

RATIO

- If $\frac{a}{b} = \frac{p}{q}$
 $\Rightarrow a = px$ and $b = qx$ [where x is the common factor]
Note: If a question involves multiple ratios, their common factor need not be same.
- If $\frac{a}{b} = \frac{p}{q}$ and $a + b = T$
 $\Rightarrow a = \frac{p}{p+q} \times T$ and $b = \frac{q}{p+q} \times T$
- If $ax = by$
 $\Rightarrow \frac{a}{b} = \frac{y}{x}$
Example: If $3x = 5y \Rightarrow \frac{a}{b} = \frac{5}{3}$
- If $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$
 $\Rightarrow a : b : c = x : y : z$
 [Ratio of numerators is same as ratio of denominators]
Example: If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} \Rightarrow a : b : c = 2 : 3 : 4$
- If $xa = yb = zc$
 \Rightarrow To calculate $a : b : c$ we divide all the terms by LCM(x, y, z)
Example: If $2a = 3b = 4c \Rightarrow \frac{2a}{12} = \frac{3b}{12} = \frac{4c}{12} \Rightarrow \frac{a}{6} = \frac{b}{4} = \frac{c}{3} \Rightarrow a : b : c = 6 : 4 : 3$
- If $a : b = \frac{p}{x} : \frac{q}{y}$
 \Rightarrow To simplify RHS we multiply all terms on RHS with LCM of denominators, i.e., LCM(x, y)
Example: If $a : b = \frac{2}{3} : \frac{7}{5} \Rightarrow a : b = \frac{2}{3} \times 15 : \frac{7}{5} \times 15 = 10 : 21$
- If $\frac{a}{b} = \frac{n_1}{d_1}, \frac{b}{c} = \frac{n_2}{d_2}, \frac{c}{d} = \frac{n_3}{d_3}$ and $\frac{d}{e} = \frac{n_4}{d_4}$
 $\Rightarrow a : b : c : d : e = (n_1 \times n_2 \times n_3 \times n_4) : (d_1 \times n_2 \times n_3 \times n_4) : (d_1 \times d_2 \times n_3 \times n_4) : (d_1 \times d_2 \times d_3 \times n_4) : (d_1 \times d_2 \times d_3 \times d_4)$
- Inverse/Reciprocal ratio of $a : b : c = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

RATIO OF INEQUALITIES

- If $\frac{a}{b} > 1$
 $\Rightarrow \frac{a+x}{b+x} < \frac{a}{b}$
 If same number is added to both N^r and D^r , the new ratio decreases.
- If $\frac{a}{b} < 1$
 $\Rightarrow \frac{a+x}{b+x} > \frac{a}{b}$
 If same number is added to both N^r and D^r , the new ratio increases.
- If $\frac{a}{b} = 1$
 $\Rightarrow \frac{a+x}{b+x} = \frac{a}{b}$

PROPORTIONS

- If a, b, c and d are in proportion

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow a \times d = b \times c$$

CONTINUOUS PROPORTION

- If a, b and c are in continuous proportion

$$\Rightarrow \frac{a}{b} = \frac{b}{c}$$

$$\Rightarrow b^2 = a \times c$$

PROPERTIES

If $\frac{a}{b} = \frac{c}{d}$, then

- $ad = bc$
- $\frac{b}{a} = \frac{d}{c}$
- $\frac{a+b}{b} = \frac{c+d}{d}$
- $\frac{a-b}{b} = \frac{c-d}{d}$
- $\frac{b}{a+b} = \frac{d}{c+d}$
- $\frac{b}{a-b} = \frac{d}{c-d}$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

- $k = \frac{a+c+e}{b+d+f}$
- $k = \frac{pa+qc+re}{pb+qd+re}$ [p, q, r are real numbers]

VARIATION

DIRECT VARIATION

If x is directly proportional to y

- $\Rightarrow x \propto y$
- $\Rightarrow x = ky$
- $\Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

INVERSE VARIATION

If x is inversely proportional to y

- $\Rightarrow x \propto 1/y$
- $\Rightarrow x = k/y$
- $\Rightarrow x_1 \times y_1 = x_2 \times y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

Example: Man Days

JOINT VARIATION

If x is proportional to two or more variable, then it is proportional to product of all those variables.

Example: If x directly proportional to p and q while inversely proportional to r

$$\Rightarrow x \propto \frac{pq}{r} \Rightarrow \frac{x_1 r_1}{p_1 q_1} = \frac{x_2 r_2}{p_2 q_2}$$

INDIRECT VARIATION

$x = \text{Fixed part} + \text{Variable Part}$

- Variable part is proportional to some given quantity

Example: Electricity bill, Phone bill etc.

PARTNERSHIPS

WHEN INVESTMENT DOES NOT CHANGE

- $P_A : P_B : P_C = I_A \times T_A : I_B \times T_B : I_C \times T_C$

WHEN INVESTMENT CHANGES

- $P_A : P_B = (I_{A1} \times T_{A1} + I_{A2} \times T_{A2} + \dots) : (I_{B1} \times T_{B1} + I_{B2} \times T_{B2} + \dots)$

REMEMBER

- Profit is divided amongst partners after salary is deducted for any partner (if specified in the question).

