INFO 6105 ASSIGNMENT 1 – Jane Akpang

Question 1

Create a matrix array A and vector array in Numpy with **random** integers using random method. For example
A=[5−2−3−179],x=[46]. Your A matrix is **5x2** and your x vector is **2x1**. Show your code and result.
Create a diagonal matrix B, with 1,2,3,4,5 in the diagonal. The order doesn't matter. You can have
B=[341]. Note your diagonal matrix is 5x5.
Calculate Ax, and BA. Show your code and result

Matrix A (5x2):
```
[[ -3   7]
 [  5  -2]
 [ -9  10]
 [  1  -8]
 [  6   4]]
```

Vector x (2x1):
```
[[ 4]
 [-7]]
```

The Prompt - Create a matrix array A and vector array in Numpy with random integers using random method.

5 x5 Diagonal Matrix B, with values 1,2,3,4,5:

Code:

```
B = np.diag([1, 2, 3, 4, 5])
print("\nMatrix B (5x5 diagonal):\n", B)
```

Matrix B (5x5 diagonal):

```
[[1 0 0 0 0]
 [0 2 0 0 0]
 [0 0 3 0 0]
 [0 0 0 4 0]
 [0 0 0 0 5]]
```

Compute Ax and BA

The process

- Ax - Multiply A by x (matrix-vector multiplication (result is 5×1).
- BA is multiplying the 5×5 diagonal matrix with A, B by A.

Matrix A (5x2):
[-4 9]
 [4 0]
 [-3 10]
 [-4 8]
 [0 0]

Vector x (2x1):
 [[10]
 [-7]]

Calculation:
(-4) × (10) + (9) × (-7) = 40 + (-63) = -103
(4) × (10) + (0) × (-7) = 40 + (0) = 40
(-3) × (10) + (10) × (-7) = 30 + (-70) = -100
(-4) × (10) + (8) × (-7) = 40 + (-56) = -96
(0) × (10) + (0) × (-7) = 0 + (0) = 0

Ax =
[-103]
[40]
[-100]
[-96]
[0] (5x1)

BA:

Matrix B (5x5 diagonal):
[[1 0 0 0 0]
 [0 2 0 0 0]
 [0 0 3 0 0]
 [0 0 0 4 0]
 [0 0 0 0 5]]

Matrix A (5x2):
[-4 9]
 [4 0]
 [-3 10]
 [-4 8]
 [0 0]

BA = [-4, 9] × 1 = [-4, 9]
[4, 0] × 2 = [8, 0]
[-3, 10] × 3 = [-9, 30]
[-4, 8] × 4 = [-16, 32]
[0, 0] × 5 = [0, 0]

BA (result of B * A):
 [[-4   9]
 [8   0]
 [-9 30]
 [-16 32]
 [0   0]]

2. The rank of Matrix A and matrix B

Matrix A (5x2):
[-4 9]
 [4 0]
 [-3 10]
 [-4 8]
 [0 0]

The maximum possible rank: min(5, 2) = 2
Actual/Full rank: 2 (Non-zero singular values).

This means Matrix A has 2 linearly independent rows and columns.

Matrix B (5x5)

Maximum possible rank: min(5, 5) = 5
Actual rank: 5

This means Matrix B has 5 Diagonal elements: [1 2 3 4 5] (Non-zero diagonal elements)
5 Singular values of B: [5. 4. 3. 2. 1.] Non-zero singular values: 5.

What about
the rank of BA? Are they different? Why?

BA = B × A =

[1×(-4) 1×9] = [-4 9]
[2×4 2×0] = [8 0]
[3×(-3) 3×10] = [-9 30]
[4×(-4) 4×8] = [-16 32]
[5×0 5×0] = [0 0]

Yes, the ranks are different! Here's why:

Rank(B) = 5 (full rank diagonal matrix)
Rank(A) = 2 (has a zero row)
Rank(BA) = 2 (same as A, not B)

Question 3 by hand:

Find the following expressions by hand. Show your steps.
(a) AB
(b) BA
(c) AB – BA
(d) ABC

Given Matrices:

A = [-2 1 8] B = [5 0 -7] C = [6 3 -1]
[-1 -1 7] [6 3 -9] [2 4 5]
[3 0 4] [-2 -2 0] [-1 -1 8]

