X’=X and Y’=Y and Z’=-Z
- Reflection relative to YZ plane : Y’=y and Z’=Z and X’=-X
- Reflection relative to XZ plane : X’=X and Y’=-Y and Z’=Z

Reflection Relative to XY Plane:

This reflection is achieved by using the following reflection equations-

- X’ = X
- Y’ = Y
- Z’= -Z

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix
(Reflection Relative to XY plane)

Figure 32: 3D Reflection Matrix relative to XY plane

Reflection Relative to YZ Plane:

This reflection is achieved by using the following reflection equations-

- X’ = -X
- Y’ = Y
- Z’ = Z

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to YZ plane)

Figure 33: 3D Reflection Matrix relative to YZ plane

Reflection Relative to XZ Plane:

This reflection is achieved by using the following reflection equations-

- X’ = X
- Y’ = -Y
- Z’ = Z

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XZ plane)

Figure 34: 3D Reflection Matrix relative to XZ plane

O' = R * O

Example

Given a 3D triangle with coordinate points A(3, 4, 1), B(6, 4, 2), C(5, 6, 3). Apply the reflection on the XY plane and find out the new coordinates of the object.

A(3,4,1) : x'=x, y'=y and z'=-z :A(3,4,-1)

B(6,4,2) : B(6,4,-2)

C(5,6,3) : C(5,6,-3)

<u>Solution-</u>

Given-

- Old corner coordinates of the triangle = A (3, 4, 1), B(6, 4, 2), C(5, 6, 3)
- Reflection has to be taken on the XY plane
- After Reflection across XY plane new coordinates are A'(3,4,-1), B(6,4,-2) AND C(5,6,-3)

YZ: x=-x

ZX:y=-y

For Coordinates A(3, 4, 1)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = Y_{old} = 4$
- $Z_{new} = -Z_{old} = -1$

Thus, New coordinates of corner A after reflection = (3, 4, -1).

For Coordinates B(6, 4, 2)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = Y_{old} = 4$
- $Z_{new} = -Z_{old} = -2$

Thus, New coordinates of corner B after reflection = (6, 4, -2).

<u>For Coordinates C(5, 6, 3)</u>

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = Y_{old} = 6$
- $Z_{new} = -Z_{old} = -3$

Thus, New coordinates of corner C after reflection = (5, 6, -3).

Thus, New coordinates of the triangle after reflection = A (3, 4, -1), B(6, 4, -2), C(5, 6, -3).

<u>Problem-02:</u>

Given a 3D triangle with coordinate points A(3, 4, 1), B(6, 4, 2), C(5, 6, 3). Apply the reflection on the XZ plane and find out the new coordinates of the object.

<u>Solution-</u>

Given-

- Old corner coordinates of the triangle = A (3, 4, 1), B(6, 4, 2), C(5, 6, 3)
- Reflection has to be taken on the XZ plane

For Coordinates A(3, 4, 1)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$
- $Z_{new} = Z_{old} = 1$

Thus, New coordinates of corner A after reflection = (3, -4, 1).

For Coordinates B(6, 4, 2)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$
- $Z_{new} = Z_{old} = 2$

Thus, New coordinates of corner B after reflection = (6, -4, 2).

For Coordinates C(5, 6, 3)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$
- $Z_{new} = Z_{old} = 3$

Thus, New coordinates of corner C after reflection = (5, -6, 3).