ECEN 4632 Problem set 7

Problem 1

1. Because N is odd $H(-z) = -\frac{z^{-1}}{2}\left(G(z^2) - z^{-N}\right)$

so $H(z) + H(-z) = z^{-N-1} = z^{-2K}$

2. $$G(e^{j\omega}) = \sum_0^N g[n] e^{-j\omega n}$$

$$= \sum_0^{K-1} g[n] e^{-j\omega n} + \sum_K^{2K-1} g[n] e^{-j\omega n}.$$

$$= \sum_0^{K-1} g[n] e^{-j\omega n} + \sum_0^{K-1} g[2K-1-n] e^{-j\omega(2K-1-n)}$$

$$= \sum_0^{K-1} g[n]\left(e^{-j\omega n} + e^{-j\omega(2K-1-n)}\right)$$

$$= 2\sum_0^{K-1} g[n] e^{-j(K-1/2)\omega} \frac{1}{2}\left\{ e^{j\omega(n-K+1/2)} + e^{-j\omega(n-K+\frac{1}{2})}\right\}$$

$$\underline{G(e^{j\omega}) = A(\omega) e^{-j(K-\frac{1}{2})\omega}}$$

where $A(\omega) = \sum_0^{K-1} 2g[n]\cos\left(n-(K-\tfrac{1}{2})\right)\omega$

We note that A is an even function.

$$G(e^{j2\omega}) = A(2\omega)\, e^{-j(2K-1)\omega}$$

So $$H(e^{j\omega}) = \frac{e^{-j\omega}}{2} \left\{ A(2\omega) e^{-j(2K-1)\omega} + e^{-j(2K-1)\omega} \right\}$$

$$= \frac{e^{-j\omega}}{2} e^{-jN\omega} \left\{ A(2\omega) + 1 \right\}.$$

$$H(e^{j\omega}) = \left\{ \frac{A(2\omega)+1}{2} \right\} e^{-j(N+1)\omega}$$

$$H(e^{j\omega}) = \left[\frac{A(2\omega)+1}{2} \right] e^{-j2K\omega}$$

We have $A(0) = 1$ since G is a low pass filter and $G(1)=1$

But $G(1) = G(e^{j2\pi}) = 1 = A(2\pi) e^{+j\pi} = -A(2\pi)$

So $A(2\pi) = -1$

Therefore $A(2\pi)+1 = 0$ or $H(e^{j\pi}) = 0$.

So H is lowpass.

3) If $\omega \in [0, \omega_p/2]$ then

$$1 - \frac{\delta_p}{2} \leq \frac{A(2\omega) + 1}{2} \leq \frac{2 + \delta_p}{2} = 1 + \frac{\delta_p}{2}$$

If $\omega \in (\pi - \omega_p/2, \pi]$ then

$$1 + \delta_p \leq -A(2\omega) \leq (1 + \delta_p).$$

and. $$-\frac{\delta_p}{2} \leq \frac{1 + A(2\omega)}{2} \leq \frac{\delta_p}{2}$$

So we created a stop band.

Problem 2

Let $f[n] = \frac{1}{2}(h[n] + h[N-n])$

and $g[n] = \frac{1}{2}(h[n] - h[N-n])$.

$$F(e^{j\omega}) = \frac{1}{2}\left(\sum_{n=0}^{N} h[n]e^{-j\omega n} + \sum_{n=0}^{N} h[N-n]e^{-j\omega n}\right)$$

$$= \frac{1}{2}\left(\sum_{0}^{N} h[n]e^{-j\omega n} + \sum_{0}^{N} h[n]e^{-j\omega(N-n)}\right)$$

$$= \frac{1}{2}\left(e^{-j\omega N/2} \sum_{0}^{N} h[n]e^{-j\omega(n-N/2)} + \right.$$

$$\left. + e^{-j\omega N/2} \sum_{0}^{N} h[n]e^{j\omega(n-N/2)}\right)$$

$$F(e^{j\omega}) = e^{-j\omega N/2} \sum_{0}^{N} h[n] \cos[\omega(n-N/2)]$$

F has linear phase.

Similarly

$$G(e^{j\omega}) = e^{-j\omega N/2} \sum_{0}^{N} h[n] \left(\frac{e^{-j\omega(n-N/2)} - e^{j\omega(n-N/2)}}{2}\right)$$

$$G(e^{j\omega}) = -je^{-j\omega N/2} \sum_{0}^{N} h[n] \sin[(n-N/2)\omega]$$

$$G(e^{j\omega}) = e^{-j(\omega N/2 + \pi/2)} \sum_{n=0}^{N} h[n] \sin[(n-N/2)\omega]$$

G has generalized linear phase.