AxB =

Row 1 of A × Column 1 of B = (-2) x (5) + (1) x (6) + (8) x (-2) = -10 + 6 - 16 = **-20**

Row 1 of A × Column 2 of B = (-2) x (0) + (1) x (3) + (8) x (-2) = 0 + 3 - 16 = **-13**

Row 1 of A × Column 3 of B = (-2) x (-7) + (1) x (-9) + (8) x (0) = 14 - 9 + 0 = **5**

Row 2 of A × Column 1 of B = (-1) x (5) + (-1) x (6) + (7) x (-2) = -5 - 6 - 14 = **-25**

Row 2 of A × Column 2 of B = (-1) x (0) + (-1) x (3) + (7) x (-2) = 0 - 3 - 14 = **-17**

Row 2 of A × Column 3 of B = (-1) x (-7) + (-1) x (-9) + (7) x (0) = 7 + 9 + 0 = **16**

Row 3 of A × Column 1 of B = (3) x (5) + (0) x (6) + (4) x (-2) = 15 + 0 - 8 = **7**

Row 3 of A × Column 2 of B = (3) x (0) + (0) x (3) + (4) x (-2) = 0 + 0 - 8 = **-8**

Row 3 of A × Column 3 of B = (3) x (-7) + (0) x (-9) + (4) x (0) = -21 + 0 + 0 = **-21**

Result:

AB = [-20 -13 5]

[-25 -17 16]

[7 -8 -21] (3 x 3)

Calculate BA

B x A = Row 1 of B × Column 1 of A = (5)(-2) + (0)(-1) + (-7)(3) = -10 + 0 - 21 = **-31**

Row 1 of B × Column 2 of A = (5)(1) + (0)(-1) + (-7)(0) = 5 + 0 + 0 = **5**

Row 1 of B × Column 3 of A = (5)(8) + (0)(7) + (-7)(4) = 40 + 0 - 28 = **12**

Row 2 of B × Column 1 of A = (6)(-2) + (3)(-1) + (-9)(3) = -12 - 3 - 27 = **-42**

Row 2 of B × Column 2 of A = (6)(1) + (3)(-1) + (-9)(0) = 6 - 3 + 0 = **3**

Row 2 of B × Column 3 of A = (6)(8) + (3)(7) + (-9)(4) = 48 + 21 - 36 = **33**

Row 3 of B × Column 1 of A = (-2)(-2) + (-2)(-1) + (0)(3) = 4 + 2 + 0 = **6**

Row 3 of B × Column 2 of A = (-2)(1) + (-2)(-1) + (0)(0) = -2 + 2 + 0 = **0**

Row 3 of B × Column 3 of A = (-2)(8) + (-2)(7) + (0)(4) = -16 - 14 + 0 = **-30**

Result:

BA = [-31 5 12]

[-42 3 33]

[6 0 -30]

Calculate AB - BA

Subtract corresponding elements: (AB - BA)[i,j] = AB[i,j] - BA[i,j]

AB = [-20 -13 5] BA = [-31 5 12]

[-25 -17 16] - [-42 3 33]

[7 -8 -21]] [6 0 -30]

AB - BA = [-20-(-31) -13-5 5-12]

[-25-(-42) -17-3 16-33]

[7-6 -8-0 -21-(-30)]

Result:

AB - BA = [11 -18   -7]

[17 - 20 -17]

[1   -8   9]

Calculate ABC

AB = [-20 -13   5]

[-25 -17   16]

[7   -8 -21]

C = [6   3 -1]

[2   4   5]

[-1 -1   8]

ABC =   Row 1 of AB × Column 1 of C = (-20) x (6) + (-13) x (2) + (5) x (-1) = -120 - 26 - 5 = **-151**

Row 1 of AB × Column 2 of C = (-20) x (3) + (-13) x (4) + (5) x (-1) = -60 - 52 - 5 = **-117**

Row 1 of AB × Column 3 of C = (-20) x (-1) + (-13) x (5) + (5) x (8) = 20 - 65 + 40 = **-5**

Row 2 of AB × Column 1 of C = (-25) x (6) + (-17) x (2) + (16) x (-1) = -150 - 34 - 16 = **-200**

