Decision Tree

$10 +$ $5-$

$H_I = -\left(\frac{10}{15}\log\frac{10}{15} + \frac{5}{15}\log\frac{5}{15}\right) = .918$

O: S   O   R
      6   4   5
      3+ 3-   4+ 0-   3+ 2-

$H_{A=O} = -\frac{6}{15}\left(\frac{3}{6}\log\frac{3}{6} + \frac{3}{6}\log\frac{3}{6}\right)$

$\phantom{H_{A=O}} -\frac{4}{15} * \left(\frac{4}{4}\log\frac{4}{4} + \frac{0}{4}\log\frac{0}{4}\right)$

$\phantom{H_{A=O}} -\frac{5}{15}\left(\frac{3}{5}\log\frac{3}{5} + \frac{2}{5}\log\frac{2}{5}\right) = .724$

T: H   M   C
    5   6   4
    3+ 2-   5+ 1-   2+ 2-

$H_{A=T} = -\frac{5}{15}\left(\frac{3}{5}\log\frac{3}{5} + \frac{2}{5}\log\frac{2}{5}\right)$

$\phantom{H_{A=T}} -\frac{6}{15}\left(\frac{5}{6}\log\frac{5}{6} + \frac{1}{6}\log\frac{1}{6}\right)$

$\phantom{H_{A=T}} -\frac{4}{15}\left(\frac{2}{4}\log\frac{2}{4} + \frac{2}{4}\log\frac{2}{4}\right) = .850$

H: H   N
    8   7
   4+4-   6+ 1-

$H_{A=H} = -\frac{8}{15}\left(\frac{4}{8}\log\frac{4}{8} + \frac{4}{8}\log\frac{4}{8}\right)$

$\phantom{H_{A=H}} -\frac{7}{15}\left(\frac{6}{7}\log\frac{6}{7} + \frac{1}{7}\log\frac{1}{7}\right) = .809$

W: W   S
    9   6
   7+2-   3+ 3-

$H_{A=W} = -\frac{9}{15}\left(\frac{7}{9}\log\frac{7}{9} + \frac{2}{9}\log\frac{2}{9}\right)$

$\phantom{H_{A=W}} -\frac{6}{15}\left(\frac{3}{6}\log\frac{3}{6} + \frac{3}{6}\log\frac{3}{6}\right) = .859$

Choose A that maximizes $\Delta H(A) = H_{initial} - H_A$

Then A = Outlook

Decision Tree AN ⟶ (Outlook)
      S / O \ R
   {1,2,8,9,11,15} {3,7,12,13} {4,5,6,10,14}

① $\{1,2,8,9,11,15\}$ ② $\{3,7,12,15\} \Rightarrow$ All + So return + ③ $\{4,5,6,10,4\}$

$3+ \quad 3-$ $4+$ $3+ \quad 2-$

$H_{I1} = -(\tfrac{3}{6}\log\tfrac{3}{6} + \tfrac{3}{6}\log\tfrac{3}{6}) = 1$

$H_{A=T} = 0$

$H_{A=H} = 0$

$H_{A=W} = -\tfrac{4}{6}(\tfrac{2}{4}\log\tfrac{2}{4} + \tfrac{2}{4}\log\tfrac{2}{4})$
$\qquad\qquad -\tfrac{2}{6}(\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{2}\log\tfrac{1}{2})$
$\qquad = 1$

T: H $\quad$ M $\quad$ C
$0+2- \quad 3+0- \quad 0+1-$

H: H $\quad$ N
$0+3- \quad 3+0-$

W: W $\quad$ S
$2+2- \quad 1+1-$

Minimize $\Delta H(A) = H_{I1} - H_A$ A can be either T or H.
Can pick Humidity bc it has more per category.

