Dynamic Skyline Queries in Metric Spaces

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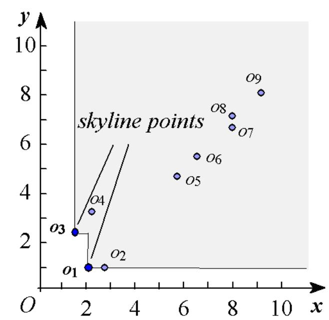
ABSTRACT

Skyline query is of great importance in many applications, such as multi-criteria decision making and business planning. In particular, a skyline point is a data object in the database whose attribute vector is not *dominated* by that of any other objects. Previous methods to retrieve skyline points usually assume static data objects in the database (i.e. their attribute vectors are fixed), whereas several recent work focus on skyline queries with dynamic attributes. In this paper, we propose a novel variant of skyline queries, namely *metric skyline*, whose dynamic attributes are defined in the metric space (i.e. not limited to the *Euclidean* space). We illustrate an efficient and effective pruning mechanism to answer metric skyline queries through a metric index. Extensive experiments have demonstrated the efficiency and effectiveness of our proposed pruning techniques over the metric index in answering metric skyline queries.



Object	Coordinates
o_1	$(2, 1)$
o_2	$(2.8, 1)$
o_3	$(1.5, 2.5)$
o_4	$(2, 3.5)$
o_5	$(5.8, 4.8)$
o_6	$(6.5, 5.5)$
o_7	$(7.8, 6.8)$
o_8	$(7.8, 7)$
o_9	$(9, 8.5)$

Figure 1: Example of Traditional Skyline

1. INTRODUCTION

Recently, *skyline queries* have attracted much attention from the database research community due to its wide applications related to multi-criteria decision making. Specifically, given a d-dimensional data set $\mathcal{D}$, a *skyline query* [2] returns all the data objects which are not *dominated* by other objects in $\mathcal{D}$. Here, we say a data object $X(X_1, X_2, ..., X_d)$ *dominates* another one $Y(Y_1, Y_2, ..., Y_d)$, if attribute X_i of X on each dimension is never greater than Y_i in Y (for all $i \in [1, d]$), and there exists at least one attribute X_j which is strictly smaller than Y_j in Y.

Figure 1 illustrates a simple example of *skyline*, in which nine data points locate in a 2-dimensional space (i.e. $d = 2$). Specifically, we say o_3 *dominates* o_4, since object $o_3(1.5, 2.5)$ has smaller coordinates than object $o_4(2, 3.5)$ in both dimensions (i.e. $1.5 < 2$ and $2.5 < 3.5$). Similarly, object $o_1(2, 1)$ *dominates* object $o_2(2.8, 1)$, since o_1 has smaller x-coordinate (attribute) than o_2 (i.e. $2 < 2.8$) and moreover y-coordinate not greater than o_2 (i.e. $1 = 1$). As objects o_1 and o_3 are not *dominated* by any other points in the space, they are called *skyline points* in the database.

In literature, many proposals have studied the efficiency issues of searching *skylines*, including *block nested loop* (BNL) [2], *divide-and-conquer* (D&C) [2], *bitmap* and *index* [23], *nearest neighbor* (NN) [14], and *branch-and-bound* (BBS) [18]. These methods assume *static* data objects in the database (i.e. attributes of each data object are *fixed*). As in the previous example of Figure 1, coordinates of each object are assumed to be *static*.

The skyline query processing with *dynamic* attributes is more challenging than that with *static* ones, in the sense that skylines have to be calculated in an online fashion, which requires effectively pruning method to reduce the computation cost. Some recent work [19, 22, 10] consider *skyline queries* with *dynamic* attributes. In particular, Papadias et al. [19] defined a *dynamic skyline*, where attributes of each data object are given by a set of *dimension functions*. Sharifzadeh and Shahabi [22] proposed the *spatial skyline query*, where attribute values of each data object are dynamically calculated as *Euclidean distances* from query points to this object. Deng et al. [10] presented the *multi-source skyline query*, in which attributes are defined as the shortest path lengths on road networks from data to query objects. In their proposed approaches [19, 22, 10], however, they all assume data objects in the form of vectors and utilize the geometric property of data objects in the *Euclidean* space to facilitate the pruning. However, there exist applications where the data from them can not be represented as vectors. For example, in the bioinformatics, the DNA sequences are usually modeled as strings. Therefore, the previous methods restricted their solutions to a specific application domain.

In this paper, we propose a more generic *skyline query* with *dynamic* attributes, namely *metric skyline*, which retrieves *skyline* points with *dynamic* attributes in the *metric* space. Specifically, the input of a *metric skyline query* is a set of data objects, $\mathcal{D} = \{o_1, o_2, \ldots, o_m\}$, and a set of query objects, $Q = \{q_1, q_2, \ldots, q_n\}$, in the *metric* space. The output includes those data objects in $\mathcal{D}$

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whose *attribute vectors* are not *dominated* by those of other data objects, where the attribute vector of a data object is defined as an n-dimensional vector consisting of *metric* distances from this object to n query points.

Metric skyline queries have many important applications. For example, consider a business plan of opening a number of shops near some residential areas. The candidate locations for opening a shop can be viewed as data objects in the database, whereas residential areas that the shop is targeting at are query points. The manager has to decide appropriate shop locations for the convenience of customers living in the targeted areas. In particular, the candidate locations of shop must be *interesting* in terms of the *traveling distance* from a shop to all its targeted customers (areas), and any candidate location should satisfy the condition that no other location is closer to *all* its targeted areas than itself. Note that, here the computation of the *traveling distance* between a candidate location and a residential area should take into account the underlying road network, for instance, the shortest road path between two places, which is a *metric* distance function.

Unlike previous work such as *dynamic skyline* [19], *spatial skyline* [22] or *multi-source skyline* on road networks [10], our *metric skyline* is more generic in that it does not require data objects be either vectors or in the *Euclidean* space. Thus, it can be applied to a much broader application areas, such as geographic information systems (GIS), traffic networks and molecular biology, where data objects in these areas are often modeled as polygons or sequences and different *metric* distance functions are used to compute dynamic attribute vectors of data objects. Due to this generality of *metric skyline*, previous methods [19, 22, 10] specifically designed in the *Euclidean* space cannot be used to retrieve *metric skyline* with arbitrary format of data (e.g. numerical vector, string, etc.) and in arbitrary *metric* spaces.

Therefore, in the paper, we propose an efficient and effective approach to answer generic *metric skyline query* in the metric space, without any assumption of the format of data objects and *metric* distance functions. In particular, we present a novel pruning method and retrieve *metric skyline* points over M-tree index [8]. However, the proposed methodology is not limited to M-tree only, and it can be applied to other *metric* index as well.

The contributions of this paper are summarized below.

1. We formally define in Section 3 the problem of *metric skyline query*, and present in Section 4 the foundation of our pruning method, namely *triangle-based pruning*, in *metric* spaces.
2. We propose in Section 5 an efficient query procedure to answer *metric skyline queries* over M-tree index [8].
3. Last but not least, we demonstrate in Section 6 the efficiency and effectiveness of our proposed approach for answering *metric skyline queries* through extensive experiments, compared with the *linear scan*.

In the sequel, Section 2 briefly reviews the previous work on *skyline* and *spatial skyline* queries. We discuss in Section 5 query processing of *metric skyline* queries. Finally, Section 7 concludes this paper.

2. RELATED WORK

2.1 Traditional Skyline Processing

The *skyline* operator was first introduced into the database community by Borzsonyi et al. [2]. In their work, they also proposed solutions based on *block nested loop* (BNL) and *divide-and-conquer* (D&C). BNL is a straightforward approach to compute *skyline* points. Specifically, each data object is compared with all the other objects in the data set and output if it is not *dominated* by others. However, BNL is computationally expensive, since the cost of dominance checks for each data object is proportional to the total database size. D&C approach recursively divides the data space into partitions until each partition can fit in memory. Then, *partial skyline* points in each partition are calculated and merged into a final skyline set.