Row 2 of AB × Column 2 of C = (-25) x (3) + (-17) x (4) + (16) x (-1) = -75 - 68 - 16 = **-159**

Row 2 of AB × Column 3 of C = (-25) x (-1) + (-17) x (5) + (16) x (8) = 25 - 85 + 128 = **68**

Row 3 of AB × Column 1 of C = (7) x (6) + (-8) x (2) + (-21) x (-1) = 42 - 16 + 21 = **47**

Row 3 of AB × Column 2 of C = (7) x (3) + (-8) x (4) + (-21) x (-1) = 21 - 32 + 21 = **10**

Row 3 of AB × Column 3 of C = (7) x (-1) + (-8) x (5) + (-21) x (8) = -7 - 40 - 168 = **-215**

Result:

ABC = [-151 -117   -5]

[-200 -159   68]

[47   10 -215]

Summary of Results:

AB = [[-20 -13   5]

[-25 -17   16]

[7   -8 -21]]

BA = [[-31   5   12]

[-42   3   33]

[6   0 -30]]

AB - BA = [[11 -18   -7]

[17 -20 -17]

[1   -8   9]]

ABC = [[-151 -117   -5]

[-200 -159   68]

[47   10 -215]]

Question 4

Calculate the eigenvalues and eigenvectors of matrix A above by hand. Show your steps below. You can use external tools to solve for a polynomial equation only. Show the trace of matrix A.

Given Matrix A = [-2   1   8]

[-1 -1   7]

[3   0   4]

To find Eigenvalues λ where $\det(A - \lambda I) = 0$

Calculate $A - \lambda I$:

$$A - \lambda I = \begin{bmatrix} -2-\lambda & 1 & 8 \\ -1 & -1-\lambda & 7 \\ 3 & 0 & 4-\lambda \end{bmatrix}$$

Find the determinant using the first row expansion:

$$\det(A - \lambda I) = (-2-\lambda) \times \begin{vmatrix} -1-\lambda & 7 \\ 0 & 4-\lambda \end{vmatrix} - 1 \times \begin{vmatrix} -1 & 7 \\ 3 & 4-\lambda \end{vmatrix} + 8 \times \begin{vmatrix} -1 & -1-\lambda \\ 3 & 0 \end{vmatrix}$$

2×2 determinant:

$$(-1-\lambda) \times (4-\lambda) - 7\times0 = (-1-\lambda)(4-\lambda)$$
$$(-1) \times (4-\lambda) - 7\times3 = -4+\lambda-21 = \lambda-25$$
$$(-1)\times0 - (-1-\lambda)\times3 = 3+3\lambda$$

$$\det(A - \lambda I) = (-2-\lambda) \times (-1-\lambda)(4-\lambda) - (\lambda-25) + 8(3+3\lambda)$$
$$= (-2-\lambda) \times (-4+\lambda-4\lambda+\lambda^2) - \lambda + 25 + 24 + 24\lambda$$
$$= (-2-\lambda)(\lambda^2-3\lambda-4) + 23\lambda + 49$$

By Expansion:

$$= -2\lambda^2 + 6\lambda + 8 - \lambda^3 + 3\lambda^2 + 4\lambda + 23\lambda + 49$$
$$= -\lambda^3 + \lambda^2 + 33\lambda + 57$$

$$\lambda^3 - \lambda^2 - 33\lambda - 57 = 0$$

Using calculator/computer: $\lambda_1 = 6$, $\lambda_2 = -3$, $\lambda_3 = -2$

To Find Eigenvectors
For each eigenvalue, solve $(A - \lambda I)v = 0$

For $\lambda = 6$:

$$A - 6I = \begin{bmatrix} -8 & 1 & 8 \\ -1 & -7 & 7 \\ 3 & 0 & -2 \end{bmatrix}$$

$-8x + y + 8z = 0$ ... (1)
$-x - 7y + 7z = 0$ ... (2)
$3x + 0y - 2z = 0$ ... (3)

equation (3): $3x = 2z$, so $x = (2/3)z$
From equation (1): $-8(2z/3) + y + 8z = 0$ $-16z/3 + y + 8z = 0$ $y = 16z/3 - 8z = -8z/3$
Choose $z = 3$: $x = 2$, $y = -8$, $z = 3$ Eigenvector: $v_1 = [2, -8, 3]$