$H_{I3} = -(\tfrac{3}{5}\log\tfrac{3}{5} + \tfrac{2}{5}\log\tfrac{2}{5}) = 0.971$

$H_{A=T} = -\tfrac{1}{5}(0)$
$\qquad\quad -\tfrac{2}{6}(\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{2}\log\tfrac{1}{2})$
$\qquad\quad -\tfrac{2}{5}(\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{2}\log\tfrac{1}{2})$

T: H $\quad$ M $\quad$ C
$1+0- \quad 1+1- \quad 1+1-$

$H_{A=H} = -\tfrac{2}{5}(\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{2}\log\tfrac{1}{2})$
$\qquad\quad -\tfrac{3}{5}(\tfrac{2}{3}\log\tfrac{2}{3} + \tfrac{1}{3}\log\tfrac{1}{3})$

H: H $\quad$ N
$1+1- \quad 2+1-$

$H_{A=W} = 0$

W: W $\quad$ S
$3+0- \quad 0+2-$

Then $A = W$

So our Decision Tree is:

Outlook?
S → Humidity?   O → YES $\{3,7,12,13\}$ R → Windy?

Humidity?: H → NO $\{1,2,8\}$ N → YES $\{9,11,15\}$

Windy?: W → YES $\{4,5,10\}$ S → NO $\{6,4\}$

5b

(1,6)
(1,5)
(3,3)
(-1,3)
(0,2)
(0,1)
(0,0)

5b

7(a)

V_0:
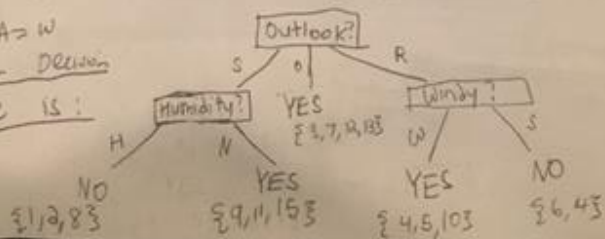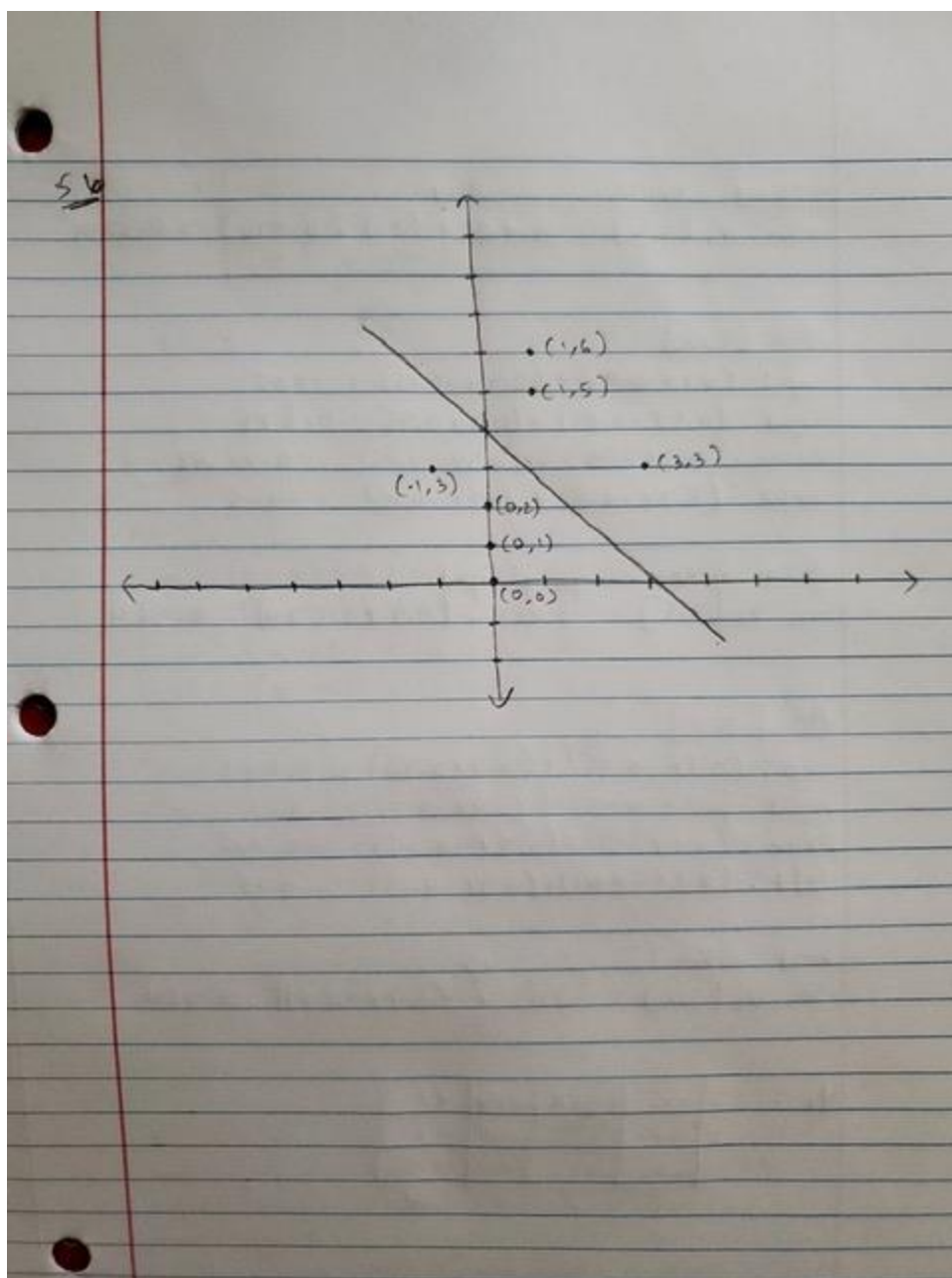
	1	2	3	4
3	0	0	0	+1
2	0		0	-1
1	0	0	0	