A variant of BNL, *sort-filter-skyline* (SFS), was proposed by Chomicki et al. [6], where the basic idea is to sort the input data with respect to some monotonic score function, and then compute the *skyline* by scanning the sorted list. Later, Tan et al. [23] proposed two progressive processing algorithms, *bitmap* and *index*. Specifically, *bitmap* encodes all the information that can determine a *skyline* point, and then simply applies bitwise "AND" operation to obtain *skyline* points. The *index* approach partitions the entire data set into several lists, each of which is in ascending order of their minimum coordinate along that dimension. *Local skyline* is calculated in each list and merged into a global one.

Kossmann et al. [14] proposed a *nearest neighbor* (NN) approach to obtain *skyline* with the aid of a spatial index, for example, R-tree [12]. Specifically, the NN algorithm identifies *skyline* by repeating the NN search on the partitioned spaces. Papadias et al. [18] improved the NN method by using the idea of *branch-and-bound* (BBS). Unlike the NN approach, which searches R-tree many times, BBS only traverses the R-tree once. In particular, the algorithm maintains a list to store skyline points and use a minimum heap to traverse the R-tree in the *best-first* manner. Entries in the heap are sorted in ascending order of a key, which is defined as the minimum L_1-norm distance from the origin to an R-tree node or data point. Whenever an R-tree node is encountered, BBS checks the dominance of the node as well as entries in it, with respect to *skyline* points in the list. Then, those entries that cannot be pruned are inserted into the heap. The traversal procedure terminates when the heap is empty. BBS algorithm has been proved to be I/O optimal [18].

Recently, Lee et al. [15] utilized the close relationship between Z-order curve and skyline processing strategy to index data objects and efficiently answer skyline queries. In particular, they encode data objects with Z-order curve and construct a novel index structure called *ZBtree*. Then, the skyline search can be conducted over *ZBtree* based on the pruning property of Z-order curve. Morse et al. [17] considered the skyline computation in the case where attributes are drawn from low-cardinality domains. A *lattice skyline algorithm* (LS) was proposed with such property.

2.2 Skyline Variants

There are many variants of the traditional skyline query. Pei et al. [21] and Yuan et al. [29] proposed methods to compute *skylines* in all possible subspaces. Tao et al. [24] gave an efficient algorithm to calculate *skylines* in a specific subspace. Chan et al. [4] defined a k-*dominant skyline* which extends the *dominant* concept of traditional *skyline* to k-*dominant*. Given a d-dimensional data space, an object p is said to k-*dominate* another one q if there are $k(\leq d)$ dimensions in which p *dominates* q. Dellis and Seeger [9] proposed a *reverse skyline query*, which obtains those objects that have the query point as skyline, where each attribute is defined as the absolute difference from objects to query point along each dimension.

In the context of uncertain databases, Pei et al. [20] proposed the probabilistic skyline over uncertain data, which returns a number of objects that are expected to be skylines with probability higher than a threshold. Khalefa et al. [13] studied the skyline query processing in the presence of missing attributes. Furthermore, *skyline* has also been studied in some constrained environment, such as on data streams [16], in distributed environment [1] and in the partial-order domain [3].

The most relevant problems to our work are the *dynamic skyline* [19], *spatial skyline* [22] and *multi-source skyline* on road networks [10]. Specifically, Papadias et al. [19] applies BBS algorithm to retrieve skyline points, where dynamic attributes of data objects are computed by a set of *dimension functions*. However, only *Euclidean distance* were considered for *dimension functions*. For the *spatial skyline* [22], given a database $\mathcal{D}$ containing data objects o_i ($i \in [1, m]$), a *spatial skyline query* takes as input an arbitrary number (e.g. n) of query points q_1, q_2, ..., and q_n in the vector space. For each data object $o_i \in \mathcal{D}$, vector $\langle L_2(q_1, o_i)$, $L_2(q_2, o_i)$, ..., $L_2(q_n, o_i)\rangle$ contains n spatial derived attributes, where $L_2(q_j, o_i)$ is the *Euclidean distance* between two data objects q_j and o_i. A *spatial skyline query* retrieves a set of data objects $o_i \in \mathcal{D}$ such that their attribute vectors are not *dominated* by those of other objects. Note that, the *spatial skyline query* requires all data objects be in a *Euclidean* space. Thus, the proposed solutions by Sharifzadeh and Shahabi [22] can utilize the geometric property of data objects in the database to prune the search space. In particular, two important theorems are given, that is, *spatial skyline points* are those data objects either within the *convex hull* of query points or having their own *Voronoi cells* intersect with boundaries of the *convex hull* of query points. All these assumptions and properties, however, do not hold in a more general *metric* space, which makes their methods inapplicable to our *metric skyline* scenario.

Similarly, the method proposed for *multi-source skyline* on road networks [10] also utilizes geometric information of data objects during the pruning, which is thus limited to road network application and cannot be used for generic *metric skyline* retrieval in other *metric* space.

In summary, previous studies on skyline variants are limited to either Euclidean space or metric space for a specific application. In contrast, our work focuses on the generic metric skyline search in the metric space.

3. PROBLEM DEFINITION

In this section, we formally define a variant of *skyline* with dynamic attributes, namely *metric skyline*, where attributes are defined in the *metric* space. Specifically, given a database $\mathcal{D}$ with m data objects o_i ($i \in [1, m]$) in a *metric* space, assume that all the pairwise distances $dist(o_i, o_j)$ between data objects o_i and o_j are known in advance, where $i, j \in [1, m]$, and $dist(\cdot, \cdot)$ is a *metric* distance function, satisfying four properties below: $\forall x, y, z \in \mathcal{D}$,

1. $dist(x, y) > 0$,
2. $dist(x, y) = 0 \Leftrightarrow x = y$,
3. $dist(x, y) = dist(y, x)$, and
4. $dist(x, z) \leq dist(x, y) + dist(y, z)$.

Symbol	Description
$\mathcal{D}$	a database of size m
o_i	the data object in $\mathcal{D}$
q_j	the j-th query point of the *metric skyline query*
m	the number of data objects
n	the number of query points
$dist(\cdot, \cdot)$	the *metric* distance function

Figure 2: Meanings of Symbols Used

Given a set of query points, $Q = \{q_1, q_2, ..., q_n\}$, a *metric skyline query* retrieves all data objects o_i such that their attribute vectors in the form $\langle dist(o_i, q_1), dist(o_i, q_2), ..., dist(o_i, q_n)\rangle$ are not *dominated* by those of other objects. In other words, a data object $o_i \in \mathcal{D}$ is in the answer to a *metric skyline* query if and only if it holds that:

$$\forall o_{cand} \in \mathcal{D} \wedge o_{cand} \neq o_i,$$
$$(\exists q_j \in Q, s.t.\ dist(q_j, o_i) < dist(q_j, o_{cand})).$$

Note that, since we consider the *metric skyline* problem in the *metric* space, no geometric information can be utilized to guide the pruning during query processing. Thus, previous methods, for example, the one that retrieves *spatial skyline* points [22] in Euclidean space or uses specific measure (e.g. the shortest distance on road networks [10]), are inapplicable to the generic *metric skyline* scenarios. Fortunately, we have one important tool (probably the only one) to facilitate the *metric skyline* search, that is, the *triangle inequality* (the fourth property for the *metric* distance function above), which will be discussed in the next section. Figure 2 illustrates the commonly-used symbols in this paper.

4. THE PRUNING FOUNDATION

In this section, we propose an effective pruning mechanism to facilitate answering *metric skyline queries* in the *metric* space. We first illustrate the basic pruning heuristics resulting from the definition of *metric skyline*, and then propose our pruning method with the help of the *triangle inequality* in the *metric* space.

4.1 Preliminary

As mentioned earlier, given a database $\mathcal{D}$ and a set, Q, of query points, data object $o_i \in \mathcal{D}$ is in the result of a *metric skyline query*, if and only if for any data object $o_k \in \mathcal{D}\backslash\{o_i\}$, there exists at least one query point $q_j \in Q$ such that $dist(q_j, o_i) < dist(q_j, o_k)$, where $dist(\cdot, \cdot)$ is a *metric* distance function. In other words, data object o_i can be safely pruned, if and only if there exists one object $o_k \in \mathcal{D}$ such that $dist(q_j, o_i) \geq dist(q_j, o_k)$ for *all* $j \in [1, n]$ (i.e. the attribute vector of o_i is *dominated* by that of o_k), for $q_j \in Q$, which is our basic pruning heuristics to answer *metric skyline queries*.