For $\lambda = -3$:

$$A + 3I = \begin{bmatrix} 1 & 1 & 8 \\ -1 & 2 & 7 \\ 3 & 0 & 7 \end{bmatrix}$$

Solve the system:
$x + y + 8z = 0$ ... (1)
$-x + 2y + 7z = 0$ ... (2)
$3x + 0y + 7z = 0$ ... (3)

From (1) and (2): $3y + 15z = 0$, so $y = -5z$
From (1): $x = -y - 8z = 5z - 8z = -3z$

$z = 1$: $x = -3$, $y = -5$, $z = 1$ Eigenvector: $v_2 = [-3, -5, 1]$

For $\lambda = -2$:

$$A + 2I = \begin{bmatrix} 0 & 1 & 8 \\ -1 & 1 & 7 \\ 3 & 0 & 6 \end{bmatrix}$$

$0x + y + 8z = 0$ ... (1)
$-x + y + 7z = 0$ ... (2)
$3x + 0y + 6z = 0$ ... (3)

From (1): $y = -8z$
From (3): $3x + 6z = 0$, so $x = -2z$
$z = 1$: $x = -2$, $y = -8$, $z = 1$ Eigenvector: $v_3 = [-2, -8, 1]$

FINAL ANSWER:

Eigenvalues and their Eigenvectors:

$\lambda_1 = 6$, eigenvector = [2, -8, 3]
$\lambda_2 = -3$, eigenvector = [-3, -5, 1]
$\lambda_3 = -2$, eigenvector = [-2, -8, 1]

b. Trace of Matrix A:
For matrix A:

$$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix}$$

Trace(A) = diagonal elements added together
Trace(A) = (-2) + (-1) + (4) = 1

Verification with Eigenvalues**:** Trace(A) = Sum of all eigenvalues
From our previous calculation:

$\lambda_1 = 6$
$\lambda_2 = -3$
$\lambda_3 = -2$

Sum of eigenvalues = 6 + (-3) + (-2) = 1

STATISTICS

Question 1

Roll a six-sided die 5 times. What is the probability of rolling a six in all 5 rolls? If rolling the die 5 times is considered one trial, perform 500 trials. What is the probability of rolling a six in all 5 rolls in exactly one of these 500 trials? What about rolling a six in all 5 rolls in at least one of the 500 trials?

Probability of rolling a six on one roll
P(six) = 1/6

Probability of 5 sixes in 5 rolls

$P(5 \text{ sixes}) = P(\text{six}) \times P(\text{six}) \times P(\text{six}) \times P(\text{six}) \times P(\text{six})$
$P(5 \text{ sixes}) = (1/6)^5$

$(1/6)^5 = 1^5 / 6^5 = 1 / 7776$

$P(5 \text{ sixes in one trial}) = 1/7776 \approx 0.0001286$
-Exactly one success in 500 trials

This is a binomial probability problem where:
- $n = 500$ (number of trials)
- $p = 1/7776$ (probability of success in each trial)
- We want exactly $k = 1$ success

Using Binomial probability method
$P(X = k) = C(n,k) \times p^k \times (1-p)^{(n-k)}$
$P(X = 1) = C(500,1) \times (1/7776)^1 \times (1 - 1/7776)^{(500-1)}$
$= C(500,1) = 500!/(1! \times 499!) = 500$

$p^1 = 1/7776$

$(1-p)^{499} = (1 - 1/7776)^{499} = (7775/7776)^{499}$

Using the approximation $(1-x)^n \approx e^{(-nx)}$ for small x:
$(7775/7776)^{499} = (1 - 1/7776)^{499} \approx e^{(-499/7776)} \approx e^{(-0.0642)} \approx 0.9378$

$P(X = 1) = 500 \times (1/7776) \times 0.9378$
$P(X = 1) = 500 \times 0.9378 / 7776$
$P(X = 1) = 468.9 / 7776 \approx 0.0603$

Result: $P(\text{exactly 1 success in 500 trials}) \approx 0.0603$

-At least one success in 500 trials

Using the complement rule
$P(\text{at least 1}) = 1 - P(\text{none})$
$P(\text{at least 1}) = 1 - P(0 \text{ successes})$

