Problem 1

1) the overal filter has frequency response

$$H_1(e^{j\omega})H_2(e^{j\omega}) = A_1(\omega)A_2(\omega)e^{j(\phi_1+\phi_2)}$$
$$e^{-j(N_1/2+N_2)\omega}$$

Thus the overal filter has linear phase

with amplitude

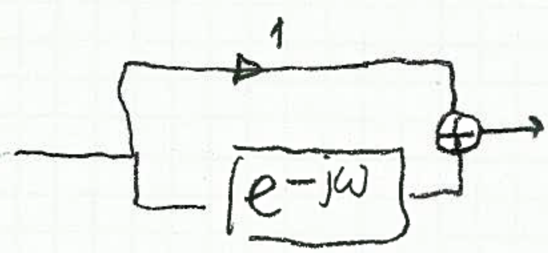
$$\begin{cases} -A_1(\omega)A_2(\omega) & \text{if } \phi_1+\phi_2=\pi \\ +A_1(\omega)A_2(\omega) & \text{otherwise} \end{cases}$$

and initial phase

$$\begin{cases} 0 & \text{if } \phi_1=\phi_2 \\ \frac{\pi}{2} & \text{otherwise} \end{cases}$$

the order is N_1+N_2.

2) The overal filter corresponding to this parallel connection



is $H(e^{j\omega}) = 1+e^{-j\omega}$

We know that the phase is $-\operatorname{atan} \frac{\sin\omega}{1+\cos(\omega)}$

which is not a linear function of ω.

Now if H_1 and H_2 have the same order

and the same initial phase, then the overall filter

$$H_1(e^{j\omega}) + H_2(e^{j\omega}) = [A_1(\omega) + A_2(\omega)]\, e^{+j(\phi - N\omega/2)}$$

has linear phase.

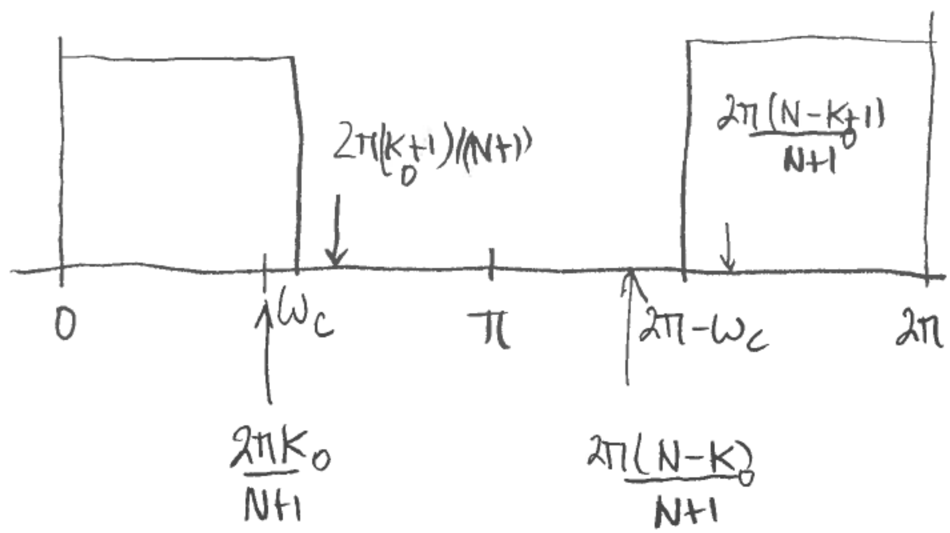
Problem 2

1) $$H(e^{j\omega}) = \sum_{n=0}^{N} h[n] e^{-j\omega n}$$

$$= \frac{1}{N} \sum_{k=0}^{N} H_i[k] \sum_{n=0}^{N} e^{j\left(\frac{2\pi k}{N+1} - \omega\right) n}$$

$$= \frac{1}{N} \sum_{k=0}^{N} H_i[k] \cdot \frac{e^{-j(N+1)\omega} - 1}{e^{j(2\pi k/(N+1) - \omega)} - 1}$$

2)



Let $K_0 = \frac{\omega_c (N+1)}{2\pi}$

then $$H_i(k) = \begin{cases} 1 & \text{if } k = 0 \ldots K_0,\ N-K_0+1, \ldots, N \\ 0 & \text{otherwise.} \end{cases}$$

$$H(e^{j\omega}) = \sum_{k=0}^{K_0} \frac{1 - e^{-j(N+1)\omega}}{1 - e^{j(2\pi k/(N+1) - \omega)}} + \sum_{k=N-K_0+1}^{N} \frac{1 - e^{-j(N+1)\omega}}{1 - e^{j(2\pi k/(N+1) - \omega)}}$$

