Financial Markets

Lecture Notes Week 2

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This Week: Market Liquidity

The 3 dimensions of market liquidity

Measuring liquidity in practice

Amihud's (2002) illiquidity measure

Roll's (1984) bid-ask spread estimator

Market Liquidity

Degree to which securities can be <u>quickly</u> bought or sold in possibly <u>large</u> quantities at prices that are <u>close to fundamentals</u>

The 3 Dimensions of Market Liquidity

1. **Market Tightness:** How small is the spread?
2. **Market Depth:** What's the price impact of (large) orders?
   - deep market = little price impact
3. **Market Resiliency:** How quickly is liquidity replenished?
   - how fast do new quotes arrive?

Airbus, 10/08/2020, 9:36

#	Shares	Bid price	Ask price	Shares	#
1	63	70.46	70.49	302	4
3	320	70.44	70.50	425	7
1	118	70.43	70.51	645	6
5	550	70.42	70.52	507	6
4	474	70.41	70.53	767	8
...	...	...	...	...	...

- "Best bid and offer" (BBO):

$$b = 70.46 \text{ and } a = 70.49$$

- "Quoted" spread: $S = a - b = 0.03$
- Round-trip cost of **small** transaction

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- Midprice $m = (70.46 + 70.49)/2 = 70.475$
- Relative spread $s = 0.03/70.475 = 0.0426\%$
  - Small? $\Rightarrow$ "tight" market
- Price impact of **small** trade (relative to m):

$$s/2 = 0.0213\%$$

Weighted Average Bid-Ask Spread

- For larger trades, transaction costs can be much larger
- Weighted-average bid-ask spread:

$$S(q) = \bar{a}(q) - \bar{b}(q)$$

  where:
  - $\bar{a}(q)$: execution price buy market order of size q
  - $\bar{b}(q)$: execution price sell market order of size q
- Inverse of "slope" of $S(q)$: market **depth**
  - how sensitive is the price to changes in order size?

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- $\bar{a}(500) = (302 \times 70.49 + 198 \times 70.50)/500 = 70.4937$
- $\bar{b}(500) = 70.4402$
- $S(500) = 0.05378$, $s(500) = S(500)/m = 0.0763\%$
- price impact buy order ($q = 500$): 0.0269%
- price impact sell order ($q = 500$): 0.0494%

Airbus, 10/08/2020: $S(q)$

- Which side of the market is deeper (ask or bid side)?

Effective Half-Spread

- LOB at price points beyond the BBO may not be readily observable
- A trading cost measure that uses prices actually obtained by investors is the **effective half-spread**:

$$S_e = d_t(p_t - m_t)$$

where:

- p_t: execution price of market order
- m_t: midprice just before execution ($(a_t + b_t)/2$)
- d_t: order direction indicator

$$d_t = \begin{cases} 1 & \text{buy order} \\ -1 & \text{sell order} \end{cases}$$

Effective Half-Spread

- Effective half-spread measures transaction cost incurred by "liquidity demander," relative to m
  - e.g., Airbus, buy order 500 shares,
    $S_e = p_t - m_t = 70.4937 - 70.475 = 0.01896$
  - in % terms: $s_e = S_e/m = 0.0269\%$ (as above)
- Does S_e measure "liquidity supplier's" profit?
  - not necessarily!
  - orders may exert lasting pressure on prices, to the detriment of liquidity suppliers (e.g., dealers) who absorb such orders into their inventories

Effective Half-Spread

- Say, BBO is $b = 99$ and $a = 101$
- Dealer (with $b = 99$, $a = 101$) receives sell order for 50 shares, so dealer buys 50 shares for 99
- What's his profit?
  - If dealer can sell for 101, profit is 2 per share (the spread) [before operating costs]
  - More realistically, suppose dealer is equally likely to unwind his position at 101 (his ask) or 99 (somebody else's bid):

$$1/2 \times (101 - 99) + 1/2 \times (99 - 99) = 1$$

  - Not very realistic either: orders often have lasting price impact, i.e., prices move in the direction of the trade!

Effective Half-Spread

- Say, prices decline to $a = 100$ and $b = 98$
- Dealer's profit:

$$1/2 \times (100 - 99) + 1/2 \times (98 - 99) = 0$$

  which can be rewritten as

$$m_{t+\Delta} - p_t = (100 + 98)/2 - 99 = 0$$

  where:
  - $m_{t+\Delta}$: new midprice
  - p_t: transaction price
- This is called the **realized** half-spread

Realized Half-Spread

- **Realized** half-spread:

$$S_r = d_t(p_t - m_{t+\Delta})$$

  - $m_{t+\Delta}$: midprice some time **after** the transaction

- We have:

$$\begin{aligned} S_r &= d_t(p_t - m_{t+\Delta}) = \underbrace{d_t(p_t - m_t)}_{=S_e} - \underbrace{d_t(m_{t+\Delta} - m_t)}_{\text{midprice revision}} < S_e \\ &= -(99 - 99) = -(99 - 100) - (-)(99 - 100) = 0 \end{aligned}$$

Amihud's (2002) Illiquidity Measure

- Widely used in the finance literature

$$IlliQ = \frac{1}{T}\sum_{1}^{T}\frac{|r_t|}{VOL_t}$$

where:

  - T: number of days of sample period (e.g., month, year)
  - $|r_t|$: absolute value of daily return
  - VOL_t: daily trading volume

$\approx$ "price response" per \$1 of trading volume

- Comparable across stocks?

