

Financial Markets(6314M0278Y)

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MSc Finance

Group Assignment 1

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(a) For each transaction, compute (but do not report in your solution) the (i) quoted spread in absolute terms; (ii) the quoted spread in relative terms; (iii) the effective half-spread in absolute terms; (iv) the effective half-spread in relative terms.

Compute (and report in your solution) the averages of these measures for the trading day. How does the average effective half-spread compare with the average quoted half-spread? How would you explain this finding (investigate your claim)?

(i) Quoted spread in absolute terms

$$S = a_t - b_t$$

Where a_t is the ask price and b_t is the bid price at trade round t . According to this formula, we can get the quoted spread for trade round t . Then, the average quoted spread in absolute terms is 0.0244.

(ii) Quoted spread in relative terms

$$S = (a_t - b_t) / m_t$$

Where m_t is the mid-price at the trade round t , which can be computed by $(a_t + b_t) / 2$. Hence, the average quoted spread in relative terms is 0.80%.

(iii) Effective half-spread in absolute terms

$$S_e = d_t \cdot (p_t - m_t)$$

where d_t is the direction (+1 for buy orders or -1 for sell orders) and p_t is the transaction price. Hence, the average effective half-spread in absolute terms: 0.0067

(iv) Effective half-spread in relative terms

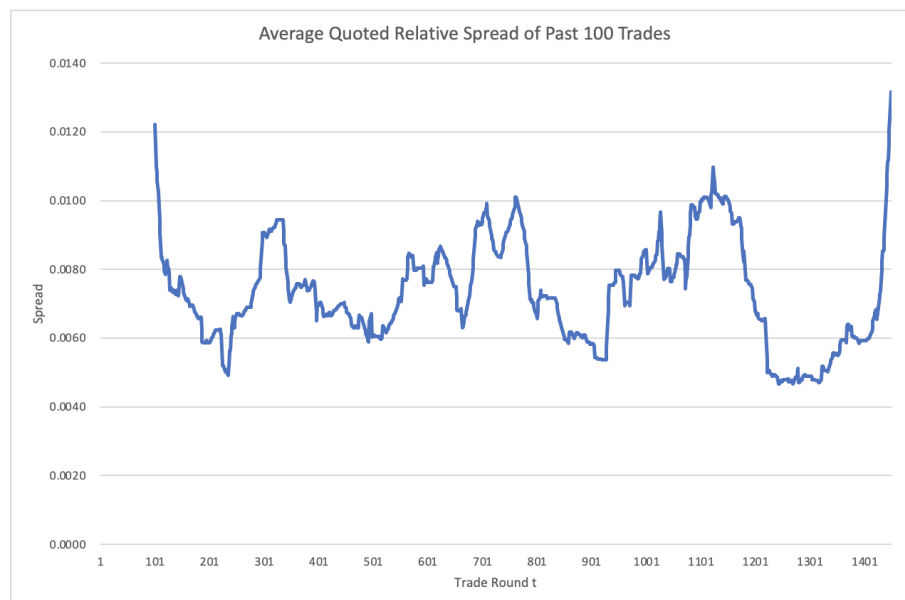
$$S_e = S_e / m_t$$

where S_e is the effective half-spread in absolute terms calculated in the previous step. Hence, the average effective half-spread in relative terms is 0.22%.

Quoted Spread		Effective Half-Spread	
Absolute	Relative	Absolute	Relative
0.0244	0.80%	0.0067	0.22%

Based on the results above, the average quoted half-spread in absolute terms and the average quoted half-spread in relative terms are 0.0122 ($0.0244/2$) and 0.40% ($0.80\%/2$) respectively. The average effective half-spread is smaller than the average quoted half-spread for both absolute terms ($0.0067 < 0.0122$) and relative terms ($0.22\% < 0.40\%$). In our conclusion, there's a price improvement since most orders are filled with the best price. In other words, the realized average transaction costs are lower than the best price.

(b) For each trade, starting from the 101-th trade, compute the average of the (quoted) relative spread for the past 100 trades. Plot a graph of your results. What kind of pattern emerges over the day?



The quoted relative spread starts at a high value and decreases sharply at the beginning. Then, the spread fluctuates for most of this trading day. Finally, it soars to the same level as the beginning of the day. The relative quoted spread can be related to the tightness of the financial market. Based on the graph, the market is relatively less tight at the beginning and at the end of the day, which can be attributed to the deficient liquidity caused by low volume of trading and intense price competition on both sides (buy orders and sell orders). Additionally, investors' behavior patterns can be inferred through the above graph. Most investors start investing between 9:30 and 10:00. And it can be seen that from about 15:45, investors start to stop trading. In addition, it can be seen that the trade volume between the beginning and the end is not constant and repeats increasing and decreasing.

(c) Starting from the second trading round ($t = 2$), compute the change in the mid-price from time $t - 1$ to time t , $\Delta m_t = m_t - m_{t-1}$ (e.g., for the second trading round you have $\Delta m_2 = m_2 - m_1$), and the change in the transaction price from time $t-1$ to time t , $\Delta p_t = p_t - p_{t-1}$. Run two OLS regressions (e.g., in Excel):

$$\Delta m_t = \alpha_m + \beta_m \cdot \Delta m_{t-1} + \varepsilon_t \text{ and } \Delta p_t = \alpha_p + \beta_p \Delta p_{t-1} + \varepsilon_t$$

Report the coefficients and t-stats (note: the dataset for the regressions starts in $t = 3$ since you have no Δp_{t-1} for $t = 2$). What signs do the beta coefficients have, and how would you interpret/explain these findings?

