Jay Patel
Exam 2A

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Q1] $\{1, e^{Ax}, e^{Bx}\}$

Let $f_1 = 1$, $f_2 = e^{Ax}$, $f_3 = e^{Bx}$

$$W(f_1, f_2, f_3)(x) = \begin{vmatrix} 1 & e^{Ax} & e^{Bx} \\ 0 & Ae^{Ax} & Be^{Bx} \\ 0 & A^2e^{Ax} & B^2e^{Bx} \end{vmatrix}$$

$$= 1[AB^2 e^{(A+B)x} - A^2Be^{(A+B)x}] - e^{Ax}(0) + e^{Bx}(0)$$

fA $= e^{(A+B)x} AB[B-A]$

fA * So, $W=0$ $A=B$ only, function f_1, f_2, f_3 are not linear independent

fA * $W \neq 0$ $A \neq B$ function f_1, f_2, f_3 are linear independent, this follows above work.

Q2] $y'' - iy' + 12y = 0$

So, $-i \dfrac{dy(x)}{dx} + \dfrac{d^2y(x)}{dx^2} + 12y(x) = 0$

$$\frac{d^2}{dx^2}(e^{\lambda x}) - i\frac{d}{dx}(e^{\lambda x}) + 12e^{\lambda x} = 0$$

Substituting $\dfrac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$ and

$\dfrac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} - i\lambda e^{\lambda x} + 12e^{\lambda x} = 0$$

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factor out $e^{\lambda x}$:

$(\lambda^2 - i\lambda + 12)e^{\lambda x} = 0$

Since $e^{\lambda x} \neq 0$

$\lambda^2 - i\lambda + 12 = 0$

So we get

$\lambda^2 + 3i\lambda - 4i\lambda + 12 = 0$

$\lambda(\lambda + 3i) - 4(i\lambda + 3) = 0$

So, $(\lambda + 3i)(\lambda - 4i) = 0$

fA So $\lambda = -3i$ $\lambda = 4i$

To show that $y = e^{4x}$ is a solution

$y' = 4ie^{4ix}$

$y'' = (4i)(4i)e^{4ix} = -16e^{4ix}$

Therefore,

$y'' - iy' + 12y = -16e^{4ix} - i(4i)e^{4ix} + 12e^{4ix}$

fA $= -16e^{4ix} + 4e^{4ix} + 12e^{4ix} = 0$

So,

fA $y = e^{4ix}$ is a solution of differential equation

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Q3] $y'' - y' - 2y = e^{3x} \sin 2x$

So,

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = -1$ or $m = 2$

$y_h(x) = c_1 e^{-x} + c_2 e^{2x}$

$y_p = Ae^{(3x)} \sin(2x) + Be^{(3x)} \cos(2x)$

$y' = 2Ae^{(3x)} \cos(2x) + 3Be^{(3x)} \cos(2x) + 3Ae^{(3x)} \sin(2x) - 2Be^{(3x)} \sin(2x)$

$y'' = 12Ae^{(3x)} \cos(2x) + 5Be^{(3x)} \cos(2x) + 5Ae^{(3x)} \sin(2x) - 12Be^{3x} \sin(2x)$

$y'' - y' - 2y = e^{3x} \sin 2x$

$12Ae^{(3x)} \cos(2x) + 5Be^{(3x)} \cos(2x) + 5Ae^{(3x)} \sin(2x) - 12Be^{(3x)} \sin(2x) - (2Ae^{3x} \cos(2x) + 3B^{(3x)} \cos(2x) + 3Ae^{(3x)} \sin(2x) - 2Be^{(3x)} \sin(2x)) - 2(Ae^{(3x)} \sin(2x) + Be^{(3x)} \cos(2x)) = e^{(3x)} \sin(2x)$

$-10e^{(3x)} (A\cos(2x) + B\sin(2x)) = e^{3x} \sin(2x)$

$-10e^{(3x)} A\cos(2x) = 0$

$-10A = 0 \Rightarrow A = 0$

$-10e^{(3x)} B\cos(2x) = e^{3x} (\sin 2x)$

$-10B = 1 \qquad B = -1/10$

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$y_p = Ae^{(3x)}\sin(2x) + Be^{(3x)}\cos(2x)$

$y_p = 0e^{(3x)}\sin(2x) - (1/10)e^{3x}\cos(2x)$

$y_p = -(1/10)e^{3x}\cos(2x)$

So,

fA = $y(x) = c_1 e^{(-x)} + c_2 e^{(2x)} - (1/10)e^{(3x)}\cos(2x)$

Q4] $y'' - 2y' - 8y = 3e^{-2x}$

So,

$m^2 - 2m - 8 = 0$

$m^2 - 4m + 2m - 8 = 0$

$m(m-4) + 2(m-4) = 0$

$(m+2)(m-4) = 0$

$m = -2$ or $m = 4$

$y_c = c_1 e^{4x} + c_2 e^{-2x}$

particular,

$y_1 = e^{4x}$, $y_2 = e^{-2x}$, $g(x) = 3e^{-2x}$

$$W(y_1, y_2) = \begin{vmatrix} e^{4x} & e^{-2x} \\ 4e^{4x} & -2e^{-2x} \end{vmatrix} = -2e^{2x} - 4e^{2x} = -6e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ 3e^{-2x} & -2e^{-2x} \end{vmatrix} = -3e^{-4x}$$

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$$W_2 = \begin{vmatrix} e^{4x} & 0 \\ 4e^{4x} & 3e^{-2x} \end{vmatrix} = 3e^{2x}$$

$$u_1 = \int \frac{W_1}{W}\,dx = \int \frac{-3e^{-4x}}{-6x^{2x}}\,dx = \frac{1}{2}\int e^{-6x}\,dx$$

$$= \frac{-1}{12}e^{-6x}$$

$$u_2 = \int \frac{W_2}{W}\,dx = \int \frac{3e^{2x}}{-6x^{2x}}\,dx = \frac{-1}{2}\int dx = \frac{-1}{2}x$$

$$y_p = u_1y_1 + u_2y_2$$

$$y_p = \left(\frac{-1}{12}e^{-6x}\right)(e^{4x}) + \left(\frac{-1}{2}x\right)(e^{-2x})$$

$$y_p = \underbrace{\boxed{\frac{-1}{12}e^{-2x}}}_{y_c} - \frac{1}{2}xe^{-2x}$$

So, $\frac{-1}{2}xe^{-2x}$.

So,

$$y = y_c + y_p$$

FA $\Rightarrow$ $y = c_1e^{4x} + c_2e^{-2x} - \frac{1}{2}xe^{-2x}$