However, with such heuristics, in order to prune an object o_i, we have to scan the entire database, which requires $O(m)$ *dominance checks* in the worst case, where m is the number of objects in the database. Obviously, this method is quite inefficient, especially when the computation of distance function $dist(\cdot,\cdot)$ is costly. Motivated by this, in the sequel, we propose a more efficient yet effective method, *triangle-based pruning*, by applying the (probably only) available tool, the *triangle inequality*, in *metric* spaces.

4.2 Triangle-Based Pruning

In our *metric skyline* problem, since the *metric* distance function is used to measure the similarity among data objects, a very important property, *triangle inequality* (Section 3), thus holds, which is the foundation of our *triangle-based pruning* method for answering *metric skyline queries*.

Specifically, we select a number of objects in the database $\mathcal{D}$ as so-called *pivots*. Then, for each data object $o_i \in \mathcal{D}$, we can obtain its lower and upper bounds of distance $dist(q_j, o_i)$, LB_{ij} and UB_{ij}, respectively, using the *triangle inequality*, where q_j is a query object ($j \in [1, n]$) and p is a *pivot*. Obviously, we have $LB_{ij} = |dist(q_j, p) - dist(p, o_i)|$ and $UB_{ij} = dist(q_j, p) + dist(p, o_i)$. Next, we utilize these bounds to prune unqualified objects during the *metric skyline* search. The following lemma illustrates the heuristics of our *triangle-based pruning* method.

LEMMA 4.1. (*Triangle-Based Pruning Heuristics*) *Given a database $\mathcal{D}$ containing m data objects o_1, o_2, ..., o_m, and a set, Q, of query points q_1, q_2, ..., q_n in a metric space, a data object $o_i \in \mathcal{D}$ can be safely pruned, if there exists a data object $o_k \in \mathcal{D}$ such that $UB_{kj} \le LB_{ij}$ for all $j \in [1, n]$, where $q_j \in Q$ and $dist(\cdot,\cdot)$ is a metric distance function.*

Proof. By the *triangle inequality* and the definition of *metric skyline*. □

Intuitively, by the *triangle inequality*, the distance from any data object to a query point is bounded by an interval. According the skyline definition, any data object o_i is definitely *dominated* by another one o_k, if the lower bound LB_{ij} of $dist(q_j, o_i)$ is never smaller than the upper bound UB_{kj} of $dist(q_j, o_k)$ for *all* dimensions (i.e. $LB_{ij} \ge UB_{kj}$ for *all* $j \in [1, n]$), which is exactly the pruning condition given in Lemma 4.1. Note that, in the case where metric distance function $dist(\cdot,\cdot)$ is costly, the pruning heuristics in Lemma 4.1 shows its superiority, since all the distances $dist(p, o_i)$ ($dist(p, o_k)$) from *pivot* p to objects o_i (o_k) can be pre-computed. The only required online computation is to calculate $dist(q_j, p)$ for all $j \in [1, n]$, when *metric skyline query* is issued. We summarize our *triangle-based pruning* method below.

THEOREM 4.1. *Given a metric skyline query set, Q, any data object $o_i \in \mathcal{D}$ can be safely pruned, if there exists a data object $o_{cand} \in \mathcal{D}$ such that $2 \cdot dist(q_j, p) + dist(p, o_{cand}) \le dist(p, o_i)$ holds for all $j \in [1, n]$, where q_j is a query object, p is a pivot, and $dist(\cdot,\cdot)$ is a metric distance function.*

Proof. According to Lemma 4.1, a data object o_i can be safely pruned if and only if there exists a data object o_k such that $UB_{kj} \le LB_{ij}$ for all $j \in [1, n]$, that is, $dist(q_j, p) + dist(p, o_k) \le |dist(q_j, p) - dist(p, o_i)|$. Since this pruning condition can only hold when $dist(q_j, p) < dist(p, o_i)$ (otherwise, inequality $dist(p, o_k) + dist(p, o_i) \le 0$ is contradict to the first property of a *metric* distance function that the distance is positive), we rewrite it as $dist(q_j, p) + dist(p, o_k) \le dist(p, o_i) - dist(q_j, p)$ for all $j \in [1, n]$, which is exactly the pruning condition in the theorem by letting $o_k = o_{cand}$. □

According to Theorem 4.1, we discuss one possible solution to our *metric skyline* problem. Specifically, we first select d *pivots* p_1, p_2, ..., and p_d from database $\mathcal{D}$. Next, we pre-compute pairwise distances $dist(p_l, o_k)$ from *pivot* p_l to object o_k for $l \in [1, d]$ and $k \in [1, m]$. Thus, for each data object o_k, we can obtain a d-dimensional vector $\langle dist(p_1, o_k), dist(p_2, o_k), ..., dist(p_d, o_k)\rangle$ which can be inserted into any multidimensional index structure such as R-tree [12]. Without loss of generality, assume we also pre-compute the minimum distance from each *pivot* p_l to data objects in $\mathcal{D}\backslash\{p_l\}$, denoted as $mindist(p_l, \mathcal{D}/p_l)$. Given any *metric skyline* query with n query points q_1, q_2, ..., and q_n, we first compute the pairwise distance $dist(q_j, p_l)$ between q_j and *pivot* p_l for any $l \in [1, d]$. Then, based on Theorem 4.1, we issue a *range query* on the R-tree, where the query interval along the l-th dimension is $[0, 2 \cdot max_{j=1}^{n} dist(q_j, p_l) + mindist(p_l, \mathcal{D}/p_l))$ for $l \in [1, d]$. All the returned objects of the range query are *metric skyline* candidates. However, this method has the defect as follows. In the case where query points follow the same distribution as data objects, the distance $dist(p_l, o_k)$ from *pivot* p_l to an object o_k is very likely to be smaller than $2 \cdot max_{j=1}^{n} dist(q_j, p_l) + mindist(p_l, \mathcal{D}/p_l)$. In other words, we have to issue a large range query which essentially accesses nearly all the objects in the database. Thus, this method is quite inefficient and may perform even worse than a *linear scan*.

5. METRIC SKYLINE QUERY

Up to now, we have illustrated our pruning foundation, *triangle-based pruning*, for answering *metric skyline queries*. Note that, without the help of indexes, we have to sequentially scan the entire database, which is not efficient. In this subsection, we discuss query processing of *metric skyline* via indexes in the *metric* space, which can significantly reduce the search space by filtering out the unqualified data objects as earlier as possible.

Specifically, we use M-tree [8] for illustration, since it is a widely-used data structure to index and search data objects in the *metric* space, and moreover it is the only metric index structure considering I/O cost. Our proposed approach for answering *metric skyline query*, however, can be applied to other *metric* indexes as well. Figure 3 depicts a small M-tree, which contains 9 data objects o_1, o_2, ..., and o_9. Only for the sake of clear presentation, we use *Euclidean distance* as the similarity measure. Figure 3(a) is the visualization of hierarchical M-tree structure in Figure 3(b). Similar to other multidimensional indexes in the vector space such as R-tree [12], data objects in the M-tree are recursively grouped together by *minimum bounding hyperspheres* (the circles in Figure 3(a)), until finally only one large sphere (i.e. root node bounding e_1 and e_2) is obtained. Each entry e_i in an intermediate node of M-tree consists of three components, a *routing point* $e_i.piv$ (a selected *pivot* in the subtree of e_i), a *covering radius* $e_i.r$, and a *parent distance* $dist(e.piv, e_i.piv)$ where $e.piv$ is the selected *pivot* (*routing point*) in the parent node e of e_i. Note that, once the M-tree is constructed on the data, all the pivots in nodes are fixed. Moreover, through the M-tree index, as long as an intermediate node is filtered out, we can avoid accessing all the objects under this node, which significantly reduces the search space of *metric skyline queries*.

In the sequel, we first introduce the rationale of our *metric skyline* query processing. Then, we illustrate the mechanism of pruning an intermediate node in M-tree index. Finally, we present the detailed procedure of our *metric skyline* query processing.

