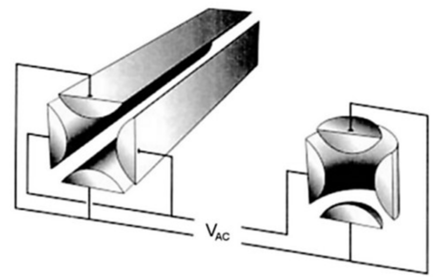
Ion Trapping and its Mechanical Analogue

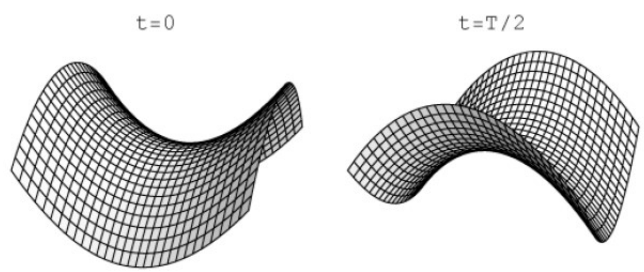
2D Quadrupole Ion Trapping

Earnshaw's Theorem: Gauss's law states that in region with zero charge density, div(E)=0, and so it is impossible to create stable equilibrium point.



The best one can do is create a saddle potential (top right) – trapping along one horizontal axis and anti-trapping along the other. Examples of electrodes used to generate these potentials are shown at lower right, with a 2D quadrupole trap (to be discussed here) on the left, and a 3D quadrupole trap on the right.

Potential oscillates between trapping and anti-trapping, with the different axes remaining 180 degrees out of phase. For oscillation of certain frequency, as an ion subject to potential begins to move along an anti-trapping axis, the anti-trapping potential along that axis switches to trapping, thereby "catching" the ion.



The Quadrupole Trap Potential

With the DC bias set to zero, a 2D trapping potential with frequency Ω, trap depth U_{RF}, and radius r_0, takes the simplified form:

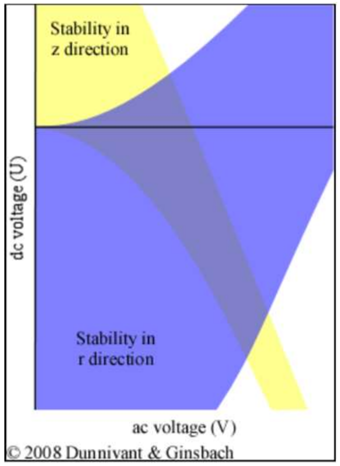
$$\Phi(x, y, t) = \frac{U_{RF}}{r_0} \cos(\Omega t)\left(x^2 - y^2\right)$$

Equations of motion are Mathieu equations -- solutions not analytic, but are either stable/bound oscillatory solutions or exponentially growing unstable solutions. In this case, “largest region of stability is given by the [following requirement]”:

$$|q| = \frac{4e|U_{RF}|}{m r_0^2 \Omega^2} < 0.908\ldots$$

Need to choose frequency and U_{RF} according to m, e, and r_0 (physical characteristics of ion and trap) to satisfy stability condition.

Regions of stability in *r* and *z* are mapped out in a stability diagram (basic example at right), and parameters are chosen such that the regions of stability overlap in each dimension. Axes of diagram are dependent on RF amplitude and DC bias (in our example, set to 0).



The Mechanical Analogue

Electric saddle potential can be visualized as a saddle-shaped surface – i.e., the following gravitational potential:

$$U(x', y') = \frac{mgh_0}{r_0^2}\left(x'^2 - y'^2\right)$$

Oscillating behavior of trapping and anti-trapping regions is achieved by rotation of the saddle. If saddle rotates with frequency Ω, then in lab frame coordinates (*x*,*y*), the potential takes the following form:

$$U(x, y) = \frac{mgh_0}{r_0^2}\left[(x^2 - y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)\right]$$

Height of the trap h_0 serves as analogue to the RF potential depth from the electric case, and r_0 is again the radius of the trap. The dependence on 2Ω is due to the symmetry under $R_z(\pi)$ of the potential.

Difference from the electric case is obvious, and results from the trap rotating, rather than inverting or ("flapping") – the cross-term results in the potential being present for all t, rather than going to zero periodically as in the case of Φ(x,y,t).

Stability of the Mechanical Trap

The equations of motion for a mass in the frictionless mechanical trap in are analytic, and are either stable/bound oscillatory or exponentially growing solutions. To ensure behavior which is stable, the following condition must be satisfied:

$$|\bar{q}| = \frac{gh_0}{\Omega^2 r_0^2} \leq \frac{1}{2}$$

Note the lack of dependence on mass – it cancels out due to $U(x,y) \propto m$.

For small q (~ less than 0.15), the potential is well-approximated by a time average of U(x,y,t) – time averaged position of particle (secular motion) then resembles a harmonic oscillator with frequency $q\Omega$.

Stability seems to depend only on trap parameters, and not on initial conditions of the mass (with exception to the **r**=0, **v**=0 case) – any trap satisfying the stability condition is predicted to have infinite trapping lifetime. Barring a failure to satisfy the above inequality, the lifetime is constrained by the presence of friction.

This behavior is explained by introducing friction – friction contributes a real exponential factor to the motion of the particle, leading to finite trapping lifetime proportional to the coefficient of friction in the system.

With friction added in, the particle lifetime can be calculated in terms of the initial starting position r=R, and is seen to depend on a factor $\ln(r_0/R)$ – the trap lifetime is greatly increased by placing the mass closer to the center of the trap.