Jay Patel

HW3

CS 230

Q1.

F: R → R

F9x) = 1+ $e^{2\sin^2 x+3}$

So, g (x) = 1 + e^x $\forall x \in R$

h (x) = $2\, sin^2 x + 3$ $\forall x \in R$

f (x) = g∘ $h\ (x) = g\big(h(x)\big)$

f(x) = g∘ $h(x)$

Q2. Let $a_1 \in A'$

a exist in A such that k (a) = a_1

$k^2 = k$

k (k (a)) = k(a)

k (a) = a_1

k(a_1) = a_1

a_1 *exist for every in* A'

k (a_1) = a_1

k k(a_1) = $k\ a_1$

$k^2 = k$

k is deompotent

Jay Patel

HW3

CS 230

Q3.

a. Using the claim from the notes 3.2

Let f: x $\rightarrow y$

f is 1-1 because $A_1\ A_2\ \subseteq x\ [f\big(A_1\ \cap\ A_2) = f(A_1)\ \cap f(A_{2)})\big]$

Let x $\in\ A_1\ \cap\ A_2$ Then x $\in A_1\ and\ x\ \in\ A_2\ where\ f(x) \in f(A_1) and\ f(A_2)$

f(x) $\in f\,(A_1) \cap (A_2)$

So let, y $\in f\,(A_1) \cap f(A_2)$

Since, y $\in f(A_1) and\ y \in f(A_2)$

So, x $\in A_1\ \cap\ A_2\ with\ f\ (x) =\ y\ and\ y\ \in f(A_1 \cap A_2)$

Therefore, f$(A_1)\ \cap f(A_2) \subseteq f(A_1\ \cap A_2)$

Hence, x[f$(A_1\ \cap\ A_2) = f(A_1)\ \cap f\ (A_2)$]

b. Using the claim from the notes 3.3

$\forall\ A_1, A_2\ \subseteq x\ [f(A_1 -\ A_2) = f(A_1) - f\,(A_2)]$

Let x $\in A_1 -\ A_2\ \ hence, x\ \in\ A_1\ and\ x\ \notin A_2\ \exists\ y = f(x)$

So, f(x) $\in f\,(A_1), f(x) \notin$ f$(A_2),\ f(x) \in f(A_1)$ – f(A_2)

Where, $\exists x\ \in A_1\ such\ that\ x\ \in\ A_1 and\ x\ \notin A_2$

x $\in (A_1 -\ A_2)$

f$(A_1) - f\,(A_2) \subseteq f(A_1 - A_2)$

f$(A_1) - f\,(A_2) =\ f(A_1) -\ f(A_2)$

Jay Patel

HW3

CS 230

f$(A_1 - A_2) = f(A_1) - f(A_2)$

c. f is 1-1 $\rightarrow \forall A_1, A_2 \subseteq x[f(A_1 \oplus A_2) = f(A_1) \oplus f(A_2)]$
   so, x $\in A_1 \oplus A_2$ $Then, x \in (A_1 \cap A_2) \cup (A_2 \cap A_1)$
   x $\in (A_1 \cap A_2) or\ x \in A_2 \cap A_1$
   x $\in A_1, x \in A_2\ or\ x \in A_2\ and\ x \in A_1$
   f(x) $\in A_1, f(x) \in A_2\ or\ f(x) \in A_2\ and\ f(x) A_1$
   f(x) $\in f(A_1 \oplus A_2)\ since\ we\ now\ know\ that\ f\ is\ 1-1$
   f$(A_1 \oplus A_2) \subseteq f(A_1) \oplus f(A_2)$
   f$(A_1) \oplus f(A_2) \subseteq f(A_1 \oplus A_2)$
   f$(A_1 \oplus A_2) = f(A_1) \oplus f(A_2)$