2). When N is odd F is type II, and G is type IV

N is even F —— I, —————— III

3)

$$H(e^{j\omega}) = F(e^{j\omega}) + G(e^{j\omega})$$

$$H(e^{j\omega}) = e^{-j\omega N/2} A(\omega) + j e^{-j\omega N/2} B(\omega)$$

with
$$A(\omega) = \sum_{0}^{N} h[n] \cos[\omega(n - N/2)]$$
$$B(\omega) = -\sum_{0}^{N} h[n] \sin[\omega(n - N/2)]$$

$$H(e^{j\omega}) = e^{-j\omega N/2} \{ A(\omega) + j B(\omega) \}$$

therefre

$$|H(e^{j\omega})| = \sqrt{|A(\omega)|^2 + |B(\omega)|^2}$$

$$|H(e^{j\omega})| = \left(|F(e^{j\omega})|^2 + |G(e^{j\omega})|^2\right)^{1/2}$$

5.26

(a) $h[n]$ real-valued? $\boxed{\textbf{YES}}$ **NO**

(b) $\sum_{n=-\infty}^{\infty} |h[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega = 1$

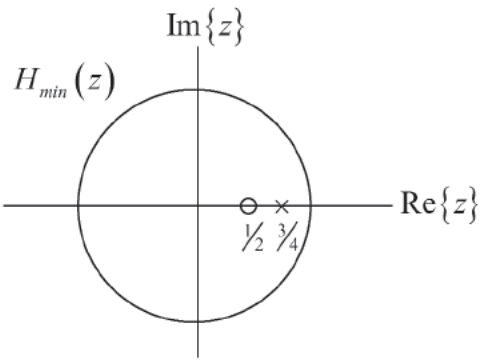
(c) Response of the system: $y[n] = s[n] \cos(\omega_c n - \frac{\pi}{2})$

5.36.

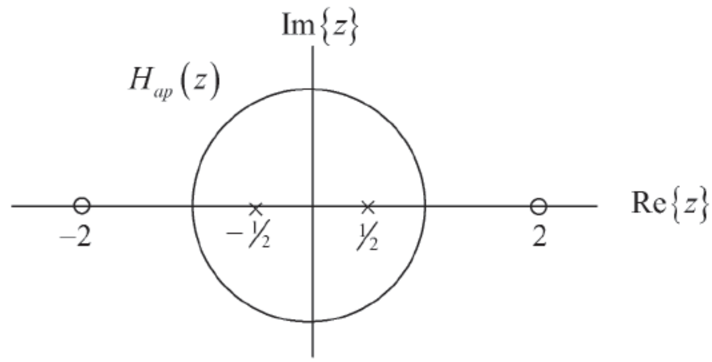
$$H(z)=\frac{\left(1-2z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1-\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

$$=\frac{\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{3}{4}z^{-1}\right)}\times\frac{\left(1-2z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

$$=H_{min}(z)H_{ap}(z).$$



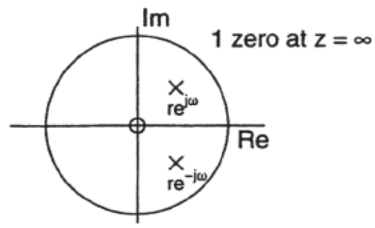
ROC: $|z|>3/4$



ROC: $|z|>1/2$

The regions of convergence reflect the requirement that both $H_{min}(z)$ and $H_{ap}(z)$ must be stable.

5.31. (a) A labeled pole-zero diagram appears below.



The table of common z-transform pairs gives us

$$(r^n \sin \omega_0 n)u[n] \longleftrightarrow \frac{(r \sin \omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}, \quad |z|>r$$

which enables us to derive $h[n]$.

$$h[n] = \left(\frac{1}{\sin \omega_0}\right)(r^n \sin \omega_0 n)u[n]$$

(b) When $\omega_0 = 0$

$$H(z) = \frac{rz^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}} = \frac{rz^{-1}}{(1-rz^{-1})^2}, \quad |z|>r$$

Again, using a table lookup gives us

$$h[n] = nr^n u[n]$$