For all homework throughout the semester you must do the following:

1. Explain in your own words what is being asked.
2. State your strategy for arriving at the solution.
3. Execute your strategy noting the steps.
4. ✎ Write legibly and in a logical order.

For each problem, we provide the approximate percentage of points.

Problem 1 [40 %]

1. The Fourier transform of a discrete time signal is

$$X\left(e^{j\omega}\right) = \cos(\omega) + \sin(2\omega) \tag{1}$$

   Compute $x[n]$.

2. Let $x[n]$ be the signal defined by

$$\begin{cases} x[-3] = x[3] = 1; \\ x[-2] = x[2] = -1; \\ x[-1] = x[1] = -2; \\ x[0] = 4; \\ x[n] = 0 \quad \text{if} \quad |n| \geq 4. \end{cases} \tag{2}$$

   Compute the following quantities without evaluating the Fourier transform of x, $X\left(e^{j\omega}\right)$.

   (a) $X\left(e^{j0}\right)$
   (b) $\angle X\left(e^{j\omega}\right)$.
   (c) $\displaystyle\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) d\omega$
   (d) $X\left(e^{j\pi}\right)$
   (e) $\displaystyle\int_{-\pi}^{\pi} |X\left(e^{j\omega}\right)|^2 d\omega$
   (f) $\dfrac{d}{d\omega} X\left(e^{j\omega}\right)(0)$

Problem 2 [40 %]

Let $X\left(e^{j\omega}\right)$ be the Fourier transform of a signal $x[n]$.

1. What is the Fourier transform of $(-1)^n x[n]$?
2. Express $\sum_{n=-\infty}^{\infty} (-1)^n x[n]$ in terms if $X\left(e^{j\omega}\right)$.
3. We consider the new signal $y[n]$ defined by its Fourier transform

$$Y\left(e^{j\omega}\right) = X\left(e^{j\omega}\right) + X\left(e^{j(\omega-\pi)}\right). \tag{3}$$

4. Is the system $x[n] \longrightarrow y[n]$ linear? time invariant?
5. Prove that $y[n]$ only depends on the even samples $x[2n], n \in \mathbb{Z}$, and is independant of the odd samples $x[2n+1], n \in \mathbb{Z}$.

Problems from the textbook [5 x 4% = 20%, or 7 x 2.85% = 20%]

Solve the following problems from the textbook:

- 2.47
- 2.48
- 2.55
- 2.67
- 2.71

For graduate students:

- 2.72
- 2.73