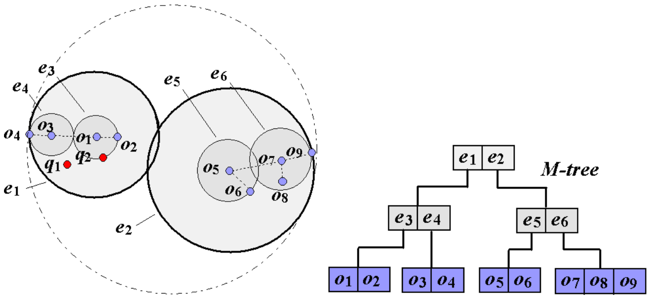


Figure 3: **An Example of M-tree**

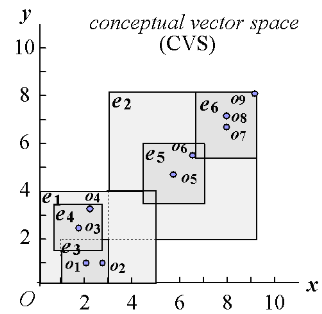


Figure 4: **Illustration of Metric Skyline in CVS**

5.1 Rationale of Query Processing

In order to clarify our *metric skyline* query processing, we *conceptually* map each entry e_i of M-tree onto a hyperrectangle in an n-dimensional vector space, namely *conceptual vector space* (CVS), in which each dimension is related to one *dynamic* attribute. Specifically, the j-th dimension of the hyperrectangle (from entry e_i) corresponds to interval $[LB(q_j, e_i), UB(q_j, e_i)]$, where $LB(q_j, e_i)$ and $UB(q_j, e_i)$ are lower and upper bounds of distance $dist(q_j, o_x)$, respectively, for any object o_x in entry e_i. Formally, by applying the *triangle inequality*, the distance $dist(q_j, o_x)$ ($j \in [1, n]$) from query point q_j to any object $o_x \in e_i$ is bounded by: $dist(q_j, o_x) \in$

$$\begin{cases} [0, dist(q_j, e_i.piv) + e_i.r] & \text{if } dist(q_j, e_i.piv) \le e_i.r, \\ [dist(q_j, e_i.piv) - e_i.r, dist(q_j, e_i.piv) + e_i.r] & \text{otherwise.} \end{cases} \quad (1)$$

where $o_x \in e_i$.

In the example of Figure 3(a), assume we have two query points q_1 and q_2. Let us first discuss upper and lower bounds of distance $dist(q_1, o_x)$ ($dist(q_2, o_x)$) from q_1 (q_2) to any object o_x in entry e_1. Since both query points q_1 and q_2 are within the circle of entry e_1, that is, $dist(q_1, e_1.piv) < e_1.r$ and $dist(q_2, e_1.piv) < e_1.r$, based on Eq. (1), we have $dist(q_1, o_x) \in [0, dist(q_1, e_1.piv) + e_1.r]$ and $dist(q_2, o_x) \in [0, dist(q_2, e_1.piv) + e_1.r]$, for any $o_x \in e_1$. As another example, we consider lower and upper bounds of distance $dist(q_1, o_y)$ ($dist(q_2, o_y)$) from q_1 (q_2) to any object o_y in entry e_6. Since it holds that $dist(q_1, e_6.piv) > e_6.r$, according to Eq. (1), we have $dist(q_1, o_y) \in [dist(q_1, e_6.piv) - e_6.r, dist(q_1, e_6.piv) + e_6.r]$, for any $o_y \in e_6$. Similarly, for query point q_2, we have $dist(q_2, o_y) \in [dist(q_2, e_6.piv) - e_6.r, dist(q_2, e_6.piv) + e_6.r]$, for any $o_y \in e_6$.

Figure 4 illustrates the resulting CVS *conceptually* transformed from Figure 3(a) with respect to q_1 and q_2. In particular, since the distance from q_1 (q_2) to entry e_1 is bounded by $[0, 5]$ ($[0, 4]$) using Eq. (1), entry e_1 corresponds to a rectangle in the 2D CVS with two diagonal corner points $(0, 0)$ and $(5, 4)$. Similarly, entry e_3 (e_4) is represented by a rectangle with corner points $(1, 0)$ and $(3, 2)$ ($(0.5, 1.5)$ and $(2.5, 3.5)$); entry e_2 is transformed to rectangle with corner points $(3, 2)$ and $(9, 9)$, and; entry e_5 (e_6) to that with corner points $(4.5, 3.5)$ and $(7, 6)$ ($(6.5, 5.5)$ and $(9, 8)$). The transformed hyperrectangles in CVS have the following property:

LEMMA 5.1. *Given a set, Q, of n query points, if an entry e in the M-tree is conceptually transformed to a hyperrectangle HR in an n-dimensional CVS, and its child node e_i is also transformed to a hyperrectangle HR_i similarly, then it holds that HR contains HR_i (i.e. $HR_i \subseteq HR$) in CVS.*

Proof. Without loss of generality, we consider the j-th dimension of hyperrectangles HR_i and HR, where $j \in [1, n]$. According to the conversion (indicated by Eq. (1)), the j-th side of HR_i (HR) is within the interval $[max\{0, dist(q_j, e_i.piv) - e_i.r\}, dist(q_j, e_i.piv) + e_i.r]$ ($[max\{0, dist(q_j, e.piv) - e.r\}, dist(q_j, e.piv) + e.r]$). Since entry e_i is fully contained in e, we have $dist(e_i.piv, e.piv) + e_i.r \le e.r$. Furthermore, according to the *triangle inequality* that $dist(q_j, e_i.piv) - dist(q_j, e.piv) \le dist(e_i.piv, e.piv)$, it holds that $dist(q_j, e_i.piv) + e_i.r \le dist(q_j, e.piv) + e.r$. Similarly, we have $max\{0, dist(q_j, e_i.piv) - e_i.r\} \ge max\{0, dist(q_j, e.piv) - e.r\}$. In other words, for each $j \in [1, n]$, the interval of HR_i is always contained in that of HR along the j-th dimension. Hence, we have $HR_i \subseteq HR$. $\square$

As a simple example in Figure 4, since entry e_1 is the parent of e_3 and e_4, the transformed rectangle from e_1 contains those from both e_3 and e_4. The same case occurs to entries e_2 and e_5 (e_6). Moreover, since data objects o_1 and o_2 are in a child node of e_3, the converted rectangle from e_3 also contains the transformed objects, where the coordinate of object o_1 (o_2) in CVS along the j-th dimension is defined as the real *metric* distance from o_1 (o_2) to q_j for $j \in [1, n]$.

Recall that, a data object is a *metric skyline* point if and only if its attribute vector is not *dominated* by that of other objects in the database. Intuitively, each dimension of our CVS corresponds to one attribute of data objects. Moreover, any intermediate node in the M-tree has its transformed hyperrectangle containing those of its children in CVS. Therefore, our *metric skyline* problem in *metric* space can be reduced to a classical *skyline search* in CVS.

Note, however, that, for different sets of query points given by different *metric skyline queries*, the resulting CVS' are also different, in terms of coordinates of hyperrectangles or even the dimensionality (due to different numbers, n, of query points). Thus, it is quite inefficient to materialize the *metric* space to a CVS for every incoming *metric skyline query*, and then issue a classical *skyline query* in CVS. Motivated by this, we propose a novel approach that can directly answer *metric skyline queries* in the *metric* space, through *metric* index. Although we do not explicitly convert (or materialize) CVS, which is the reason that we call CVS "*conceptual*", our query procedure can inherently perform a *skyline query* in CVS.

