Advanced Risk Management

Exercises Week 1

1. We have observations on the daily log-returns of the S&P 500 index from January 2001 through December 2010, in total $T = 2514$ observations. This implies that in this period, there were on average 251 trading days per year. The average and standard deviation of the daily returns over this period were $\hat{\mu} = 0.0056\%$ and $\hat{\sigma} = 1.3771\%$.

   (a) Under the assumption that the daily returns are i.i.d., obtain a 95% confidence interval for the mean daily return. Does the mean return differ significantly from 0?

   (b) If the daily returns R_{t+1} are i.i.d. $N(\mu, \sigma^2)$, show that the K-period return $R_{t+1:t+K} = \sum_{k=1}^{K} R_{t+k}$ has mean $K\mu$ and standard deviation $\sqrt{K}\sigma$.

   (c) Recalling that a year corresponds to $K = 251$ days, obtain an estimate of the annual mean return and annual standard deviation of the S&P500 index. Does the annual mean return differ significantly from 0?

2. Suppose that we use the last 100 daily returns on a portfolio ($m = 100$) to obtain the VaR, either using Historical Simulation or using the RiskMetrics method. The previous 100 returns $R_{t-100}, \ldots, R_{t-1}$ all lie between -2% and and 2%, and the current estimate of the standard deviation for today is $\sigma_t = 1\%$. At the end of day t, we obtain a return R_t of either -5% or $+5\%$.

   (a) Compare the effect of $R_t = -5\%$ with the effect of $R_t = 5\%$ on tomorrow's 1% Value at Risk $\mathit{VaR}_{t+1}^{0.01}$ using Historical Simulation. You may assume that $R_{t-100} = 0$.

   (b) Compare the effect of $R_t = -5\%$ with the effect of $R_t = 5\%$ on tomorrow's 1% Value at Risk $\mathit{VaR}_{t+1}^{0.01}$ using RiskMetrics. Discuss the difference with your answer to (a).

3. Consider two returns R_1 and R_2, which have the same mixed continuous discrete distribution:

$$\begin{aligned} \Pr(a \leq R_i \leq b) &= 0.01(b-a), \qquad -1 \leq a < b \leq 0, \\ \Pr(R_i = 0.1) &= 0.99. \end{aligned}$$

   In words, the return is either uniformly distributed between -1 and 0, with 1% probability, or equal to 0.1, with 99% probability.

   (a) Show that the 1% VaR of both R_1 and R_2 is given by 0.

   (b) Let $R = R_1 + R_2$, and assume that the two returns are independent. Show that

$$\Pr(R < 0) > 0.01.$$

   This implies that the 1% VaR of $R_1 + R_2$ is larger than 0, and hence larger than $\mathit{VaR}(R_1) + \mathit{VaR}(R_2)$. This example illustrates that VaR is not sub-additive.

4. Explain why calculating the Expected Shortfall using Historical Simulation will require a very long historical period (i.e., a very large m).