Jay Patel

Q1] By second equation of motion

$x = ut + \frac{1}{2}at^2$

$x = 1500m \Rightarrow 1.5km = 1500m$

$u = 25m/sec$

$t = 20sec$

$a = ?$

$1500 = 25 * 20 + 400 * a/2$

$1000 = 400 * a/2$

$a = 5 m/s^2$

To find velocity,

$\frac{dx}{dt} = u(1) + \frac{1}{2} a (2t)$

$u = 25 m/s$

$a = 5 m/s^2$

$t = 20 sec$

$\frac{dx}{dt} = 25 + 5(20)$

$= 100 + 25$

FA $\boxed{= 125 m/s}$

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Q2] $\tan x \frac{dy}{dx} = (1+y)^2$

$$\frac{dy}{dx} = \frac{(1+y)^2}{\tan(x)}$$

$$\Rightarrow \frac{dy}{(1+y)^2} = \frac{dx}{\tan(x)}$$

$$\frac{dy}{(1+y)^2} = \cot(x)\,dx.$$

$$\frac{dy}{(1+y)^2} = \frac{\cos(x)}{\sin(x)}\,dx$$

By integrating on both sides.

$$\int \frac{dy}{(1+y)^2} = \int \frac{\cos(x)}{\sin(x)}\,dx$$

let $\sin(x) = t$

$\Rightarrow \cos(x)\,dx = dt$

$$\int \frac{dy}{(1+y)^2} = \int \frac{1}{t}\,dt$$

$$\frac{-1}{1+y} = \ln(t) + c$$

FA $\Rightarrow \frac{-1}{1+y} = \ln(\sin(x)) + c.$

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Q3] $(3x^2y + 4y^2 + 2)dx + (x^3 + 8yx)\,dy = 0$

$\Longrightarrow$ $Mdx + N\,dy = 0$

$M = 3x^2y + 4y^2 + 2$

$N = x^3 + 8yx$

$$\frac{\partial M}{\partial y} = 3x^2 + 8y$$

$$\frac{\partial N}{\partial x} = 3x^2 + 8y$$

So, $\frac{\partial M}{\partial y} = \frac{\delta N}{\partial x}$, the equation is exact.

Let the solution be $u(xy) = 0$.

$$\frac{\partial u}{\partial x} = M = 3x^2y + 4y^2 + 2$$

$u(x, y) = x^3y + 4y^2x + 2x + g(x)$

$g(y)$ is a function of y alone

So, $\frac{\partial u}{\partial y} = x^3 + 8yx + g'(x)$

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But $\frac{\partial u}{\partial y} = N = x^3 + 8yx$

$g'(x) = 0$

$g'(y) = c$, "c" is constant

$\Rightarrow u(x/y) = x^3y + 4y^2x + 2x + c$

Solution $u(x,y) = 0$

FA $\Rightarrow x^3y + 4y^2x + 2x + c = 0$

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Q4] $Cy' - Ae^{-2bx} y = Be^{-2bx}$

$C\frac{dy}{dx} + (-Ae^{-2bx})y = Be^{-2bx}$

$p(x) = e^{\int P(x)\,dx}$.

$p(x) = -Ae^{-2bx}$

$p(x) = e^{-\int Ae^{-2bx}}$

$= e^{\frac{Ae^{-2bx}}{+2b}}$

$= \boxed{e^{\frac{Ae^{-2bx}}{2b}}}$

$Ce^{Ae^{-2bx}/2b}\frac{dy}{dx} + (-Ae^{-2bx})(e^{Ae^{-2bx}/2b})y$

$= Be^{-2bx} e^{A^{-2bx}/2b}$

$$\frac{Ce^{Ae^{-2bx}/2b} - Be^{-2bx}e^{Ae^{-2x}/2b\,b}}{CAe^{-2xb}\cdot e^{Ae^{-2bx}/2b}} = -y$$

Integrating both Sides

$$\int \frac{C e^{Ae^{-2bx}/2b} - Be^{-\frac{4b^2x+Ae^{-2xb}}{2b}}}{Ae^{-\frac{4b^2x+Ae^{-2bx}}{2b}}}$$

$dx = \int -y\cdot dy$

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$$\int\left(\frac{C}{A}e^{Ae^{-2bx}/2bx} - C\,\frac{-4b^2x + Ae^{-2/3x}}{2b} - \frac{Be^0}{A}\right)$$

$$dx = \int -y\,dy$$

$$\int\left(\frac{C}{A}\,\frac{e^{\overset{2}{\cancel{4b^2}}x}}{2b} - \frac{B}{A}\right)dx = \int -y\,dy$$

$$\frac{C}{A}\,\frac{e^{2bx}}{2b} - \frac{B}{A}x = \frac{-y^2}{2} + C$$

$$\frac{2Ce^{2bx}}{2b} - 2Bx = A(-y)^2 + C$$

FA $\Rightarrow$ $Ce^{2bx} - 2Bx = -Ay^2 + C$

$p(x) = Ae^{-2bx/C}$

$q(x) = Be^{-2bx/C}$