For 0 successes in 500 trials:

P(X = 0) = (1-p)^500 = (7775/7776)^500

Using the approximation:

(7775/7776)^500 = (1 - 1/7776)^500 ≈ e^(-500/7776) ≈ e^(-0.0643) ≈ 0.9377

P(at least 1) = 1 - 0.9377 = 0.0623

P(at least 1 success in 500 trials) ≈ 0.0623

Final result:

1. P(5 sixes in one trial) = (1/6)^5 = 1/7776 ≈ 0.000129

2. P(exactly 1 success in 500 trials) = 500 × (1/7776) × (7775/7776)^499 ≈ 0.0603

3. P(at least 1 success in 500 trials) = 1 - (7775/7776)^500 ≈ 0.0623

Question 2

A study found that the average amount of coffee consumed by college students is 3 cups per day. Assuming this consumption follows a normal distribution with a standard deviation of 0.8 cups, what is the probability that a randomly selected college student drinks between 2.2 and 3.8 cups of coffee per day?

Given Information:

- Mean (μ) = 3 cups per day
- Standard deviation (σ) = 0.8 cups
- Want: P(2.2 < X < 3.8)

Convert to Standard Normal (Z-scores)

Formula: Z = (X - μ) / σ

For X = 2.2: Z_1 = (2.2 - 3) / 0.8 = -0.8 / 0.8 = -1

For X = 3.8: Z_2 = (3.8 - 3) / 0.8 = 0.8 / 0.8 = +1

We need: $P(-1 < Z < 1)$

Using the standard normal table or the empirical rule:

- $P(Z < 1) = 0.8413$
- $P(Z < -1) = 0.1587$

$P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$ $P(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826$

Answer:

The probability that a randomly selected college student drinks between 2.2 and 3.8 cups of coffee per day is 68.26% or 0.6826.

Question 3:

6 Digital Camera Prices The prices (in dollars) for a particular model of digital camera with 18.0 megapixels and a f/3.5−5.6 zoom lens are shown here for 10 randomly selected online retailers. Estimate the true mean price for this particular model with 95% confidence.

Given Data:
Prices: [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]
Sample size (n) = 10

Calculate Sample Mean ($\bar{x}$)
Sum = 999 + 1499 + 1997 + 398 + 591 + 498 + 798 + 849 + 449 + 348 = 9426
$\bar{x} = 9426 \div 10 = \$942.60$

Calculate Sample Standard Deviation (s), finding deviations from mean:

$(999 - 942.6)^2 = 56.4^2 = 3{,}181$
$(1499 - 942.6)^2 = 556.4^2 = 309{,}581$
$(1997 - 942.6)^2 = 1054.4^2 = 1{,}111{,}759$

$(398 - 942.6)^2 = (-544.6)^2 = 296{,}589$
$(591 - 942.6)^2 = (-351.6)^2 = 123{,}623$
$(498 - 942.6)^2 = (-444.6)^2 = 197{,}669$
$(798 - 942.6)^2 = (-144.6)^2 = 20{,}909$
$(849 - 942.6)^2 = (-93.6)^2 = 8{,}761$
$(449 - 942.6)^2 = (-493.6)^2 = 243{,}641$
$(348 - 942.6)^2 = (-594.6)^2 = 353{,}549$

Sum of squared deviations = 2,669,262
$s^2 = 2{,}669{,}262 \div (10-1) = 2{,}669{,}262 \div 9 = 296{,}584.7$
$s = \sqrt{296{,}584.7} = \$544.41$

Critical t-value
For 95% confidence with df = n - 1 = 9:
$t_{0.025,9} = 2.262$

Margin of Error = $t \times (s/\sqrt{n})$
$ME = 2.262 \times (544.41/\sqrt{10})$
$ME = 2.262 \times (544.41/3.162)$
$ME = 2.262 \times 172.15 = \$389.45$

95% Confidence Interval = $\bar{x} \pm ME$
Lower limit = 942.60 - 389.45 = \$553.15
Upper limit = 942.60 + 389.45 = \$1,332.05

Answer:
We are 95% confident that the true mean price for this digital camera model is between \$553.15 and \$1,332.05.

