Advanced Risk Management

Exercises Week 2

1. In addition to the GARCH(1,1) model on Slide 15 of Week 2, we have estimated a GARCH(2,1) model, which can be written as

$$\sigma_{t+1}^2 = \omega + \alpha_1 R_t^2 + \alpha_2 R_{t-1}^2 + \beta\sigma_t^2.$$

The Stata output is as follows:

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ARCH family regression

Sample: 03jan2001 - 31dec2010                   Number of obs     =      2,514
Distribution: Gaussian                          Wald chi2(.)      =          .
Log likelihood = -3707.484                      Prob > chi2       =          .

------------------------------------------------------------------------------
             |                 OPG
           R |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
R            |
       _cons |    .042164    .018291     2.31   0.021     .0063143    .0780136
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |  -.0054058   .0113747    -0.48   0.635    -.0276999    .0168883
         L2. |    .107387   .0144182     7.45   0.000     .0791278    .1356462
             |
       garch |
         L1. |   .8854413   .0111009    79.76   0.000      .863684    .9071986
             |
       _cons |   .0180067   .0029191     6.17   0.000     .0122853    .0237281
------------------------------------------------------------------------------
```

(a) Test whether this model is significantly better than the GARCH(1,1) model using a likelihood ratio test. Does the outcome agree with the t-statistic (labelled z) of α_2?

(b) The unconditional variance in this model is $\sigma^2 = \omega/(1 - \alpha_1 - \alpha_2 - \beta)$. Evaluate this expression and compare it to the estimate of σ^2 from the GARCH(1,1) model.
[Note that $\hat{\alpha}_1$ is negative, because Stata does not impose $\alpha_i \geq 0$; apparently this does not lead to a problem with $\hat{\sigma}_t^2$ being positive.]

2. To allow for a (constant) non-zero mean return, the GARCH(1,1) model is formulated as

$$\begin{aligned} R_{t+1} &= \mu + \sigma_{t+1} z_{t+1}, \\ \sigma_{t+1}^2 &= \omega + \alpha(R_t - \mu)^2 + \beta\sigma_t^2. \end{aligned}$$

Because z_t are i.i.d. with mean zero and variance 1, the model implies $E_t(R_{t+1}) = E(R_{t+1}) = \mu$. Using $\mathrm{Var}(R_{t+1}) = E[(R_{t+1} - \mu)^2]$ and $\mathrm{Var}_t(R_{t+1}) = E_t[(R_{t+1} - \mu)^2]$, show that in this model, $\mathrm{Var}(R_{t+1}) = \sigma^2 = \omega/(1 - \alpha - \beta)$, just like in the model with $\mu = 0$.

3. There are different ways to obtain an asymmetric News Impact Curve in GARCH models. Christoffersen discusses the so-called non-linear GARCH model (with zero mean return):

$$
\begin{aligned}
R_{t+1} &= \sigma_{t+1} z_{t+1}, \\
\sigma^2_{t+1} &= \omega + \alpha (R_t - \theta \sigma_t)^2 + \beta \sigma^2_t.
\end{aligned}
$$

By comparing the NIC of this model with the NIC of the GJR-GARCH model, discuss which one gives a better representation of the leverage effect.

4. We wish to obtain a 10-day Value at Risk from a volatility model. We assume that the K-day returns $R_{t+1:t+K}$ have a normal distribution with mean 0 and variance $\mathrm{Var}(R_{t+1:t+K}) = \sum_{k=1}^{K} E_t(\sigma^2_{t+k})$, so that

$$
\mathit{VaR}^p_{t+1:t+K} = -\sqrt{\textstyle\sum_{k=1}^{K} E_t(\sigma^2_{t+k})} \times \Phi_p^{-1}.
$$

   (a) Show that for the RiskMetrics model $\sigma^2_{t+1} = 0.06 R^2_t + 0.94 \sigma^2_t$, we have $\mathrm{Var}(R_{t+1:t+10}) = 10 \times \sigma^2_{t+1}$, so that $\mathit{VaR}^p_{t+1:t+10} = \sqrt{10} \times \mathit{VaR}^p_{t+1}$.

   (b) On one of the worst days of the 2008 financial crisis, both the RiskMetrics model and a GARCH model indicated that $\sigma^2_{t+1} = 25$. Use the formulas on Slide 7 of Week 2 to obtain a 10-day 1% VaR based on a GARCH model with $\hat{\omega} = 0.01$, $\hat{\alpha} = 0.08$, $\hat{\beta} = 0.91$, and compare it to the RiskMetrics 10-day 1% VaR.