5.2 Pruning Intermediate Entries

Before we provide the detailed query procedure for the *metric skyline* retrieval, in this subsection, we present heuristics of pruning entries in an intermediate node of M-tree, which can avoid accessing data objects under entries and thus reduce the search cost. Assume that we have obtained a *metric skyline* candidate o_{cand} in the database $\mathcal{D}$. Our goal is to find the condition of pruning an entry e_i by candidate o_{cand}. Obviously, if the attribute vector of candidate o_{cand} can *dominate* that of any point o_x in entry e_i, then we can safely prune entry e_i. Specifically, we have the theorem below:

THEOREM 5.1. *Given a set, Q, of query points q_1, q_2, ..., and q_n, and a candidate skyline point $o_{cand} \in \mathcal{D}$, for any entry e_i in the M-tree, entry e_i can be safely pruned by candidate o_{cand} if it holds that $dist(q_j, o_{cand}) \leq LB(q_j, e_i)$ for all $j \in [1, n]$, where $dist(\cdot,\cdot)$ is a metric distance function and $LB(q_j, e_i)$ is the minimum possible distance between query point q_j and any data object in e_i.*

Proof. By contradiction, assume that although it holds $dist(q_j, o_{cand}) \leq LB(q_j, e_i)$ for all $1 \leq j \leq n$, there still exists a data object o_x in entry e_i that is a *metric skyline* point. Therefore, according to the definition of *metric skyline*, since candidate o_{cand} is in the database $\mathcal{D}$, there exists a query point q_j such that $dist(q_j, o_x) < dist(q_j, o_{cand})$. However, this is contrary to the fact that $dist(q_j, o_{cand}) \leq LB(q_j, e_i)$ (i.e. $dist(q_j, o_{cand}) \leq dist(q_j, o_x)$ since $o_x \in e_i$) for all $j \in [1, n]$, which completes our proof. □

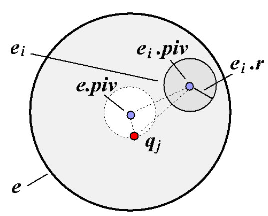


Figure 5: Illustration of Theorem 5.2

Theorem 5.1 illustrates the criterion of pruning an entry e_i using candidate $o_{cand} \in \mathcal{D}$, which requires to obtain lower bound distance $LB(q_j, e_i)$. As given by Eq. (1), the calculation of $LB(q_j, e_i)$ involves computing $dist(q_j, e_i.piv)$, which is costly in the case where the time complexity of $dist(\cdot,\cdot)$ is high. Moreover, it is quite inefficient to perform such calculation for every intermediate entry encountered during query processing. To reduce the cost, we present a more efficient way to prune data objects with *parent distances* stored in entries.

THEOREM 5.2. *Given a set, Q, of query points q_1, q_2, ..., and q_n, and a candidate skyline point $o_{cand} \in \mathcal{D}$, for any entry e_i in the M-tree, entry e_i can be safely pruned by o_{cand} if it holds that $dist(q_j, o_{cand}) \leq dist(e.piv, e_i.piv) - dist(q_j, e.piv) - e_i.r$ for all $j \in [1, n]$, where $dist(\cdot,\cdot)$ is a metric distance function and entry e is parent of entry e_i.*

Proof. From Theorem 5.1, entry e_i can be safely pruned if $dist(q_j, o_{cand}) \leq LB(q_j, e_i)$ for all $j \in [1, n]$. Therefore, it is sufficient to prove that $dist(e.piv, e_i.piv) - dist(q_j, e.piv) - e_i.r \leq LB(q_j, e_i)$ for all $1 \leq j \leq n$. Since it holds that $LB(q_j, e_i) = max\{0, dist(q_j, e_i.piv) - e_i.r\}$ (Eq. (1)) and $dist(e.piv, e_i.piv) - dist(q_j, e.piv) \leq dist(q_j, e_i.piv)$ (*triangle inequality* as illustrated in Figure 5), we can infer that $dist(e.piv, e_i.piv) - dist(q_j, e.piv) - e_i.r \leq dist(q_j, e_i.piv) - e_i.r \leq LB(q_j, e_i)$, which completes our proof. □

Since distances $dist(q_j, e.piv)$ between query points q_j and *pivot* $e.piv$ in parent node e have been computed before accessing entry e_i and moreover *parent distances* $dist(e.piv, e_i.piv)$ are stored in entry e_i, we can utilize these information to quickly check the pruning condition given in Theorem 5.2, which requires much lower cost than that of directly calculating the metric distance. In brief, we check the condition of pruning entries in an intermediate node as follows. First, we use Theorem 5.2 to prune entries in the node. Then, in case entry e_i cannot be pruned, we compute the distance between q_j and $e_i.piv$ and perform the pruning with Theorem 5.1. Finally, if entry e_i cannot be pruned by both Theorems 5.2 and 5.1, we have to access the subtree of e_i since e_i may contain *metric skylines*.

Pruning over Other Metric Index. Note that, our proposed methodology to prune intermediate entries can be applied to other *metric* indexes as well, such as VP-tree [5] or SS-tree [27]. Taking VP-tree [5] as an example, since intermediate nodes in a VP-tree also have *pivots* (called *vantage points*), we can use the same pruning idea as that in Theorem 5.1 and moreover derive light-weighted pruning conditions similar to Theorem 5.2, which can be integrated into our *metric skyline* query processing.

5.3 Query Processing

In this subsection, we present the procedure of our *metric skyline* query processing, namely MSQ, in Figure 6, which is directly processed in the *metric* space via *metric* index, M-tree. Specifically, similar to query processing on multidimensional indexes like R-tree [12], we search *metric skyline* points in a *best-first* manner by initializing an empty min-heap $\mathcal{H}$ and set rlt (recording *metric skyline* points). We define the key in heap $\mathcal{H}$ as $\sum_{j=1}^{n} LB(q_j, e)$ for any entry e, where q_j is the query point for $j \in [1, n]$. Intuitively, the smaller the key is, the more likely entry e contains *metric skyline* points. Then, we insert all entries of root into $\mathcal{H}$ (lines 1-3). Each time we pop out one entry e from heap $\mathcal{H}$ (lines 4-5) and by applying Theorem 5.1, check whether or not e can be pruned by points in rlt (lines 6-7). If the answer is yes, we do nothing; otherwise, process e as follows.

If e is a data point, we simply add it to rlt (lines 8-11). In case entry e is a leaf node, for each object o_i in e, which cannot be pruned by Theorem 4.1, we further verify whether or not it can be pruned by points in rlt and insert it into heap $\mathcal{H}$ when the answer is negative (lines 12-15). Similarly, in the case where entry e is an intermediate node, for each entry e_i in e, we first perform a quick pruning with rlt based on Theorem 5.2, followed by another verification with Theorem 5.1 if the former one fails to prune entry e_i (lines 17-19). When entry e_i cannot be pruned by both theorems, we need to insert e_i into heap $\mathcal{H}$ for further filtering. The query procedure repeats until heap $\mathcal{H}$ is empty.

As a example in Figure 3, assume a *metric skyline query* specifies two query points q_1 and q_2, and aims to retrieve all the *metric skyline* points through M-tree $\mathcal{I}$ constructed over $\mathcal{D}$, which contains nine data objects o_1, o_2, ..., and o_9. We first initialize set rlt and min-heap $\mathcal{H}$ which accepts entries in the form (e, key). Figure

```
Procedure MSQ {
  Input: M-tree I, n query points q_1, q_2, ..., and q_n
  Output: a set rlt of metric skyline points
  initialize a min-heap H accepting entries in the form (e, key)
  rlt = Φ;
  insert all entries of root(I) into heap H
  while H is not empty
     (e, key) = de-heap H
     if e can be pruned by some point in rlt (Th. 5.1)
        do nothing
     else //e is not dominated
        if e is a data point
           add e to rlt
        else
           if e is a leaf node
              for each data object o_i in e that cannot be pruned by Th. 4.1
                 if o_i is not dominated by any point o_cand ∈ rlt
                    insert o_i into heap H
           else // intermediate node
              for each entry e_i in e
                 if e_i cannot be pruned by Th. 5.2
                    if e_i cannot be pruned by Th. 5.1
                       insert e_i into heap H
  return rlt
}
```

Figure 6: Metric Skyline Query Processing (MSQ)

heap operation	heap status	rlt
access $root(\mathcal{I})$	$(e_1, 0)$, $(e_2, 5)$	Φ
expand e_1	$(e_3, 1)$, $(e_4, 2)$, $(e_2, 5)$	Φ
expand e_3	$(e_4, 2)$, $(o_1, 3)$, $(o_2, 3.8)$, $(e_2, 5)$	Φ
expand e_4 (de-heap o_1, o_2)	$(\mathbf{o_1}, \mathbf{3})$, $\cancel{(o_2, 3.8)}$, $(o_3, 4)$, $(e_2, 5)$, $(o_4, 5.5)$	$\{o_1\}$
de-heap o_3	$(\mathbf{o_3}, \mathbf{4})$, $(e_2, 5)$, $(o_4, 5.5)$	$\{o_1, o_3\}$
expand e_2	$\cancel{(o_4, 5.5)}$, $\cancel{(e_5, 8)}$, $\cancel{(e_6, 12)}$	$\{o_1, o_3\}$

Figure 7: Heap Contents During Metric Skyline Retrieval

Data sets	Data size	Dim.	Measure	Page size
SF	$174K$	2	L_1-norm	$1KB$
$sstock$	$50K$	4	L_2-norm	$10KB$
sat	$200K$	5	L_∞-norm	$10KB$
$signature$	$50K$	64	*Edit distance*	$10KB$

Figure 8: Characteristics of the Four Tested Data Sets

7 illustrates the heap contents of query processing in each step of procedure MSQ.

