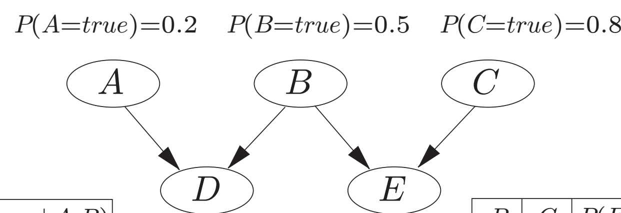
Guidelines - For this assignment, you only need to submit a PDF file with your NETID(s), e.g., NETID.pdf or NETID1_NETID2.pdf.

Problem 1. Marie is getting married tomorrow at an outdoor ceremony in NYC. In recent years, it rained about once every three days. The weatherman has predicted there will be no rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time. What is the probability that it will rain on the day of Marie's wedding according to Bayes' rule?

Problem 2. Consider the Monty Hall problem that we discussed in class. Instead of having three doors with one door having a price, now suppose there are four doors with two doors hiding goats behind them and one door having a price behind it. The other door has nothing behind it. To play the game, the guest will pick a door and the host will then reveal another door with a goat behind. The guest can then choose whether to stick to the original choice or switching. Should the guest switch? What is the probability of winning if the guest switches?

Problem 3. Consider the following Bayesian network, where variables A through E are all Boolean valued:



A	B	$P(D{=}true \mid A,B)$
F	F	0.9
F	T	0.6
T	F	0.5
T	T	0.1

B	C	$P(E{=}true \mid B,C)$
F	F	0.2
F	T	0.4
T	F	0.8
T	T	0.3

a) What is the probability that all five of these Boolean variables are true?

b) Compute $P(A, E \mid B)$ (yes, this is a lot of work).

c) Compute $P(A = true \mid B = false, C = false, D = false, E = false)$.

d) Compute $P(C = true, D = false \mid B = false)$.

e) Compute $P(B = true, C = false, D = true \mid A = false)$.

Problem 4. Using information gain as the criteria for deciding the node to split, learn the decision tree to depth 2 (i.e., the root should have grandchildren) using the data provided in Table 1. Show your work, i.e., how do you decide which feature to use for each split. For each split, clearly mark how the samples are grouped. For this purpose, you may use the "Day" as the identifier of the samples. As an example, if you start with samples for day 01, 02, and 03, after a split, you may have two children nodes, one with $\{01, 03\}$ and the other with $\{02\}$.

Problem 5. Consider building a linear classifier and then an SVM for the following two-class training data:

Table 1: Previous tennis playing decisions based on outlook, temperature, humidity, and wind

Day	Outlook	Temperature	Humidity	Wind	Played Tennis?
01	Sunny	Hot	High	Weak	No
02	Sunny	Hot	High	Strong	No
03	Overcast	Hot	High	Weak	Yes
04	Rain	Mild	High	Weak	Yes
05	Rain	Cool	Normal	Weak	Yes
06	Rain	Cool	Normal	Strong	No
07	Overcast	Cool	High	Strong	Yes
08	Sunny	Cool	High	Weak	No
09	Sunny	Mild	Normal	Weak	Yes
10	Rain	Hot	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No
15	Sunny	Mild	Normal	Weak	Yes

Positive class:

$$[-1\ 3]^T, [0\ 2]^T, [0\ 1]^T, [0\ 0]^T$$

Negative class:

$$[1\ 5]^T, [1\ 6]^T, [3\ 3]^T$$

a) Compute a linear classifier using $\alpha = 0.8$ and using the samples in the following order $[-1, 3], [1, 6], [0, 1], [1, 5], [0, 2], [0, 0], [3, 3]$. Show your work for the first seven iterations and also provide the final $w = (w_0 = b, w_1, w_2)$.

b) Plot the training points and, by inspection, draw a linear classifier that separates the data with maximum margin.

c) This linear SVM is parametrized by $h(x) = w^T x + b$. Estimate the parameters w and b for the classifier that you have drawn.

d) Suppose you observe an additional set of points, all from the positive class:

$$[-2\ 0]^T, [-2\ 1]^T, [-2\ 3]^T, [-1\ 0]^T, [-1\ 1]^T, [0\ 0]^T$$

What is the linear SVM (in terms of w and b) now?

Problem 6. What is the VC-dimension of (a) two half-planes (i.e., two linear separators)? (b) the inside of a triangle? Explain.

Problem 7. Carry out policy iteration over the MDP example covered in class with R given in Table 2 and $\gamma = 0.9$. For a state s, if $R(s) = \pm 1$, s is a terminal state. For the transition model, assume that the agent has 0.9 probability of going to the intended direction and 0.1 probability of moving to the left. For example, if the agent is at the lower left corner (coordinates $(1, 1)$) and intends to go right, then it will reach $(2,1)$ with 0.9 probability and $(1,2)$ with 0.1 probability. If a target cell is not reachable, then the corresponding probability goes back to the current cell. For example, if the agent is at $(3,3)$ and is trying to go up, then with 0.1 probability it goes to $(2,3)$ and with 0.9 probability it is stuck at $(3,3)$. For your answer you should provide:

a) The first two iterations of your computation.

b) The converged rewards and the extracted policy. For this problem, you need to provide last two iterations showing that the value changes are within 0.001 for all cells.

Table 2: Reward R for a 4×3 grid world

-0.05	-0.05	-0.05	+1
-0.05	OBS	-0.05	-1
-0.05	-0.05	-0.05	-0.05

As a suggestion, you should complete the first question manually to make sure you will be able to do so, for obvious reasons :). For solving the second, it is perhaps better to do it using a program, perhaps using Python or excel.