Question 4:

The average number of books read by a person in a year is reported to be 12. A 'reader' is defined as a person who reads at least one book in a year. A random sample of 50 readers from a local community library showed that the average number of books read per person was 13.4. The population standard deviation is 4.5 books. At the 0.01 level of significance, can it be concluded that this sample represents a significant difference from the national average?

Given Information:

Population mean (μ_0) = 12 books
Sample mean ($\bar{x}$) = 13.4 books

Sample size (n) = 50
Population standard deviation (σ) = 4.5 books
Significance level (α) = 0.01

Hypotheses
H_0: $\mu = 12$ (no difference from national average)
H_1: $\mu \neq 12$ (there is a difference) - Two-tailed test

Test Z-score
$Z = (\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
Standard Error: $SE = \sigma/\sqrt{n} = 4.5/\sqrt{50} = 4.5/7.071 = 0.636$
Z-statistic: $Z = (13.4 - 12) / 0.636 = 1.4 / 0.636 = 2.20$

Find Critical Values
For $\alpha = 0.01$ (two-tailed test):
Critical values = ± 2.576

Make Decision
Decision Rule: Reject H_0 if $|Z| > 2.576$
Our result: $|Z| = |2.20| = 2.20$
Since $2.20 < 2.576$, we fail to reject H_0

Conclusion:
At the 0.01 level of significance, we cannot conclude that this sample represents a significant difference from the national average of 12 books per year. The difference between 13.4 and 12 books is not statistically significant at the 1% level. The observed difference could reasonably be due to random sampling variation.

Question 5:

A statistics professor is used to having a variance in his class grades of no more than 100. He feels that his current group of students is different, and so he examines a random sample of midterm grades as shown. At $\alpha = 0.05$, can it be concluded that the variance in grades exceeds 100?
The grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]

Result:

Grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]
Sample size (n) = 15
Claimed variance (σ_0^2) = 100
Significance level (α) = 0.05

Hypotheses
H_0: $\sigma^2 \leq 100$ (variance does not exceed 100)
H_1: $\sigma^2 > 100$ (variance exceeds 100) - Right-tailed test

Calculate Sample Mean
Sum = 92.3 + 89.4 + 76.9 + 65.2 + 49.1 + 96.7 + 69.5 + 72.8 + 67.5 + 52.8 + 88.5 + 79.2 + 72.9 + 68.7 + 75.8 = 1117.3
$\bar{x} = 1117.3 \div 15 = 74.49$

Calculate Sample Variance (s^2) - Calculate squared deviations:

$(92.3 - 74.49)^2 = 318.0$
$(89.4 - 74.49)^2 = 222.3$
$(76.9 - 74.49)^2 = 5.8$
$(65.2 - 74.49)^2 = 86.3$
$(49.1 - 74.49)^2 = 643.8$
$(96.7 - 74.49)^2 = 494.3$
$(69.5 - 74.49)^2 = 24.9$
$(72.8 - 74.49)^2 = 2.9$
$(67.5 - 74.49)^2 = 48.8$
$(52.8 - 74.49)^2 = 469.5$
$(88.5 - 74.49)^2 = 196.3$
$(79.2 - 74.49)^2 = 22.2$
$(72.9 - 74.49)^2 = 2.5$
$(68.7 - 74.49)^2 = 33.5$
$(75.8 - 74.49)^2 = 1.7$

Sum of squared deviations = 2,572.8
$s^2 = 2{,}572.8 \div (15-1) = 2{,}572.8 \div 14 = 183.77$

Chi-Square Test Statistic
Formula: $\chi^2 = (n-1)s^2 / \sigma_0^2$
$\chi^2 = (15-1) \times 183.77 / 100 = 14 \times 183.77 / 100 = 25.73$

Finding Critical Value
Degrees of freedom = n - 1 = 14
For $\alpha = 0.05$ (right-tailed): $\chi^2_{0.05,14} = 23.685$

Decision Rule: Reject H_0 if $\chi^2 > 23.685$
Our result: $\chi^2 = 25.73$
Since $25.73 > 23.685$, Reject H_0

Conclusion:
At $\alpha = 0.05$, we can conclude that the variance in grades exceeds 100.