For all homework throughout the semester you must do the following:

1. Explain in your own words what is being asked.
2. State your strategy for arriving at the solution.
3. Execute your strategy noting the steps.
4. ✎ Write legibly and in a logical order.

For each problem, we provide the approximate percentage of points.

Problem 1 [40 %]

We consider an FIR low-pass filter $G(z)$ with the following specifications :

- the order of the filter is $N = 2K - 1$, $G(z) = \sum_{n=0}^{N} g[n]z^{-n}$
- the filter is symmetric: $g[n] = g[N - n]$
- the amplitude $A(\omega)$ in the pass band $[0, \omega_p)$ satisfies

$$1 - \delta_p \leq A(\omega) \leq 1 + \delta_p, \quad \omega \in [0, \omega_p) \tag{1}$$

  with $\omega_p < \pi$,
- there is no stop band.

Such a filter is called *one-band* filter. We define a new filter by

$$H(z) = \frac{z^{-1}}{2}\left[G(z^2) + z^{-N}\right] \tag{2}$$

1. Prove that
$$H(z) + H(-z) = z^{-2K} \tag{3}$$
2. Is H a low-pass, band-pass, or high-pass filter ?
3. What are the passband, and stop-band tolerances, and the band-edge frequencies of $H(z)$?

Problem 2 [40 %]

Let $H(z)$ be a finite impulse response filter (not necessarily linear phase).

1. Prove that $H(z)$ can be expressed as the sum of two FIR filters

$$H(z) = G_1(z) + G_2(z)$$

   where $G_1(z)$ and $G_2(z)$ have linear phase (exact or generalized). Furthermore, the orders of $G_1(z)$ and $G_2(z)$ are smaller or equal than the order of $H(z)$.

2. Of what types are $G_1(z)$ and $G_2(z)$? express your answer as a function of the parity of the order $H(z)$ (even vs odd).

3. Express $|H(e^{j\omega})|$ as a function of the amplitudes of $G_1(z)$ and $G_2(z)$.

Problems from the textbook [2 x 10 % = 20%]

Solve the following problems from the textbook:

- 5.26
- 5.31

For graduate students:

- 5.36