Specifically, our *metric skyline* query processing starts by inserting all entries (i.e. e_1 and e_2) in root $root(\mathcal{I})$ into heap $\mathcal{H}$, which sorts them in ascending order of keys (i.e. $(e_1, 0)$ and $(e_2, 5)$). Each time we pop out an entry (e.g. e_1) with the minimum key (0) in heap $\mathcal{H}$. Since the initial set, rlt, for *metric skyline* is empty, we expand entry e_1 and add its children e_3 and e_4 back to heap $\mathcal{H}$. Then, we further de-heap entry e_3 from $\mathcal{H}$ which contains data objects o_1 and o_2. Since o_1 and o_2 cannot be pruned by any point in rlt (empty), we insert them into heap $\mathcal{H}$. Similarly, we also expand entry e_4 and insert objects o_3 and o_4 into heap $\mathcal{H}$. After that, we encounter the first data object o_1 in the heap, which cannot be pruned by empty rlt, and thus add it to rlt. Since the attribute vector of the secondly popped data object o_2 is *dominated* by that of o_1 (as illustrated in Figure 4), we simply discard data object o_2. Furthermore, the attribute vector of the third data object o_3 is not *dominated* by o_1 in rlt, and o_3 is thus added to rlt. Finally, we expand entry e_2 and obtain its child entries e_5 and e_6. The next popped object, o_4, is discarded due to the *dominance* of its attribute vector by that of object o_1 (or o_3). Moreover, since attribute vectors (rectangles) of the remaining two entries e_5 and e_6 in heap $\mathcal{H}$ are *dominated* by that of data object o_1 (as shown in Figure 4), both of them can be pruned. Finally, the resulting points o_1 and o_3 in the set rlt are the answer to the *metric skyline query*.

Note that, procedure MSQ in Figure 6 demonstrates the search procedure of *metric skyline queries* in the *metric* space, which exactly corresponds to that of *skyline queries*, BBS [18], in CVS. In other words, given query points from a *metric skyline query*, the execution of procedure MSQ in Figure 6 is implicitly a *skyline* search in CVS. In particular, each node of M-tree can be *conceptually* converted into a hyperrectangle in CVS, whose accessing cost is one page I/O. Since BBS [18] is proved to be I/O optimal, we have similar result for our MSQ, due to its equivalence to BBS in CVS.

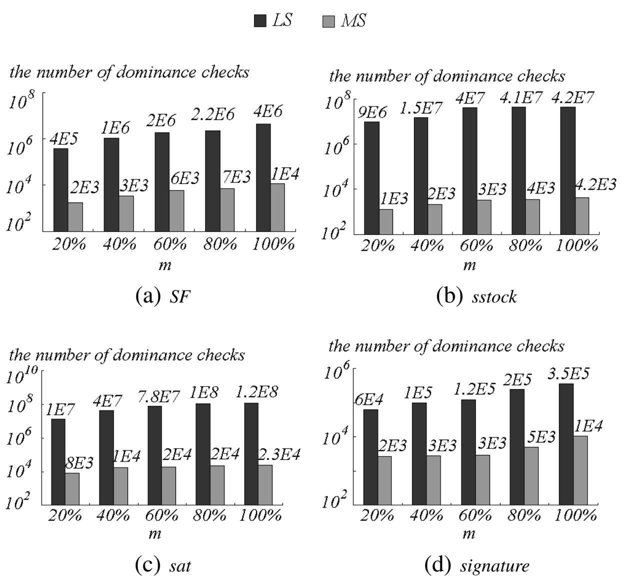
THEOREM 5.3. *The search procedure* MSQ *in Figure 6 is I/O optimal in CVS.*

Proof. Proved by the facts that, procedure MSQ in *metric* spaces corresponds to BBS in CVS and moreover BBS is I/O optimal [18]. □

6. EXPERIMENTAL EVALUATION

In this section, through extensive experiments, we demonstrate the efficiency and effectiveness of our proposed approach for *metric skyline queries*. In particular, we test our methods using both real and synthetic data sets, SF [25], $sstock$ [26], sat [11], and $signature$ [25], with four different *metric* distance functions, L_1-norm, L_2-norm, L_∞-norm, and *Edit distance*, respectively. All the selected distance functions have been widely used in many real-world applications [28, 7]. The first data set, SF, contains $174K$ 2D spatial locations in San Francisco. The second one, $sstock$, is obtained from 193 company stocks' daily closing price from late 1993 to early 1996, which consists of $50K$ truncated time series with length 4. The third data set, sat, includes $200K$ 5-dimensional satellite image data. The last one, $signature$, is a synthetic data set. Specifically, $50K$ strings, each containing 64 English letters, are randomly generated, which form about 20 clusters. Figure 8 briefly summarizes characteristics of our tested four data sets. In order to guarantee large *node capacity* for indexes, in our experiments, we set the page size of tree indexes to 1 KB for SF, and 10 KB for the other 3 data sets.

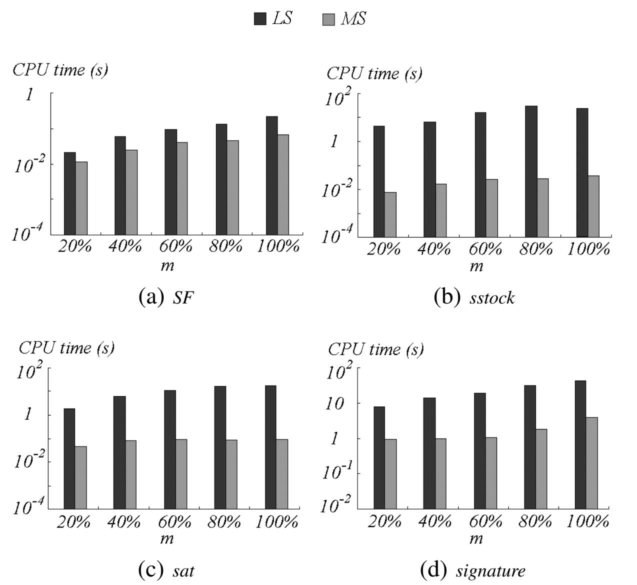
Note that, methods like *dynamic* [19], *spatial* [22], *multi-source skylines* [10] are designed for specific applications and cannot handle generic cases with arbitrary *metric* distance functions. Thus, we do not compare with them in our experiments. Moreover, our search procedure MSQ inherently corresponds to the BBS algorithm [18] in CVS, which is I/O optimal. However, it is inefficient to use BBS to answer *metric skyline queries* in CVS by first performing a space conversion and then constructing a multidimensional index (e.g. R-tree [12]) over transformed data in CVS. Therefore, in our experiment, we only compare our approach,



(a) *SF* (b) *sstock*

(c) *sat* (d) *signature*

Figure 9: The Number of Dominant Checks vs. m



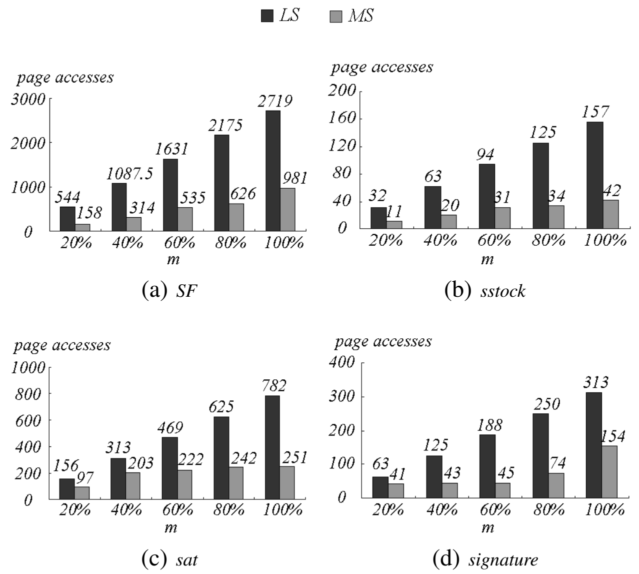
(a) *SF* (b) *sstock*

(c) *sat* (d) *signature*

Figure 10: The CPU Time vs. m

namely MS, over M-tree index [8] with the *linear scan* (LS). In particular, we consider three measures, the number of dominance checks, the CPU time, and page accesses (i.e. I/O cost). Note that, the CPU time include the cost of both accessing the index and checking the dominance relationships.

