UNIVERSITEIT VAN AMSTERDAM

Economics and Business

Academic Year 2018-2019

Exam:	**Advanced Risk Management** **6314M0351**
Date and time of the exam: **26 March 2019, 18:00–21:00** **Duration of the exam: 3 hours**	

Identification:

You have to identify yourself using your certificate of registration (UvA-identification card) and a valid proof of identity (passport, ID card) with a good resembling photograph. If you cannot identify yourself, access to the exam may be denied.

If you are not correctly registered via SIS for the course component, your exam will not be marked and registered.

Please write your name and student number on every sheet of paper you hand in.

Warning against fraud/cheating:

Students who are caught at any form of fraud/cheating will be punished.

Make sure that your mobile phone is switched off and put away in your briefcase/bag. This also applies for other audio equipment, headphones, digital watches (e.g. I-watches) and other electronic devices. Your briefcase/bag must be closed and placed on the floor beside your table.

Tools allowed:

Pen, pencil, pocket calculator

Bathroom visits:

For exams of 2 hours or less: It is not allowed to visit the bathroom during the exam.

For exams of 3 hours: During the exam it is only allowed to visit the bathroom after permission of the head invigilator.

Specific information on this exam:

This exam consists of 4 pages (including the front page). Please check if you received all pages. The exam consists of 4 questions, with 17 sub-questions. In total 100 points can be obtained; at the start of each sub-question, the maximum number of points is indicated.

Final Exam Advanced Risk Management, 26 March, 2019

Write your name and student number on all your written work. This is a closed book exam. The maximum number of points that can be obtained is given at the start of each sub-question, adding up to 100 points. Motivate your answers, and provide intermediate steps for any derivations that are asked.

1. We wish to measure the risk of a financial institution's equity portfolio, based on a set of daily historical log-returns $\{R_t, t = 1, \ldots, T\}$ on the portfolio.

   (a) [**5**] The one-day 1% Value at Risk (VaR) based on Historical Simulation (HS) is

   $$\mathit{VaR}_{t+1}^{0.01} = -\mathit{Percentile}\left(\{R_{t-j}, j = 0, \ldots, m-1\}, 1\right).$$

   Discuss the advantages and disadvantages of choosing the lag length m very large (e.g., $m = 5000$, corresponding to 20 years of daily returns).

   (b) [**5**] Suppose that $m = 100$, and that R_t is very large in absolute value: $|R_t| = 4\sigma$, where σ^2 is the daily return variance. Explain why the effect of R_t on $\mathit{VaR}_{t+1}^{0.01}$ is different depending on the sign of R_t, i.e., depending on whether $R_t = -4\sigma$ or $R_t = 4\sigma$.
   [You may assume that $|R_t|$ is much larger than the previous m absolute returns $|R_{t-j}|, j = 1, \ldots m$.]

   (c) [**5**] We now wish to backtest the VaR method, using the sequence of exceedances $\{I_t, t = 1, \ldots, T\}$, where $I_{t+1} = 1$ if $R_{t+1} < -\mathit{VaR}_{t+1}^{0.01}$, and $I_{t+1} = 0$ otherwise. Suppose that t-ratio of b_1 in the linear regression

   $$I_{t+1} = b_0 + b_1 I_t + e_{t+1}$$

   is much larger than the usual 5% critical value of 1.96. Why is this a problem for the VaR method?

   (d) [**5**] Name an extension of HS, or an alternative VaR method, that may resolve the problem indicated in question (c). Explain your choice.

   (e) [**5**] Discuss the advantages of Expected Shortfall over VaR as a measure of market risk.

2. Based on daily returns on the US dollar / Euro exchange rate over the period 2000–2018, we have estimated a GARCH(1,1) model (with non-zero mean μ):

   $$R_{t+1} = \mu + \sigma_{t+1} z_{t+1}, \qquad \sigma_{t+1}^2 = \omega + \alpha(R_t - \mu)^2 + \beta\sigma_t^2,$$

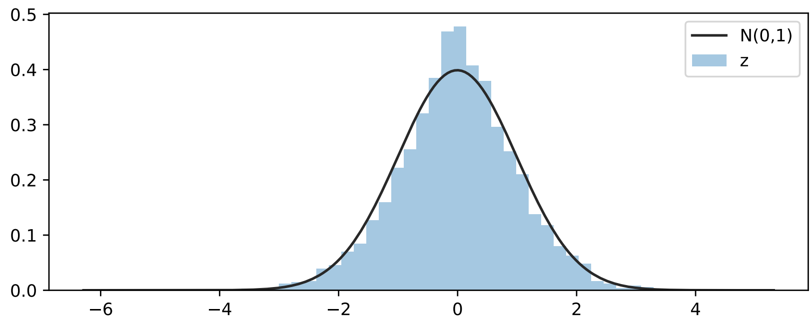
   using the `arch` package in Python, with the output given at the top of the next page.

```
                        Constant Mean - GARCH Model Results
==============================================================================
Vol Model:                      GARCH   Log-Likelihood:               -4174.80
Distribution:                  Normal   Method:             Maximum Likelihood
                                 Mean Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu         6.9892e-03  7.802e-03      0.896      0.370 [-8.302e-03,2.228e-02]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega      1.1635e-03  7.022e-04      1.657  9.754e-02 [-2.128e-04,2.540e-03]
alpha[1]       0.0299  3.998e-03      7.471  7.950e-14  [2.203e-02,3.770e-02]
beta[1]        0.9671  4.478e-03    215.965      0.000      [  0.958,  0.976]
==============================================================================
```

(a) [**5**] Based on the estimation output, do you conclude that the exchange rate returns display significant volatility clustering? Explain.

