For all homework throughout the semester you must do the following:

1. Explain in your own words what is being asked.
2. State your strategy for arriving at the solution.
3. Execute your strategy noting the steps.
4. ✎ WRITE LEGIBLY AND IN A LOGICAL ORDER.

For each problem, we provide the approximate percentage of points.

Problem 1 [40 %]

In the design of filters we often approximate a specified magnitude characteristic without particular regard to the phase. In many filtering problems, we would prefer that the phase characteristic be zero or linear. For causal filters it is impossible to have zero phase. However, for many filtering applications it is not necessary that the impulse response of the filter be zero for $n < 0$ if the processing is not to be carried out in real time.

One technique commonly used consists in filtering the signal forward in time, and then backward through the same filter.

Let $h[n]$ be the impulse response of a real causal filter with an arbitrary phase characteristic. Let $H(e^{j\omega})$ be the Fourier transform of $h[n]$:

$$H(e^{j\omega}) = A(\omega)e^{j\psi(\omega)} \tag{1}$$

1. Method A : Let $x[n]$ be the input signal. Process $x[n]$ through the filter $h[n]$ to get $v[n]$:

$$v[n] = (h * x)[n] \tag{2}$$

Then process $v[n]$ backward through $h[n]$ to get $w[n]$.

$$w[n] = (h * v^{-})[n] \tag{3}$$

where v^{-} is the signal defined by

$$v^{-}[n] = v[-n] \tag{4}$$

The output $y[n]$ is then taken to be

$$y[n] = w[-n] \tag{5}$$

   (a) Determine the overall impulse response $h_1[n]$ that relates $x[n]$ to $y[n]$.

   (b) Determine $H_1(e^{j\omega})$ the Fourier transform of h_1 in terms of $A(e^{j\omega})$ and $\psi(\omega)$, and show that $H_1(e^{j\omega})$ has zero phase.

2. Method B : Let $x[n]$ be the input signal. Process $x[n]$ through the filter $h[n]$ to get $v[n]$:

$$v[n] = (h * x)[n] \tag{6}$$

Also process $x[n]$ backward through $h[n]$ to get $u[n]$.

$$u[n] = (h * x^-)[n] \tag{7}$$

where x^- is the signal defined by

$$x^-[n] = x[-n] \tag{8}$$

The output $y[n]$ is then taken as the sum of $v[n]$ and $u[-n]$:

$$y[n] = v[n] + u[-n] \tag{9}$$

This complete sequence of operation is linear time invariant and can be represented by the impulse response $h_2[n]$.

(a) Determine the overall impulse response $h_2[n]$ that relates $x[n]$ to $y[n]$.

(b) Determine $H_2(e^{j\omega})$ the Fourier transform of h_2 in terms of $A(e^{j\omega})$ and $\psi(\omega)$, and show that $H_2(e^{j\omega})$ has zero phase.

Problem 2

If we are given a filter, it is sometimes possible to use it repetitively to implement a new filter with a sharper frequency response. One approach is to cascade the filter with itself two or more times. It can easily be showed that this approach will :

- reduce the stop band attenuation if the frequency response is less than 1 in these bands;
- increase the passband approximation error.

Another approach suggested by Tukey (the co-inventor of the FFT) is described by the block diagram in Figure 1. Tukey called this approach "twicing".

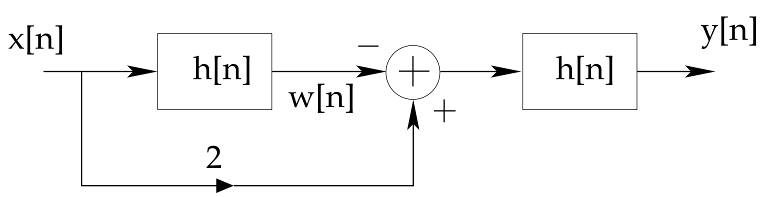

Figure 1:

Let $g[n]$ be the overall impulse of the system :

$$y[n] = (g * x)[n] \tag{10}$$

Assume that the filter h is noncausal and symmetric :

$$h[n] = \begin{cases} h[-n] & \text{if } -M \leq n \leq M \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

1. Compute the overall impulse response $g[n]$, and determine whether $g[n]$ is :

   (i) FIR

   (ii) symmetric

2. Compute $G(e^{j\omega})$, the Fourier transform of $g[n]$.

3. Let δ_p and δ_s be the tolerances in the passband and stopband respectively :

$$\begin{array}{rlll} (1-\delta_p) & \leq H(e^{j\omega}) & \leq (1+\delta_p) & \text{if } \ 0 \leq \omega \leq \omega_p \\ -\delta_s & \leq H(e^{j\omega}) & \leq \delta_s & \text{if } \ \omega_s \leq \omega \leq \pi \end{array} \tag{12}$$

   Show that the overall frequency response $G(e^{j\omega})$ satisfies the specification :

$$\begin{array}{rlll} A & \leq G(e^{j\omega}) & \leq B & \text{if } \ 0 \leq \omega \leq \omega_p \\ C & \leq G(e^{j\omega}) & \leq D & \text{if } \ \omega_s \leq \omega \leq \pi \end{array} \tag{13}$$

   and determine A, B, C and D in terms of δ_p, and δ_s.

4. If $\delta_p \ll 1$ and $\delta_s \ll 1$, what is the first order approximation to the tolerances of $G(e^{j\omega})$ in the passband and the stopband ?

Problems from the textbook [4 x 5 % = 20%]

Solve the following problems from the textbook:

- 5.21
- 5.25
- 5.28
- 5.29

For graduate students:

- 5.42
- 5.44