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CS 331
HW11
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Q1.
Remember that $EQ_{CFG}$ is co-Turing recognizable language if and only if its complement $\overline{EQ_{CFG}}$ is a Turing recognizable language.
Now, $\overline{EQ_{CFG}}$ = A U B, where
A = {w|w does not have any for <G1, G2> for some CFGs G1 and G1},
B = {<G1, G2> |G1 and G2 are CFGs and L (G1) $\neq$ L (G2)}

Now, $EQ_{CFG}$ is realized by the following TM M:
M = "on input <G1, G2> where G1 and G2 are CFG

1. Examine if G1, G2 are valid CFG. If at least 1 does not, accept
2. Transform G1, G2 into corresponding CFG G1, G2 both in CNF
3. Replicate the below step 4 for j = 1,2,3..
4. Examine if G1, and G2 produces $s_j$ if precisely one of them does accept

Hence, it is proved that $EQ_{CFG}$ is co-Turing recognizable language.

Q2.

To proof that T is undecidable use the reduction ATM $\leq m\, T$. So, a decider for T produces a decider for ATm, but it is already proved that ATM is undeciadable. So assume that T is decidable and construct a decider for ATM.
Assume R as the decider for T.
Consider a machine as follows:
M1 on input w1:

1. If input is not in the set: {01,10}, reject
2. If input is 01, accept
3. If input is 10, operate machine M on input w and accept if M accepts
4. If M halts and reject then reject.

The following machine D decides ATM:
D on input <M,w>:”

So, Run machine R on input <M> and accepts if R accepts, else rejects if R rejects
L (m1) = {01,10} if M accepts w and {01} otherwise. Machine R decides T,, so it will know when to accept and when to reject. <M1>
Using this faculty, machine D decides ATM, which is known to be undecidable.
The mapping reduction lies in this proof

Hence there is no such machine R and the language T is undecidable.

Q3.
The given problem is defined as the following language:
$USELESS_{TM}$ = {<T, q> |q is a useless state in TM T}

Show that $USELESS_{TM}$ is undecidable reducing $E_{TM}$ to $USELESS_{TM}$, where
$E_{TM}$ = {<T1> | T1 is a TM and L (T1) = ∅}
Using theorem in 5.2 we know that $E_{TM}$ is undecidable
So S = "on input <T>, where M is a TM:

1. Run TM R on input <T,$q_{accept}$ >, where $q_{accept}$ is the accept state of T.
2. If R accepts, accept. If it rejects, then reject

But, since it is known that $E_{TM}$ is undecidable and there cannot be a TM that decides $USELESS_{TM}$
Therefore, it is proved that the given problem is undecidable.

Q4.
Proof: By using contradiction, assume that the busy beaver function BB is computable. If function BB is computable then there exists a TM F for computing it. Now without loss of generality a TM F on input $1^n$ and the TM F halts with $1^{BB(n)}$ for each value on n.
Now, build a TM M, and this TM halts when it will start from a blank tape based on F.

Construction of TM:
Here M is a TM, which halts when starts from blank tape.

Steps:
1. Now the TM M writes n number of 1s on the tape.
2. TM M doubles the number of 1s on the tape.
3. Now, M executes the TM F on the input $1^{2n}$

Therefore, TM M halts from the blank tape. IN order to implement the TM M at most n number of states are required for the step 1 of the TM and c numbers of state are required for step 2 and 3, c is a constant.

Conclusion: By definition BB (n+c) is the max number of 1s on which TM with states (n+c) will halt is at least the number of 1s on which the TM M halts. Which means that BB (n+c) ≥ BB (2n) and this relationship will hold for all the values on n. Hence there is a contradiction since it is stricktly increasing function so BB (n+c) < BB (2n), therefor it is not a computable function.

Q5. [Extra Credit]

Decidability:
Assume that B = 0*1* thus it is required to prove that A is decidable iff A $\leq_m B$ Solution can be divided into two parts.

1. If A is decidable then A $\leq_m B$
2. If A $\leq_m B$ then A is decidable

If A is decidable then A $\leq_m B$.
Proof: Define a function f as follows:
f (s) = 01 if s ∈ A
f (s) = 10 otherwise

Since A is decidable, decider can be used for A to compute f.
Also, s ∈ A iff s (s) ∈ $B$.
Hence, f is mapping reduction from A to B

If A $\leq_m B$ then A is decidable
Proof: Since A $\leq_m B$ there exists a function f, such that w ∈ A iff F (w) ∈ $B$
Now consider TM M:
M = "on input w:

1. Computer f (w)
2. If f (w) is in form of 0* 1* then accept, else reject.