Roll's (1984) Bid-Ask Spread Measure

- Is there a way to estimate spreads based only on transaction prices, i.e., without observing ask prices, bid prices or the order book?
- Basic idea of Roll (1984):
  - Spread $\uparrow \Rightarrow$ Price fluctuations $\uparrow$
  - $\Rightarrow$ Price fluctuations (which we can measure empirically) may tell us something about the spread!
  - Let's see how this works!

Roll (1984): Simple Model

- Ask and bid prices at time t are:

  $$a_t = m_t + c \text{ and } b = m_t - c$$

  where c is the quoted half-spread ($c = S/2$)
- Midprice m_t proxies fundamental value
- Fundamental value follows a **random walk**:

  $$m_t = m_{t-1} + \epsilon_t$$

  where $E[\epsilon_t] = 0$, $E[\epsilon_t \epsilon_s] = 0$ for all $s \neq t$
  - ϵ_t accounts for the arrival of new information
  - as the expected value is zero, the expected return is zero, too, which is a reasonable approximation for small time intervals (e.g., seconds or much smaller)

Roll (1984): Simple Model

- Trades take place at either the ask or bid, i.e.,
  - trades are "small"
  - no price improvement (relaxed during tutorials)

- Transaction prices: $p_t = m_t + cd_t$, where:

$$d_t = \begin{cases} 1 & \text{with prob. } 1/2 \text{ (buy order)} \\ -1 & \text{with prob. } 1/2 \text{ (sell order)} \end{cases}$$

  i.e., order flow is balanced: $E[d_t] = 0$.

- In this model, how are price **changes** correlated over time?

Roll (1984)

- Price change from $t-1$ to t:

$$\begin{aligned}\Delta p_t &= p_t - p_{t-1} = \overbrace{\underbrace{m_{t-1} + \epsilon_t}_{=m_t} + cd_t}^{=p_t} - \Big(\overbrace{m_{t-1} + cd_{t-1}}^{=p_{t-1}} \Big) \\ &= c(d_t - d_{t-1}) + \epsilon_t\end{aligned}$$

Additional assumptions (can be relaxed):

1. $E[d_t d_s] = 0$ for all $s \neq t$
2. $E[d_t \epsilon_t] = 0$
3. $E[d_t \epsilon_s] = 0$ for all $s \neq t$

Roll (1984)

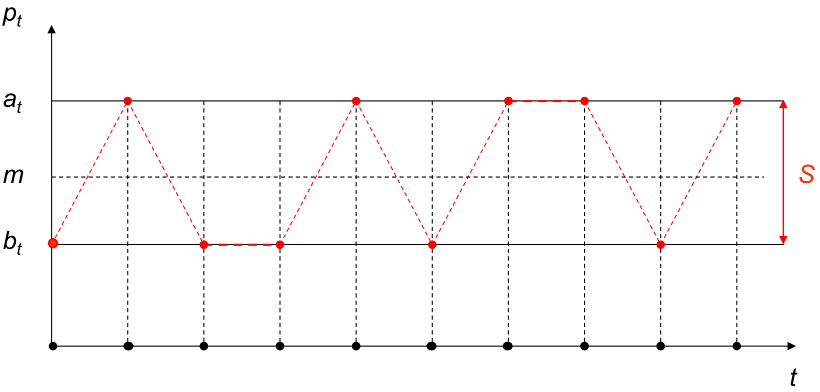
- From the previous assumptions, it follows that

$$\text{Cov}[\Delta p_t, \Delta p_{t-1}] =$$
$$\text{Cov}[c(d_t - d_{t-1}) + \epsilon_t, c(d_{t-1} - d_{t-2}) + \epsilon_{t-1}] =$$
$$-c^2 \times Var[d_{t-1}] = -c^2 \times \underbrace{\left(\overbrace{E[d_{t-1}^2]}^{=1} - \overbrace{(E[d_{t-1}])^2}^{=0} \right)}_{=\text{Var}[d_{t-1}]} = -c^2 < 0$$

$\Rightarrow$ Price changes are **negatively** correlated

- Intuition: bid-ask bounce

Roll (1984)



Roll (1984)

- From above,

$$Cov[\Delta p_t, \Delta p_{t-1}] = -c^2$$

- Solving this for c yields

$$c = \sqrt{-Cov[\Delta p_t, \Delta p_{t-1}]}$$

- Since $c = S/2$, we obtain

$$\boxed{S = 2 \times \sqrt{-Cov[\Delta p_t, \Delta p_{t-1}]}}$$

  - $Cov[\Delta p_t, \Delta p_{t-1}]$: estimated from transaction prices

Key Concepts

- Tightness, depth and resiliency
- Quoted spread
- Effective spread
- Relative spread
- Amihud's (2002) illiquidity measure
- Roll's (1984) bid-ask spread measure