Task 1
Part a) :

P(color is not green | vehicle is Truck) = ?

Calculating probability of "color is green"

P(color is Green) = 0.0195 + 0.0252 +
0.0160 + 0.0465
= 0.1072.

∴ P((color is not green) = 1 − P(color is green)
= 0.8928

P(vehicle is Truck) = 0.1070 + 0.0160
+ 0.0265
= 0.1495

P(color is not green | vehicle is Truck)

$$= \frac{P(\delta green \cap Truck)}{P(Truck)}$$

P(δGreen ∩ Truck) = 0.1070 + 0.0265
= 0.1$\overset{3}{\cancel{4}}$35

$$\therefore P(color\ \delta Green | Truck) = \frac{0.1\overset{3}{\cancel{4}}35}{0.1495}$$

= [illegible]

= 89.23 %

b) we want to prove that

$p(A) * P(B) = P(A \cap B)$

P(vehicle) * P(color)

consider, car as SUV &
color as green

P(Green) = 0.1072
P(SUV) = 0.4337

P(Green) * P(SUV) = 0.0465
&
from table, P(Green $\cap$ SUV) = 0.0465

As both values are same,
they are independent from
each other.

Task 2)

a) Given that,
there are 11 variables &
variable A has 7 values &
variable $B_1 \cdots B_{10}$ has 8 values.

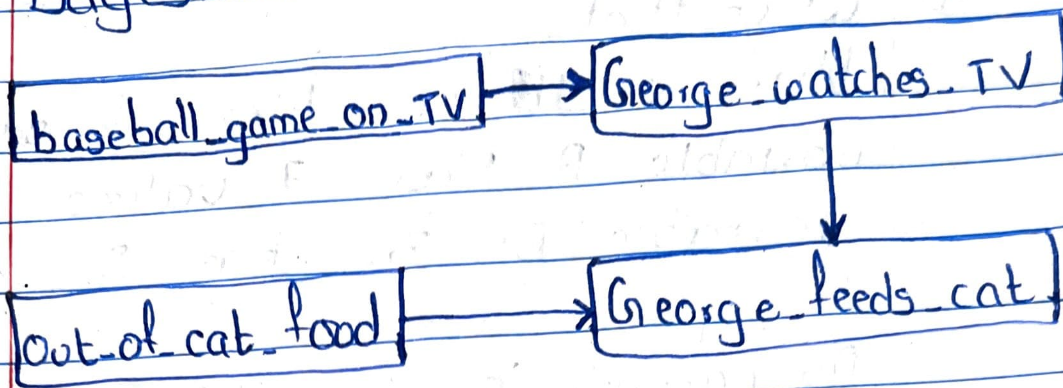
As variable B is condetionally endependent, it'll give 8^{10} values.

As variable A has 7 values.
So, total $7 * 8^{10}$ many numbers is needed to store in joint distribution table

b) we can do one thing is that by stroring only 8^{10} values.
so there will be $7+8^{10}$ values which is way less than given in a).

c) yes, senario does follow the naive-bayes model

Task 3)
Bayesian network design.



Baseball_game_on_TV has no parent

0	0.5	0.70
1	0.5	0.30
	(P)	from code

Out_of_cat_food has no parent

value	Prob	
0	0.5	0.83
1	0.5	0.17
		from code

baseball_game_on_TV is parent of Geog watches_TV

bb_g_o_TV	G_W_TV	P(bb_g_o_TU | G_W_TV)
0	0	0.88
0	1	0.12
1	0	0.07
1	1	0.93

George-watches-TV & out-of-cat-food are parents of George-feeds-cat.

G_W_TV	O_O_C_f	G_f_C	P(G_f_C | G_W_TV ∩ O_O_cf)
0	0	0	0.04
0	0	1	0.96
0	1	0	0.68
0	1	1	0.32
1	0	0	0.29
1	0	1	0.71
1	1	0	0.96
1	1	1	0.04

I've drawn tables independently, not in diagram because of space issue.

Task 5)

using inference by Enumeration.

$P(B\_g\_o\_TV \mid \neg(G\_f\_c))$

$$= \frac{P(B\_g\_o\_TV \wedge \neg G\_f\_c)}{P(\neg(G\_f\_c))}$$

$P(B\_G\_O\_TV \wedge \neg G\_f\_c) =$

$(0.3041 * 0.9279 * 0.1698 * 0.9588) +$

$(0.3041 * 0.07207 * 0.1698 * 0.6842) +$

$0.3041 * 0.9279 * 0.8301 * 0.2936) +$

$0.3041 * 0.07207 * 0.8301 * 0.0421)$

$= \underline{\underline{0.118}}$

$P(G\_f\_c) = 0.7562$

$P(\neg G\_f\_c) = 1 - 0.7562$

$= 0.2438$

$\therefore P(b\_g\_o\_TV \mid \neg(G\_f\_c))$

$$= \frac{0.118}{0.2438}$$

$= \underline{\underline{0.484}}$

Task 6)

$$\text{Entropy (class)} = -P(x) \cdot \log_2 P(x) - P(y) \cdot \log_2 P(y)$$

$$= -\frac{5}{10} \cdot \log\left(\frac{5}{10}\right) - \frac{5}{10} \log\left(\frac{5}{10}\right)$$

$$= 1$$

Information Gain of A

$$\text{Entropy(class} \mid A=1) = -\frac{3}{3} \log\left(\frac{3}{3}\right) = 0$$

$$\text{Entropy (class} \mid A=2) = -\frac{1}{4} \log_2\left(\frac{1}{4}\right) - \frac{3}{4} \log_2\left(\frac{3}{4}\right)$$

$$= 0.8113$$

$$\text{Entropy (class} \mid A=3) = -\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right)$$

$$= 0.922$$

$$\text{Gain (class} \mid A) = 1 - \left[\frac{3}{10}(0) + \frac{4}{10}(0.8113) + \left(\frac{3}{10}\right) 0.922\right]$$

$$= \underline{\underline{0.39}}$$

Information Gain of B :

$$Entropy(class \mid B=1) \Rightarrow -\frac{1}{4}\log\left(\frac{1}{4}\right) - \frac{3}{4}\log\left(\frac{3}{4}\right)$$

$$= 0.8113$$

$$Entropy(class \mid B=2) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)$$

$$= 0.8113$$

$$Entropy(class \mid B=3) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)$$

$$= 1$$

$$Gain(class \mid B) = 1 - \left[\frac{4}{10} \times 2 \times (0.8113) + \frac{2}{10}(1)\right]$$

$$= 0.151$$

Information Gain of c:

$$\text{Entropy}(\text{class} \mid c=1) = -\frac{2}{5}\log_2\left(\frac{2}{5}\right) - \frac{3}{5}\log_2\left(\frac{3}{5}\right)$$

$$= 0.97$$

$$\text{Entropy}(\text{class} \mid c=2) = -\frac{2}{4}\log\left(\frac{2}{4}\right) - \frac{2}{4}\log\left(\frac{2}{4}\right)$$

$$= 1$$

$$\text{Entropy}(\text{class} \mid c=3) = -1\log_2(1)$$

$$= 0$$

$$\text{Gain}(\text{class} \mid c) = 1 - \left[\frac{5}{10}(0.97) + \frac{4}{10}(1) + 0\right]$$

$$= \underline{\underline{0.88}}$$