For all homework throughout the semester you must do the following:

1. Explain in your own words what is being asked.
2. State your strategy for arriving at the solution.
3. Execute your strategy noting the steps.
4. ✎ Write legibly and in a logical order.

For each problem, we provide the approximate percentage of points.

Problem 1 [40 %]

In this problem we study whether the property of having linear phase is conserved when two filters are combined in parallel, or in cascade.

- Show that the filter $H(z)$ formed by connecting two generalized linear-phase filters $H_1(z)$ and $H_2(z)$ in cascade (series) has generalized linear-phase. Compute the order, the amplitude, the group delay, and the initial phase of the combined filter. Express your answer as a function of the corresponding parameters of $H_1(z)$ and $H_2(z)$.
- Does the filter $H(z)$ formed by connecting two generalized linear-phase filters $H_1(z)$ and $H_2(z)$ in parallel have generalized linear-phase? What happens when the filters $H_1(z)$ and $H_2(z)$ have the same order?

Problem 2 [40 %]

The following method, called frequency sampling, is proposed to design an FIR filter.

- Step 1: compute the ideal frequency response $H_i(e^{j\omega})$.
- Step 2: Evaluate H_i at $N+1$ equispaced frequencies,

$$H_i[k] = H(e^{j\omega_k}), \quad \text{where} \qquad \omega_k = \frac{2\pi k}{N+1}, \quad k = 0, \ldots, N$$

- Step 3: define $h[n]$ by

$$h[n] = \frac{1}{N} \sum_{k=0}^{N} H_i[k] \exp\left(j \frac{2\pi k n}{N+1}\right) \quad k = 0, \ldots, N$$

1. Prove that the frequency response of the filter $h[n]$ satisfies

$$H(e^{j\omega}) = \frac{1}{N+1} \sum_{k=0}^{N} H_i[k] \, \frac{1 - e^{-j\omega(N+1)}}{1 - e^{-j(\omega - 2\pi k/(N+1))}}$$

   for any $\omega \neq \omega_k$. The above formula can be interpreted as a frequency interpolation from the samples $H_i[k]$.

2. Compute $h[n]$ in the special case where H_i is an ideal lowpass filter with cutoff frequency ω_p.

Problems from the textbook [3 x 6.67 % = 20%]

Solve the following problems from the textbook:

- 5.32
- 5.44
- 5.45

For graduate students

- 5.33