5.32. The given causal LTI system has system function

$$H(z) = 0.5 + 0.2z^{-1} - 0.3z^{-2} + cz^{-3} + 0.75z^{-4} - z^{-5}.$$

This system will be minimum phase if all of its zeros lie inside the unit circle. Note that it is equivalent to require the zeros of the function $2H(z)$ to lie inside the unit circle. Let us write

$$\begin{aligned} A^{(5)}(z) = 2H(z) &= 1 + 0.4z^{-1} - 0.6z^{-2} + 2cz^{-3} + 1.5z^{-4} - 2z^{-5} \\ &= 1 - \left(-0.5z^{-1} + 0.6z^{-2} - 2cz^{-3} - 1.5z^{-4} + 2z^{-5}\right). \end{aligned}$$

If we were to implement the system $2H(z)$ as a lattice, we would use the expression for $A^{(5)}(z)$ to find the *k*-parameters $k_1, k_2, \ldots, k_5$. A necessary and sufficient condition for the zeros of $A^{(5)}(z)$ to lie inside the unit circle is $|k_i| < 1, \quad i = 1, 2, \ldots, 5$. Thus we can use the lattice synthesis as a way of testing the given system for the minimum phase property.

The lattice synthesis begins with $k_5 = \alpha_5^{(5)} = 2$. We see immediately that $|k_5| \geq 1$. This implies that the given system cannot be minimum phase, regardless of the value of the parameter c.

5.44. Assume that each impulse response corresponds to at most one frequency response.

A. $h_1[n]$ is a Type I FIR filter. Frequency reponse C corresponds to a Type I filter, as $\left|H_C\left(e^{j\omega}\right)\right| \neq 0$ for $\omega = 0$ or $\omega = \pi$.

B. $h_2[n]$ is a Type II FIR filter. Frequency response B corresponds to a Type II filter, as $\left|H_B\left(e^{j\omega}\right)\right| = 0$ for $\omega = \pi$.

C. $h_3[n]$ is a Type III filter. The frequency response must be D, as D is the only frequency response for which $\left|H_D\left(e^{j\omega}\right)\right| = 0$ for $\omega = 0$ and $\omega = \pi$.

D. $h_4[n]$ is a Type IV filter. Frequency response A corresponds to a Type IV filter, as $\left|H_A\left(e^{j\omega}\right)\right| = 0$ for $\omega = 0$.

5.45. A. Systems B, C, D, and E are IIR systems. All of these have poles at places other than the origin and infinity.

B. Systems A and F are FIR systems. These have poles only at the origin.

C. A causal LTI system is stable if and only if all of its poles lie inside the unit circle. Systems A, B, C, E, and F (i.e., all but D) are stable.

D. A stable causal system is minimum phase if its inverse system is also stable and causal. This means that all of the zeros as well as all of the poles must lie inside the unit circle. System E is the only minimum-phase system.

E. A system that is causal with a rational frequency response must be an FIR system to have linear phase. Both systems A and F are linear phase systems, as for both of these systems the zeros occur in reciprocal pairs or at $z = \pm 1$.

F. System C is allpass. It is the only system for which poles and zeros occur in conjugate reciprocal pairs.

G. Only System E has a stable and causal inverse. This is the only system having all of its zeros inside the unit circle.

H. System F has the shortest impulse response, with seven nonzero samples. System A has 12 nonzero samples, and the remaining systems are IIR.

I. Systems A and F are lowpass systems. Systems B and D are eliminated as they each have a zero at $\omega = 0$. System C is an allpass system. System E will have a frequency response whose magnitude tends to peak at frequencies near the system poles. None of these poles are near $\omega = 0$ and one of them is near $\omega = \pi$.

J. The "minimum group delay" property is an attribute of a minimum phase system. System E is the only minimum phase system in the given set. (Note that System E has the minimum group delay among systems with the same magnitude response. System E may not have the minimum group delay among the systems shown.)

5.33. Appears in: Fall05 PS2, Spring05 PS2, Spring04 PS2, Fall03 PS2, Spring03 PS2.

Problem

$H(z)$ is the system function for a stable LTI system and is given by:

$$H(z) = \frac{(1 - 2z^{-1})(1 - 0.75z^{-1})}{z^{-1}(1 - 0.5z^{-1})}.$$

(a) $H(z)$ can be represented as a cascade of a minimum phase system $H_1(z)$ and a unity-gain all-pass system $H_A(z)$, *i.e.*

$$H(z) = H_1(z)H_A(z).$$

Determine a choice for $H_1(z)$ and $H_A(z)$ and specify whether or not they are unique up to a scale factor.

(b) $H(z)$ can be expressed as a cascade of a minimum-phase system $H_2(z)$ and a generalized linear phase FIR system $H_L(z)$:

$$H(z) = H_2(z)H_L(z).$$

Determine a choice for $H_2(z)$ and $H_L(z)$ and specify whether or not these are unique up to a scale factor.