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HW4

CS 230

Q11. Given f: x → Y and $B \subseteq Y$

F^{-1} (B) = {x ∈ $X \mid f(X) \in B\}$

a. Suppose $B_1 \subseteq B_2$

Let, x ∈ $f^{-1}(B_1) \leftrightarrow f(x) \in B_1 \subseteq B_2$

$\rightarrow x \in f^{-1}(B_2) \leftrightarrow x \in f^{-1}(B_2)$

$Hence, f^{-1}(B_1) \subseteq f^{-1}(B_2)$

b. Let x ∈ $f^{-1}(B_1 \cup B_2)$

$\leftrightarrow f(x) \in (B_1 \cup B_2) \rightarrow (f(x) \in B_1)\ or\ (f(x) \in B_2)$

$\rightarrow \{x \in f^{-1}(B_1)\}\ or\ \{x \in f^{-1}(B_2)\}$

$\rightarrow x \in f^{-1}(B_1) \cup f^{-1}(B_2)$

c. Let x ∈ $f^{-1}(B_1 \cap B_2)$

$\leftrightarrow f(x) \in B_1 \cap B_2 \rightarrow (f(x) \in B_1)\ and\ (f(x) \in B_2)$

$\rightarrow \left(x \in f^{-1}(B_1)\right) and \left(x \in f^{-1}(B_2)\right)$

$\rightarrow x \in f^{-1}(B_1) \cap f^{-1}(B_2)$

$Hence, (B_1 \cap B_2) \subseteq f^{-1}(B_1) \cap f^{-1}(B_2)$

d. Let x $\in f^{-1}(B_1 - B_2) \leftrightarrow f(x) \in B_1 - B_2) \leftrightarrow (f(x) \in B_1)$ and $(f(x) \notin B_2)$

$\rightarrow x \in f^{1}(B_1)$ and $x \notin f^{-1}(B_2)$

$\rightarrow x \in (f^{-1}(B_1) - f^{-1}(B_2)$

Hence, $f^{-1}(B_1 - B_2) \subseteq f^{-1}(B_1) - f^{-1}(B_2)$

Q14. $\forall A, B \subseteq X \, \psi(A \cup B) = \psi(A) \cup (B)$

if (A $\subseteq B)$ $then$ $A \cup B = B,$ so we get $\psi(B) = \psi(A) \cup \psi(B) \rightarrow \psi(A) \subseteq \psi(B)$

So, A $\subseteq B \rightarrow \psi(A) \subseteq \psi(B) \ldots\ldots(a)$

Therefore, A $\cap B \subseteq \psi(A)$ and $\psi(A \cap B) \subseteq \psi(B)$

$\rightarrow \psi(A \cap B) \subseteq \psi(A)$ and $\psi(A \cap B) \subseteq \psi(B)$

$\rightarrow \psi(A \cap B) \subseteq \psi(A) \cap \psi(B)$

Q17. We know that, f: A $\rightarrow B$ is one to one $function$

Assume that, $g_1 : S \rightarrow A$ and $g_2 : S \rightarrow A$ are $such$ $that$ $fog_1 = fog_2$

Hence, $fog_1 : S \rightarrow B$

$fog_2 : S \rightarrow B$ are $functional$

Given that $fog_1 = fog_2$ $then$

$(fog_1)(x) = (fog_2)(x) \forall x \in S$

$\rightarrow f(g_1(x)) = f(g_2(x))$

$\rightarrow g_1(x) = g_2(x) \forall x \in S$

Which is one to one

$\rightarrow g_1 = g_2$