(b) [**5**] Obtain an estimate of the unconditional daily return variance from the estimation results given above. Explain your answer.

(c) [**5**] The histogram of the shocks $\hat{z}_t = (R_t - \hat{\mu})/\hat{\sigma}_t$ is given below. Would this support a risk measure based on the assumption $z_t \sim N(0,1)$? Why, or why not?



(d) [**10**] After defining $x_t = -\hat{z}_t, t = 1, \ldots, T$, we have sorted x_t in ascending order, and kept the 5% largest obervations: this gives $T_u = 138$ observations x_t which are all larger than $u = 1.69$. For these losses x_t, we assume the following Pareto distribution:

$$\Pr(x_t > x) = 0.05 \left(\frac{x}{u}\right)^{-1/\xi}, \qquad \text{for } x \geq u.$$

The Hill estimate of ξ is $\hat{\xi} = 0.25$. Show that the VaR, implied by the estimated GARCH model from (a) and the estimated Pareto distribution, is given by

$$\mathit{VaR}_{t+1}^{0.01} = -\hat{\mu} + \hat{\sigma}_{t+1} \times 2.527,$$

where $\hat{\mu} = 0.007$, and $\hat{\sigma}_{t+1}$ is the GARCH prediction of tomorrow's volatility.

3. We consider a portfolio consisting of two stock market indices, with daily returns R_{1t} and R_{2t}, and hence portfolio return $R_t = w_1 R_{1t} + w_2 R_{2t}$, with $w_1 + w_2 = 1$. The returns are assumed to have a zero mean. We wish to allow for time-varying conditional variances and covariances $\sigma_{ij,t+1} = E_t(R_{i,t+1}R_{j,t+1})$, and therefore use the following bivariate GARCH model:

$$\begin{aligned} \sigma_{11,t+1} &= 0.02 + 0.03R_{1t}^2 + 0.95\sigma_{11,t}, \\ \sigma_{22,t+1} &= 0.02 + 0.03R_{2t}^2 + 0.95\sigma_{22,t}, \\ \sigma_{12,t+1} &= 0.01 + 0.03R_{1t}R_{2t} + 0.95\sigma_{12,t}. \end{aligned}$$

   (a) [**10**] Show that for any w_1 (and hence $w_2 = 1 - w_1$), the conditional variance of the portfolio returns, $\sigma_{t+1}^2 = E_t(R_{t+1}^2)$ satisfies a GARCH(1,1) specification. What are the parameter values (ω, α, β) of this GARCH model?

   (b) [**10**] For some fixed w_1 and w_2, we wish to obtain $\mathit{VaR}_{t+1:t+2}^{0.01}$, the *two-day* 1% portfolio VaR. We assume that the portfolio shocks $z_{t+j} = R_{t+j}/\sigma_{t+j}$ are independent $N(0,1)$ distributed. Describe the steps that are needed to calculate $\mathit{VaR}_{t+1:t+2}^{0.01}$ using Monte Carlo simulation.
   You may assume that you know σ_{t+1}, and that you can let the computer draw independent $N(0,1)$ random variables $\breve{z}_{i,j}$, for $i = 1, \ldots, MC$ and $j = 1, 2$.

   (c) [**5**] A simple approximation of the two-day Value at Risk is

$$\mathit{VaR}_{t+1:t+2}^{0.01} \approx \sqrt{2} \times \mathit{VaR}_{t+1}^{0.01} = \sqrt{2} \times 2.33 \times \sigma_{t+1}.$$

   Will this approximation lead to an over-estimation or an under-estimation of the true $\mathit{VaR}_{t+1:t+2}^{0.01}$? Explain.

4. Give, for each of the following statements, a motivated choice whether they are true or false. If additional conditions are needed under which a statement may be true, then indicate this.

   (a) [**5**] Backtesting Expected Shortfall requires a larger test period than backtesting Value at Risk.

   (b) [**5**] The GJR-GARCH model is useful when the shocks $z_t = (R_t - \mu)/\sigma_t$ have an asymmetric (skewed) distribution.

   (c) [**5**] When high-frequency (intra-day) returns are available to calculate the realized variance, then we don't need GARCH models anymore for risk management.

   (d) [**5**] The Student's t copula is preferred over the Gaussian copula to allow for tail dependence between financial returns.

   (e) [**5**] In the Vasicek model, the portfolio credit risk gradually reduces to zero as the number of loans in the portfolio increases.