$\gamma = 0.9$

For V_1

At [3,3]

up: $(0.1 \times 0) + (0.9 \times 0) = 0$

right: $(0.1 \times 0) + (0.9 \times 1) = 0.9$

down: $(0.1 \times 1) + (0.1 \times 0) = 0.1$

left: $(0.1 \times 0) + (0.1 \times 0) = 0$

best action is going right

so, $V_1[3,3] = -0.05 + (0.9 \times 0.9) = 0.76$

for all other $s \in S$, $V_1(s) = -0.05$

V_1:

	1	2	3	4
3	-0.05	-0.05	0.76	+1
2	-0.05		-0.05	-1
1	-0.05	-0.05	-0.05	-0.05

For V_2

At [3,3]

up: $(0.1 \times -0.05) + (0.9 \times 0.76) = 0.679$

right: $(0.1 \times 0.76) + (0.9 \times 1) = 0.976$

down: $(0.1 \times 1) + (0.9 \times -0.05) = 0.055$

left: $(0.1 \times -0.05) + (0.9 \times -0.05) = -0.05$

7a1

best action is going right

So, $V_2 [3,3] = -0.05 + (0.9 \times 0.976) = 0.8284$

At $[2,3]$

up: $(0.1 \times -0.05) + (0.9 \times -0.05) = -0.05$
right: $(0.1 \times -0.05) + (0.9 \times 0.76) = 0.679$
down: $(0.1 \times 0.76) + (0.9 \times -0.05) = -0.00342$
left: $(0.1 \times -0.05) + (0.9 \times -0.05) = -0.05$

best action is going right
So, $V_2 [2,3] = -0.05 + (0.9 \times 0.679) = 0.5611$

At $[3,2]$

up: $(0.1 \times -0.05) + (0.9 \times 0.76) = 0.679$
right: $(0.1 \times -0.05) + (0.9 \times -0.05) = -0.05$
down: $(0.1 \times -1) + (0.9 \times -0.05) = -0.145$
left: $(0.1 \times -0.05) + (0.9 \times -0.05) = -0.05$

best action is going up
So, $V_2 [3,2] = -0.05 + (0.9 \times 0.679) = 0.5611$

V_2:

	1	2	3	4
3	-0.05	0.5611	0.8284	+1
2	-0.05		0.5611	-1
1	-0.05	-0.05	-0.05	-0.05

7a2

| 0.328 | 0.424 | 0.544 | 0.424 |

Iteration #8

0.557	0.688	0.835	+1
0.441	WALL	0.688	-1
0.336	0.429	0.545	0.429

Iteration #9

0.557	0.688	0.835	+1
0.441	WALL	0.688	-1
0.337	0.430	0.546	0.430

Iteration #10

0.557	0.688	0.835	+1
0.441	WALL	0.688	-1
0.338	0.431	0.546	0.431

The optimal policy is:

Right	Right	Right	+1
Up	WALL	Up	-1
Right	Right	Up	Left

7b