Cpr E 281 HW06 SOLUTIONS

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Binary Number Representation and Arithmetic
Assigned Date: Seventh Week
Due Date: First class of 8th week

P1. (6 points) How many bits are required to represent each of the following sets of integers to represent unsigned integers in binary?
(a) The integers from 0 to 511 inclusively: $\log_2(511+1)=9$
(b) The integers from 0 to 2,047 inclusively: $\log_2(2047+1)=11$
(c) The integers from 0 to 1,234,567 inclusively: $\lceil\log_2(1234567+1)\rceil=21$

P2. (6 points) How large a value can be represented by each of the unsigned binary quantities?
(a) A 8-bit quantity: $2^8-1=255$
(b) A 12-bit quantity: $2^{12}-1=4095$
(c) A 20-bit quantity: $2^{20}-1=1{,}048{,}576$

P3. (8 points) Convert each of the following binary numbers into decimal. Assume these quantities represent unsigned integers.
(a) 1110; (b) 10110; (c) 1001111; (d) 100000000

Answer: (a)=14; (b)=22; (c)=79; d)=2^8=256

P4. (8 points) Convert each of the following decimal numbers into binary.
(a) 10; (b) 15; (c) 201; (d) 1023
Answer:
(a): 1010
(b): 1111
(c): 11001001
(d): 1111111111

P5. (4 points) Suppose a jogger wants to use her ten fingers to count laps as she circles a track. Each finger can be in two different states to represent a binary digit. How many laps can she conveniently count? Briefly justify your answer.
Answer: She can use binary representation and count up to 2^5-1=31 Laps.

P6. (6 points) How many trinary digits are required to represent numbers in the following ranges?
(a) The integers from 0 to 255 inclusively
Answer: $\lceil\log_3(255+1)\rceil$=6
(b) The integers from 0 to 4,095 inclusively
Answer: $\lceil\log_3(4095+1)\rceil$=8
(c) The integers from 0 to 1,234,567 inclusively
Answer: $\lceil\log_3(1234567+1)\rceil$=13

P7. (6 points) Convert each of the following binary numbers into hexadecimal.
(a) 1011; (b) 11111; (c) 1011101

(a): 1011_2=B_{16};
(b): 11111_2=1_1111=$1F_{16}$
(c): 1011101_2=101_1101 =$5D_{16}$

P8 (6 points) Convert each of the following hexadecimal numbers into binary.
(a) D4; (b) 8E3; (c) ABCDEF
(a) D4 = b1101_0100_2;
(b) 8E3 = 1000_1110_0011_2
(c) ABCDEF =1010_1011_1100_1101_1110_1111_2

P9. (6 points) Convert each of the following decimal numbers into hexadecimal.
(a) 123; (b) 210; (c) 1023
Answer:
(a) 123=16*7+11=$7B_{16}$
(b) 210=16*13+2=$D2_{16}$
(c) 1023=11_1111_1111_2=$3FF_{16}$

P10. (6 points) Convert each of the following hexadecimal numbers into decimal.
(a) 5E; (b) B2; (c) 3D8
Answer:
(a) 5E = 16*5+14=94
(b) B2 = 16*11+2=178
(c) 3D8= 256*3+16*13+8=984

P11. (12 points) An expedition to Mars found the ruins of a civilization. The explorers were able to translate the mathematical equations:
$5x^2$ - 50x + 125 = 0
with the solutions: x = 5 and x = 8.
The x = 5 solution seemed okay, but x = 8 was puzzling. The problem should be because Martians were using a non-decimal number system. Therefore, “50” is not fifty, but “50” in base b (50_b=5×b+0×1=5b). The explorers reflected on the way in which Earth's number system developed. How many fingers would you say the Martians had? *Hint:* What should be the value of the base b such that both 5 and 8 are solutions of the equation?

Answer: Suppose that the Martians had b fingers, then they would represent number radix-b number system, then the equation should be rewrite to represent number in base b:

$$5x^2 - (5 \cdot b + 0)x + (1 \cdot b^2 + 2b + 5) = 0$$

Since x = 5 is a solution, we can substitute x=5 and solve for b. We obtain two solution b=13 and b = 10 in decimal.
Since x = 8 is also a solution, we can substitute x=8 and solve for b again. We obtain two solution b=13 and b = 25 in decimal.
Thus b =13 is a possible explanation for the solution.

P12. (6 points) What is the value represented by the bit string 110101 if:
(a) it is in sign-and-magnitude representation?
110101 = 1_10101=(-)10101=-21
(b) it is in 1's complement representation?
110101 =-(001010)=-10
(c) it is in 2's complement representation?
Let x=110101. Then –x=(001010)+1=11. Therefore, x=-11.

P13. (6 points)

a) 110000_2
b) 001101_2
c) 111101_2
d) 110110_2
e) 011010_2
f) 100001_2

P14. (6 points) Answer the following:

(a) In 2's complement we have only one way to represent 0. This simplifies our representation scheme.
(b) To write the number 710 using 2's complement representation you need only to convert 710 to binary.
(c) To write the number -710 using 2's complement representation you need to convert 710 to binary, complement this number, and then add 1. Remember that 2's complement exceeds 1's complement by 1.

P15. (8 points) Assume the following numbers are represented as 4-bit words in 2's complement form. Perform the following operations and identify, in each case, whether or not an overflow occurs:

	1111		1000		1111		0000
(a)	1111	(b)	1000	(c)	1111	(d)	1000
+	0001	+	1110	+	1111	+	0011
	0000		10110		1110		1011

a) This is (-1)+(1) = 0, no overflow. Also, notice last two carry bits are the same.
b) This is (-8)+(-2)=-10, and there is overflow. Also, notice last two carry bits are different.
c) The subtraction is converted to addition by finding 2's complement of 2^{nd} number. It is (-1) – (1) = -2. No overflow. Also, notice last two carry bits are the same.
d) The subtraction is converted to addition by finding 2's complement of 2^{nd} number. It is (-8) – (-3) = -5 and therefore there is no overflow. Also, notice last two carry bits are the same.