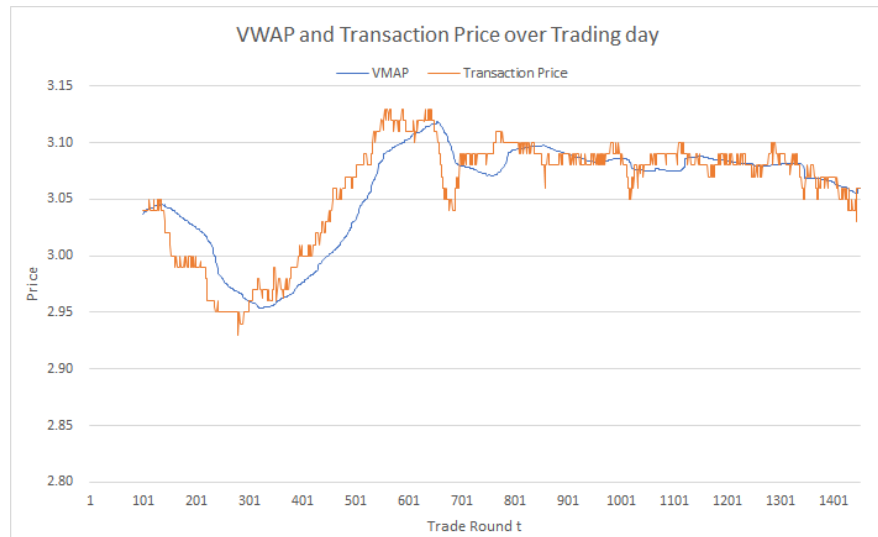
	Coefficient (β)	t-stat
Δm_{t-1}	-0.2629	-10.337
Δp_{t-1}	-0.3412	-13.849

The coefficients of both Δm_{t-1} and Δp_{t-1} are negative. In regression 1, ceteris paribus, every 1 extra unit of Δm_{t-1} is expected to decrease Δm_t by 0.2629 units. In addition, the absolute value of t-statistics is higher than 2.58 (the critical value for 99% confidence level) which indicates that the effect is statistically significant at 99% confidence level. Similarly, in regression 2, ceteris paribus, every 1 more unit of Δp_{t-1} would decrease Δp_t by 0.3412 units. Besides, this impact is also statistically significant at 99% confidence level.

(d) For each trade, starting from the 101-th trade, compute the Volume Weighted

Average Price (VWAP) over the past 100 trades:
$$VWAP = \frac{\sum p_t |q_t|}{\sum |q_t|}$$

where q_t and p_t are the size and price of the t -th trade. Plot the evolution of the transaction price and the VWAP over the trading day (starting from the 100-th trade). In practice, the VWAP is sometimes used as a benchmark to gauge the execution performance of brokers and other traders. The idea is that a trader does a good job in executing an order when the transaction price is more attractive than the VWAP of past trades (i.e., lower for buy orders and higher for sell orders). What do you think of this reasoning?



Both the transaction price and VWAP show a similar trend over the trading periods. The transaction price is either more or less attractive than the VWAP based on different trading periods. To be specific, the difference between the transaction price and VWAP will be explained. The transaction price is adjusted for the type of orders such as buy orders and sell orders, while the VWAP of past trades does not differentiate for buy orders and sell orders. In other words, the transaction price brings more incentives and motivations to the trader than the VWAP of past trades does. Therefore, the trader does a good job in executing an order when the transaction price is more attractive than the VWAP of past trades.

(e) Using the transaction price data, compute Roll's estimate of the bid-ask spread for the day. How does this estimate compare with the average quoted spread that you computed in part (a)? How would you explain this finding?

Roll believes that the market spread can be computed only by transaction prices (the covariance of price changes):

$$S = 2 \times \sqrt{-Cov[\Delta P_t, \Delta P_{t-1}]} = 2 \times c$$

where Δp_t is the difference of transaction prices between the trade round $t-1$ and t , Δp_{t-1} is the difference of transaction prices between the trade round $t-2$ and $t-1$. S is the spread and c is the half-spread. Based on this formula, we can firstly calculate the covariance of price

changes which is around - 0.00001495. Then we can get Roll's spread which is around 0.00773267.

Roll's approach is attractive in that it uses only transaction price to represent estimates. However, as can be seen from the comparison between average quoted spread (0.0244) and Roll's spread (0.0077), Roll's method underestimates the spread of the financial market. This failure of Roll's method can be attributed to the fact that the 3 additional assumptions mentioned in the lecture might not be perfectly satisfied in reality. Additionally, in Roll's model, the flows of orders and prices are regarded as 'Random Walk'. However, the flows of orders can be influenced many times within a single day, which can lead to the underestimation of price spread.

On the other hand, the positive relationship between price changes can also be problematic. As can be seen from Roll's formula, the formula cannot be established if the serial covariance is positive: Roll's spread = 0 when $\text{Cov}[\Delta p_t, \Delta p_{t-1}] < 0$. We have found that there was a modified formula of the covariate of price change made by Goyenko et al. (2009) with a consideration of positive price change correlations. The results of additional calculations through the above formula are as follows:

$\text{Cov}[\Delta p_t, \Delta p_{t-1}] = -0.00001495$ and Roll's Spread = 0.007729996. Then we need to take the mean of the transaction prices which is 3.06 into account:

$$\begin{aligned} \text{Goyenko Modified Roll's Spread} &= \text{Roll's Spread} / \text{the Mean of Price} \\ &= 0.007729996 / 3.06 = 0.02527 \end{aligned}$$

In this case, the difference between Roll's estimates and Average quoted spread is $|0.02527 - 0.0244| = 0.00087$ which is a much smaller gap. Concisely, we could check that Roll's estimate is not reliable when we have the long-term price change data. In this case, we need to use modified Roll's model for better accuracy.