For all homework throughout the semester you must do the following:

1. Explain in your own words what is being asked.
2. State your strategy for arriving at the solution.
3. Execute your strategy noting the steps.
4. ✎ Write legibly and in a logical order.

The purpose of this homework is to study and implement a filter bank based on wavelet spline filters. These filters have many interesting properties and are commonly used in signal processing.

Grading guidelines

- The theoretical part should be solved with pen and paper.
- For the experiments, you can use the programming language of your choice: C, Python, MATLAB, etc. You will not get credit for your code if you do not add comments in the code.
- You will only get credit for an experiment or a result if it includes a discussion that connects the experiment with a theoretical analysis.
- Discuss the issues that you encountered as you performed the experiments.

Theory

Consider the filter H_0 defined by

$$H_0(z) = -\frac{1}{8}z^{-2} + \frac{1}{4}z^{-1} + \frac{3}{4} + \frac{1}{4}z - \frac{1}{8}z^2 \tag{1}$$

1. Find all the zeroes and poles of H_0, and factorize the filter. Is H_0 lowpass, highpass, or band pass ?
2. Compute the frequency response $H_0(e^{j\omega})$ and show that H_0 has linear phase.
3. Plot the magnitude of the frequency response using Matlab.
4. Find the coefficients a and b of the filter
$$F_0(z) = az^{-1} + b + az \tag{2}$$
so that
$$H_0(z)F_0(z) + H_0(-z)F_0(-z) = 2. \tag{3}$$
5. Find all the zeroes and poles of F_0, and factorize the filter. Is F_0 lowpass, highpass, band pass?
6. Compute the frequency response of F_0.

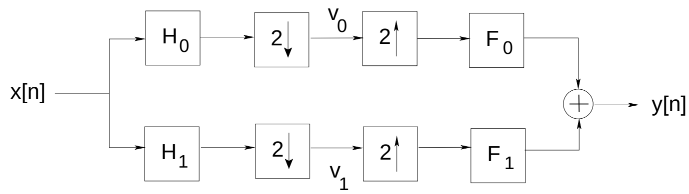

Figure 1: A maximally decimated, two channel filter bank

7. The filters H_0 and F_0 are used in the filter bank scheme described in Fig. 1. The filters H_1 and F_1 are defined by

$$F_1(z) = z^{-1}H_0(-z) \quad \text{and} \quad H_1(z) = zF_0(-z). \tag{4}$$

Give the expression of $f_1[n]$, and $h_1[n]$ in terms of $h_0[n]$ and $f_0[n]$ respectively.

8. Prove that the system provides a perfect reconstruction (up to a delay n_0 that you will determine),

$$y[n] = x[n - n_0]$$

Experiments

The goal of the experiments is to implement the filter bank scheme, and understand the concept of perceptual audio coding that is used in MP3, AAC, etc.

1. Write a MATLAB function `filtdec` that filters a sequence $x[n]$ with a filter $h[n]$, and downsamples the result by a factor 2. You should extend the signal $x[n], n = 1, \cdots, N$ on each side using the following mirroring scheme :

$$x[-n] = x[n+1], \quad x[N+n] = x[N-n+1], \quad n = 0, 1, \cdots \tag{5}$$

2. Write a MATLAB function `upfilt` that upsamples a sequence $v[n]$ by a factor 2, and filters the result with a filter $f[n]$. You should extend the signal $v[n], n = 1, \cdots, N$ on each side using the following mirroring scheme :

$$v[-n] = v[n+1], \quad v[N+n] = v[N-n+1], \quad n = 0, 1, \cdots \tag{6}$$

3. Download the audio files http://ece.colorado.edu/~fmeyer/class/ecen4632/tracks.zip

4. Load one audio file. For instance,

```
> [x,fs]=audioread('electronic1.wav');
> whos
whos
  Name           Size                Bytes  Class     Attributes

  fs             1x1                     8  double
  x         639451x1               5115608  double
```

`x` is the left channel of a stereo audio file. `fs` is the sampling rate, 22,050 Hz.

5. Apply the function `filtdec` with h_0, and h_1 to the audio signal. You obtain two signals v_0 and v_1.

6. Compute and display the magnitude of the Fourier transform of v_0 and v_1. How do the Fourier transforms compare to the frequency responses $H_0\left(e^{j\omega}\right)$ and $H_1\left(e^{j\omega}\right)$ of the respective filters? You will need to display the logarithm of the Fourier transform using, e.g. the function `log10`.

   Hint: type 'doc fft' and familiarize yourself with the cumbersome format that MATLAB uses to return the Fourier transform.

7. Play both filtered signal (at a slower rate) using the command:

   ```
   > slowrate = fs/2;
   > sound (filtered,  slowrate);
   ```

   Can you hear a difference between the signals?

8. Apply the function `upfilt` to v_0 and v_1 using the filters f_0 and f_1 respectively.

9. Construct the signal $y[n]$ (see Fig. 1), and compute the difference with $x[n]$. Can your filter bank reconstruct exactly $x[n]$ (up to a delay)?

10. Repeat questions 3. to 7. for the five audio files: `classical1.wav, electronic1.wav, jazz1.wav, rock1.wav, world1.wav'`. Compare the effect of the filter bank on the various tracks: provide an answer that is based on the appearance of the Fourier transform plot, and your auditory perception of the filtered signals.

11. Based on the plots of the Fourier transform of the original signal and the Fourier transforms of v_0 and v_1, explain for which of the five tracks we could ignore v_0 or v_1. Confirm your quantitative analysis with a perceptual validation by listening to the outputs v_0 and v_1. By discarding v_0 or v_1, you have implemented a simple perceptual audio coder.