Problem 1
Marie is getting married tomorrow at an outdoor ceremony in NYC. In recent years, it rained about once every three days. The weatherman has predicted there will be no rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 80 percent of the time. When it doesn't rain, he incorrectly forecasts rain 20 percent of the time. What is the probability that it will rain on the day of Marie's wedding according to Bayes' rule?

Solution
Let A = the event that it rains
Let B = the event that the weatherman predicts no rain

$$\begin{aligned} P(A|B) &= \frac{P(B|A) * P(A)}{P(B)} && (1)\\ &= \frac{\frac{1}{5} * \frac{1}{3}}{\frac{1}{3} * \frac{1}{5} + \frac{2}{3} * \frac{4}{5}} && (2)\\ &= \frac{\frac{1}{15}}{\frac{1}{15} + \frac{8}{15}} && (3)\\ &= \frac{\frac{1}{15}}{\frac{9}{15}} && (4)\\ &= \frac{1}{9} && (5) \end{aligned}$$

Problem 2
Consider the Monty Hall problem that we discussed in class. Instead of having three doors with one door having a price, now suppose there are four doors with two doors hiding goats behind them and one door having a price behind it. The other door has nothing behind it. To play the game, the guest will pick a door and the host will then reveal another door with a goat behind. The guest can then choose whether to stick to the original choice or switching. Should the guest switch? What is the probability of winning if the guest switches?

Solution Yes, the guest should still switch. Here's all the possible results for switching:
Pick prize, switch to remaining goat
Pick prize, switch to nothing
Pick goat number one, switch to nothing
Pick goat number one, switch to prize
Pick goat number two, switch to nothing
Pick goat number two, switch to prize
Pick nothing, switch to remaining goat
Pick nothing, switch to prize

Here's all the possible results without switching:
Pick prize
Pick goat number one
Pick goat number two
Pick nothing

As shown, switching will provide a $\frac{3}{8}$ chance to win as opposed to a $\frac{1}{4} = \frac{2}{8}$ chance at the start of the game.

Problem 3
Consider the following Bayesian network, where variables A through E are all Boolean valued.
a) What is the probability that all five of these Boolean variables are true?

b) Compute $P(A, E|B)$ (yes, this is a lot of work).
c) Compute $P(A = true|B = false, C = false, D = false, E = false)$.
d) Compute $P(C = true, D = false|B = false)$.
e) Compute $P(B = true, C = false, D = true|A = false)$.

Solution a) $0.2 * 0.5 * 0.8 * 0.1 * 0.3 = .0024$
b) $P(A) * P(E|B) = 0.2 * (.8 * .2 + .3 * .8) = 0.08$
c) $P(A = true|B = false, C = false, D = false, E = false) = P(A = true|D = false) = \frac{P(D|A)*P(A)}{P(D)} = \frac{(0.5*0.5+0.5*0.1)*0.2}{0.8*0.5*0.9+0.8*0.5*0.6+0.2*0.5*0.5+0.2*0.5*0.1} = \frac{1}{11}$
d) $P(C = true, D = false|B = false) = P(C = true)*P(D = false|B = false) = 0.8*(0.8*0.1+0.2*0.5) = 0.144$
e) $P(B = true, C = false, D = true|A = false) = 0.5 * 0.2 * 0.6 = .06$

Problem 4
Using information gain as the criteria for deciding the node to split, learn the decision tree to depth 2 (i.e., the root should have grandchildren) using the data provided in Table 1. Show your work, i.e., how do you decide which feature to use for each split. For each split, clearly mark how the samples are grouped. For this purpose, you may use the "Day" as the identifier of the samples. As an example, if you start with samples for day 01, 02, and 03, after a split, you may have two children nodes, one with 01, 03 and the other with 02

Solution See other pdf of paper and pencil work.

Problem 5
a) Compute a linear classifier using $\alpha = 0.8$ and using the samples in the following order [-1, 3], [1, 6], [0, 1], [1, 5], [0, 2], [0, 0], [3, 3]. Show your work for the first seven iterations and also provide the final $w = (w_0 = b, w_1, w_2)$. b) Plot the training points and, by inspection, draw a linear classifier that separates the data with maximum margin. c) This linear SVM is parametrized by $h(x) = w^T x + b$. Estimate the parameters w and b for the classifier that you have drawn. d) Suppose you observe an additional set of points, all from the positive class: $[-2, 0]^T, [-2, 1]^T, [-2, 3]^T, [-1, 0]^T, [-1, 1]^T, [0, 0]^T$. What is the linear SVM (in terms of w and b) now?

Solution See other pdf of paper and pencil work.
5c) w = [-0.25, 0.4], b = 4

Problem 6
What is the VC-dimension of (a) two half-planes (i.e., two linear separators)? (b) the inside of a triangle? Explain.

Solution a) The VC-dimension of two half-planes (two perceptrons) is 5. With 6 points arranged in a hexagon shape alternating between + and -, it is impossible to shatter this configuration. However, it can shatter any configuration of a set of 5 points.
b) The VC-dimension of a triangle is 7. With 8 points arranged in a octagon shape alternating between + and -, it is impossible to shatter this configuration with a triangle. With 7, if there are 3 + and 4 - alternating for example, it can surround the 3 - inside of it.

Problem 7
a) The first two iterations of your computation.
b) The converged rewards and the extracted policy. For this problem, you need to provide last two iterations showing that the value changes are within 0.001 for all cells.

Solution See other pdf of paper and pencil work.