Jay Patel

Hw #7

CS 230

Q7.

a. We can use repeatedly pascal identity

$$\binom{n+1}{m+1} = \binom{n}{m} + \binom{n}{m+1}$$

$$= \binom{n}{m} + \binom{n-1}{m+1}$$

$$= \binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \cdots + \binom{m+1}{m} + \binom{m}{n} + \binom{m-1}{m+1}$$

$$=\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \cdots + \binom{m+1}{m} + \binom{m}{n} + 0 + 0 + \cdots + \sum_{k=m}^{n}\binom{k}{m}$$

b. Let's say that there is a set P and has n elements in it and on the other side Q has n + 1 elements. We can choose k elements of Q to form a set B, if and only if B has elements of q, and then we can say that m + 1 elements come from P. So, $\binom{n}{m+1}$ ways to form B. and if B does not contain any elements of q and elements come from p then, $\binom{n}{m}$ these are the ways in order to form B. So in other words if we compare both the sides of the equation, we need to see and find number of ways to select m + 1/ n+ 1 cars.
By the sum rule we can say that, $\binom{n}{m+1} + \binom{n}{m}$
Hence proved.
$$\binom{n+1}{m+1} = \binom{n}{m+1} + \binom{n}{m} \quad \blacksquare$$

Q12.

From multinomial theorem we know that,

If $\sum_{i=1}^{k} b_i = n^1$ then, multinomial coefficient is

$$\frac{n^1!}{b_1! * b_2! * \ldots\ldots b_k!}$$

a. Let's assume, k = n, $b_1 = b_2 = \cdots = b_i = 2$ and $n^1 = (2n)$

$$\sum_{i=1}^{n} 2 = 2 + 2 + 2 + \cdots + 2 = 2n$$

Multinomial coefficient is,

$$\frac{(2n)!}{2! * 2! * 2! \ldots * 2!}$$

Therefore, 2! = 2

$= \frac{(2n)!}{2*2*2*2*2} = \frac{(2n)!}{2^n}$ → Hence using the fact that multinomial coefficient are non- negative integers we can say that, $2^n | (2n)!$

Also,

$\frac{(2n)!}{2^n} = \{n * (n-1) * (n-2) * \ldots..1\} *$
$\{(2n-1) * (2n-3) * \ldots * 1\}$

If the number are consecutive numbers and if
n $\geq$ $2, at\ least\ one\ of\ the\ number\ would\ be\ even.$

b. So, $b_1 = b_2 = b_3 = b_n = n, k = n\ and\ n^1 = n^2$

$$\sum_{i=1}^{n} n = n + n + \cdots + n = n^2$$

multinomial coefficient is $\frac{(n^2)!}{n! * n! * n! \ldots . n!} = \frac{(n^2)!}{(n)!^n}$

since we know that multinomial are integers for non-negative integers.

So, $a^m\ divides\ k!$

⇨ $a^m * a\ divides\ (k!)!$

So,

$(n!)^n\ divides\ n^2!$
$(n!)^n * n!\ divides\ (n^2!)!$
$(n!)^{n+1}\ divides\ (n^2!)!$

[by claiming n < $n^2! - 1 = n! < (n^2! - 1)! = n! | (n^2! - 1)!$]
Hence,
$(n!)^n | n^2! * n! | (n^2 - 1)!$
$= (n!)^{n+1} | n^2! (n^2! - 1)!$
$= (n!)^{n+1} | (n^2!)!$

Q14.

Given that, x + y + z + u = 10 $---\rightarrow 1$
Such that, $x_1 y_1 z_1 \; u \geq 0$
Hence, no. of variables, n = 4 and r = 10

So, no. of solutions:
$= \binom{r+n-1}{r} = r + n - 1_r$
$= \binom{10+4-1}{10} = \binom{13}{10}$
$= \frac{\lfloor 13}{\lfloor 10 \lfloor 13-10} = \frac{\lfloor 13}{\lfloor 10, \lfloor 3}$
$= \frac{13 * 12 * 11 * \lfloor 10}{\lfloor 10 * 3 * 2}$
hence we choose 3 places out of 13 $\binom{13}{3}$ *to make this partition.*