In order to evaluate the performance of our query processing, we generate a set, Q, of query points q_1, q_2, ..., and q_n as follows. We first choose a random object o in the database $\mathcal{D}$, then retrieve $max\{\lambda \cdot m, n\}$ data objects in $\mathcal{D}$ that are closest to o, and finally randomly select n points from these objects as query points, where λ is a parameter within $(0, 1)$, m is the data size, and n is the number of query objects. Note that, a similar parameter has been used in the experiments of *spatial skyline* [22] to test the query performance with different region areas covered by query points.



(a) *SF* (b) *sstock*

(c) *sat* (d) *signature*

Figure 11: The I/O Cost vs. m

Intuitively, large λ may lead to a large diameter of query points in the *metric* space. Specifically, in the example of opening shops near residential areas (Section 1), λ indicates the closeness of these areas (query points).

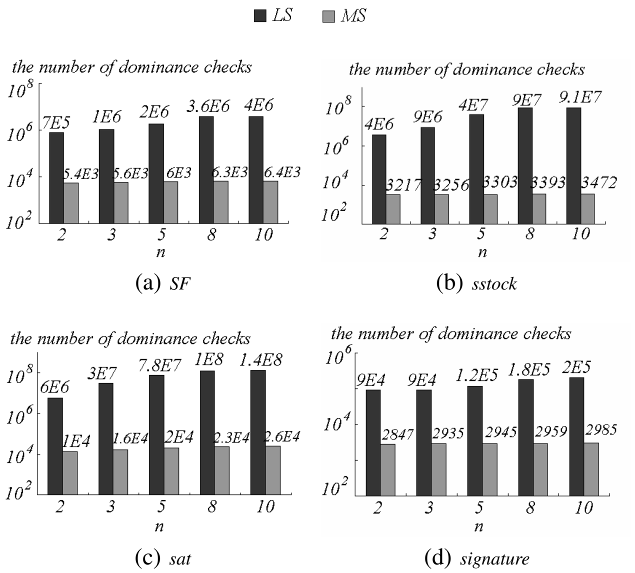
In the sequel, we compare the performance of MS and LS in answering metric skylines over four data sets. Specifically, Section 6.1 evaluates the effect of the data size m on the performance of *metric skyline queries*, whereas Section 6.2 studies the performance with respect to the query size (i.e. the number, n, of query points). Furthermore, we also present the experimental results by varying the workload of query points with respect to parameter λ. All our experiments are conducted on a Pentium IV 3.4GHz PC with 1G memory and query results are averaged over 50 runs.

6.1 Query Performance vs. Data Size

In the first set of experiments, we illustrate the performance with *metric skyline queries* under different data sizes m, by comparing MS with LS over both real and synthetic data sets. In particular, for LS, we assume that data objects are stored consecutively on disk blocks and the *metric skyline query* can be answered by sequentially scanning disk pages. For MS method, we insert each data point from data set $\mathcal{D}$ into a standard M-tree index $\mathcal{I}$, on which the *metric skyline query* is processed (i.e. procedure MSQ).

Figure 9 illustrates the number of dominance checks of LS and MS, with respect to data size m, during the *metric skyline* search, over four data sets, SF, $sstock$, sat, and $signature$, where the number, n, of query points is set to 5 and $\lambda = 0.03\%$. For all data sets, when m varies from 20% to 100% of the total data size, the number of dominance checks also increases. This is reasonable, since more data objects become candidates of *metric skyline*.

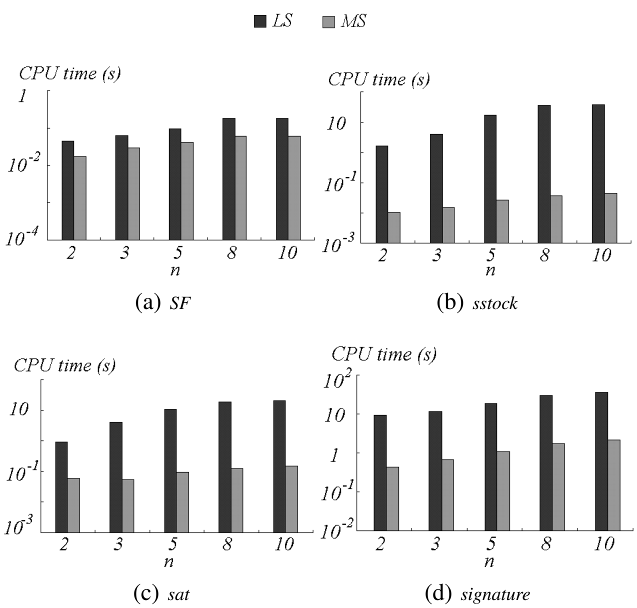
Obviously, since LS sequentially scans the data set for only one pass, it requires large number of dominance checks each time a data point is encountered, so as not to incur *false dismissals* (i.e. actual answer to *metric skyline queries* that are, however, not in the final result). On the other hand, since M-tree can facilitate pruning the search space, MS can save the cost of dominance checks. Thus,



(a) *SF* (b) *sstock*

(c) *sat* (d) *signature*

Figure 12: The Number of Dominant Checks vs. n



(a) *SF* (b) *sstock*

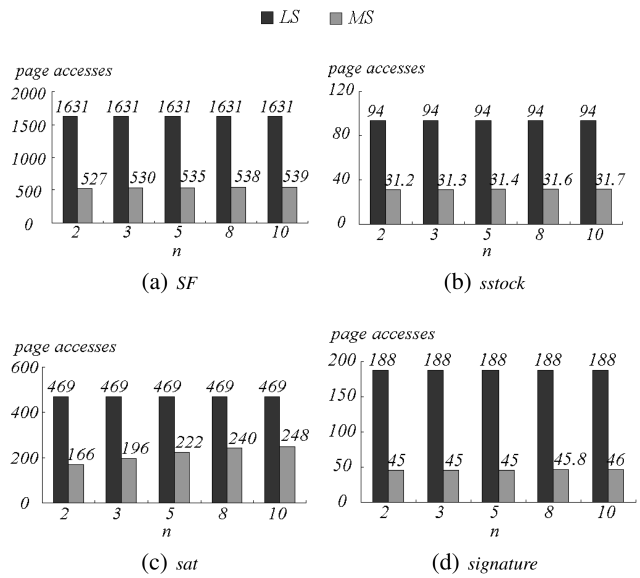
(c) *sat* (d) *signature*

Figure 13: The CPU Time vs. n

MS always outperforms LS by order of magnitude.

Correspondingly, Figure 10 shows the CPU time of LS and MS with the same experimental settings, over the four data sets. Since the major cost in the CPU time is for dominance checking and distance computation, the trends of the CPU time with respect to m are similar to that of the number of dominance checks in Figure 9. From 11, we can also find that CPU cost of finding metric skylines in $signature$ data set is much larger than costs on other tree data sets, this because the expensive distance function, Edit distance, is used in $signautre$ data set.

Figure 11 demonstrates the effect of data size m on the I/O cost of LS and MS, over four data sets, where $n = 5$ and $\lambda = 0.03\%$. For



(a) *SF* (b) *sstock*

(c) *sat* (d) *signature*

Figure 14: The I/O Cost vs. n

LS, since data objects are sequentially stored on disk, the number of page accesses (I/O cost) can be easily obtained, that is, dividing the required space for the entire data set by the page size. Specifically, in our implementation, each real number takes up 8 Bytes. Thus, the total number of pages for data set SF is 2719, that for $sstock$ is 157, for sat 782, and for $signature$ 313, respectively. As shown in our experimental results, the I/O cost of LS is higher than that of MS.

From the results demonstrated in Figures 9 to 11, we can find that larger m will lead to higher cost in finding metric skylines. Compared to LS, our proposed method, MS, not only reduces the search cost significantly but also scales smoothly with the increase of data size, which indicates that MS can be applied to very large data sets with complicated distance functions.

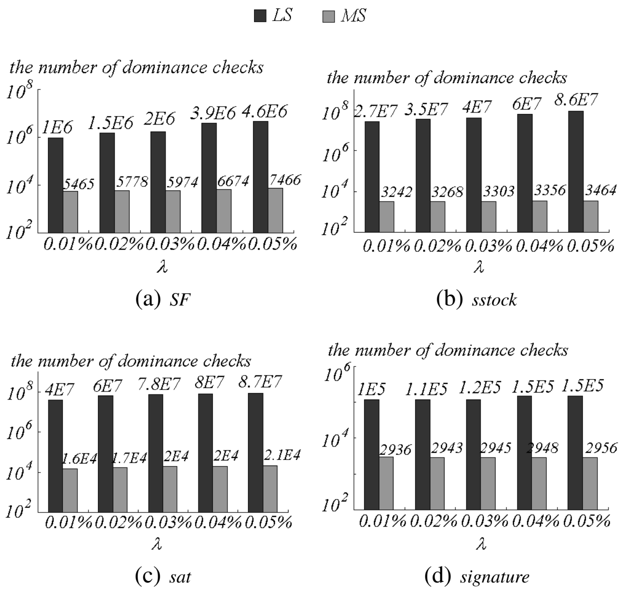
6.2 Query Performance vs. Query Size

In the second set of experiments, we evaluate the effect of query size (i.e. the number of query points, n) on the query performance, in terms of the number of dominance checks, the CPU time, and the I/O cost. Specifically, we vary the number, n, of query points from 2 to 10, and compare MS with LS.

In particular, Figure 12 illustrates the number of dominance checks with LS and MS, over four data sets SF, $sstock$, sat, and $signature$, where $n = 2, 3, 5, 8, 10$, $\lambda = 0.03\%$, and m is set to 60% of the data size that each data set has (described in Figure 8). For both methods, when n increases, the number of dominance checks also becomes higher, since more objects are included as *metric skyline* points. In general, MS requires much fewer dominance checks than LS.

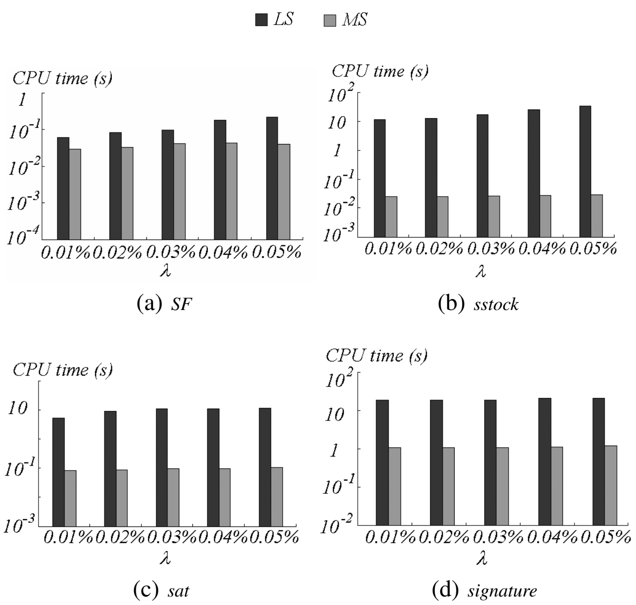
Figure 13 demonstrates the CPU time of LS and MS with the same experimental settings. Like previous results, the trends of the CPU time are similar to that of the number of dominance checks. That is, the CPU time increases with the increasing n due to higher distance computation between query points and objects/MBRs.

Next, Figure 14 presents the I/O cost of LS and MS, with the same



(a) *SF* (b) *sstock*
(c) *sat* (d) *signature*

Figure 15: **The Number of Dominant Checks vs.** λ



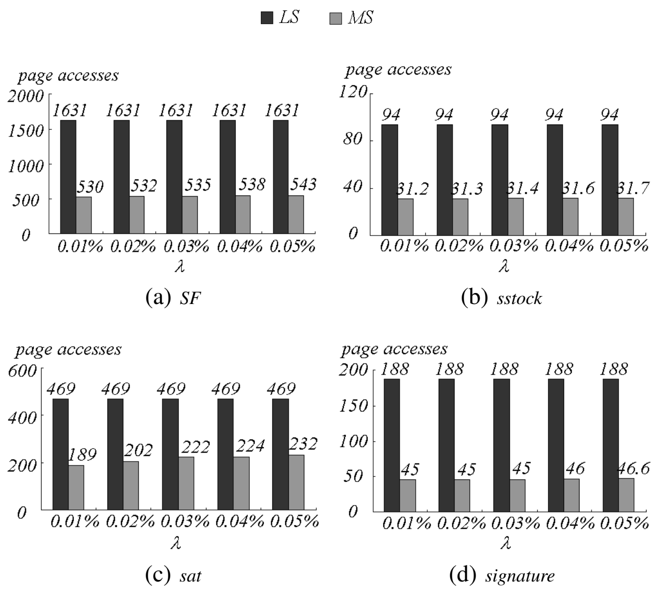
(a) *SF* (b) *sstock*
(c) *sat* (d) *signature*

Figure 16: **The CPU Time vs.** λ

settings. Specifically, when the number, n, of query points becomes larger, more *metric skyline* points are included in the answer set and more page accesses (I/O's) are thus needed for MS during the *metric skyline* search.

Again, the results reported in Figures 12 to 14 show that the search cost of MS is much less than that of LS in retrieving the metric skylines when different number of query objects are used.

Finally, we demonstrate the effect of parameter λ on the query performance. Recall that, λ can approximately indicate the diameter of query points scattered in the *metric* space, similar to that used in [22]. Figure 15, Figure 16, and Figure 17 illustrate the number of dominance checks, the CPU time, and page accesses, respectively, where m is 60% of the total data size and $n = 5$. From figures, we



(a) *SF* (b) *sstock*
(c) *sat* (d) *signature*

Figure 17: **The I/O Cost vs.** λ

find that when parameter λ becomes large (i.e. query points loosely scattered in the space), both the number of dominance checks and the CPU time increase. Furthermore, the I/O cost of both methods also increases when large λ is used. This is mainly because more *metric skyline* points are included in the answer set when query points are loosely scattered in the space, which requires more I/O's to retrieve. Similar to previous results, MS outperforms LS, in terms of both evaluated measures.

In summary, we have demonstrated through extensive experiments the efficiency and effectiveness of our proposed method, MS, over different data sets and under various metric measures, for the *metric skyline* retrieval, compared with LS.

7. CONCLUSIONS

Skyline plays an important role in a wide spectrum of applications including the business planning, multi-criteria decision making, and so on. Previous work on the *skyline* search assume data objects have either *static* attributes over Euclidean space or *dynamic* attributes designed for specific applications. However, there exist applications where the data from them can not be represented as vectors. For example, in the bioinformatics, the DNA sequences are usually modeled as strings. In these applications, other than the data themselves, the only information that we can obtain are the distance between each pair of data objects. Often, a metric distance function in used and the corresponding data space is called metric space.

In this paper, motivated by the usefulness of skyline queries, we study the skyline queries over a metric space. Specifically, we propose a generic *skyline query*, namely *metric skyline*, which retrieves *skyline* points with *dynamic* attributes defined in the *metric* space. In order to search metric skylines efficiently, we present an effective pruning mechanism and efficient query algorithm over the *metric* index. Extensive experiments have demonstrated the efficiency and effectiveness of our proposed method.

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