KEY

Name KEY

ISU Net ID _____@iastate.edu

Lab Section (circle one): A (T 6-9), **B** (W 3-6), **C** (R 4-7), **D** (F 12-3), **E** (F 3-6), **F** (T 12-3), **G** (W 6-9), **J** (W 12-3), **K** (T 3-6)

CprE 281
Digital Logic

Examination #1

3/2/2015 4:10-5:00PM

Directions: There are 7 questions in this exam. Each question is worth points indicated along with the problem. You should roughly spend 1 minute for every two point. So plan accordingly. If a problem appears to be hard, move on. Please read the questions carefully. Do not write more than what is required.

Calculator should NOT be used.

Problem	Score	
1	_____ / 12 points	} LEANDRO
2	_____ / 12 points	
3	_____ / 17 points	
4	_____ / 18 points	} JOHN
5	_____ / 10 points	
6	_____ / 12 points	} CHRIS
7	_____ / 19 points	

Total _____ (out of 100 points) } JOHN (TOTAL + ENTER INTO BB)

1. (Total 12 points)

(a) (1 points) 1010 (in binary) = 10 (in decimal)

(b) (1 points) 10011 (in binary) = 19 (in decimal)

(c) (1 points) 1010101 (in binary) = 125 (in octal)

(d) (1 points) 1010101 (in binary) = 55 (in hexadecimal)

(e) (1 points) 27 (in decimal) = 11011 (in binary)

(f) (3 points) 432 (in decimal) = 110110000 (in binary)

(g) (1 points) 57 (in decimal) = 39 (in hexadecimal)

(h) (2 points) 57 (in decimal) = 111 (in base 7)

(i) (1 points) 101 (in base 3) = 10 (in decimal)

2. (Total 12 points) Consider the truth table for function F below.

X Y Z	F	Index
0 0 0	0	0
0 0 1	0	1
0 1 0	1	2
0 1 1	1	3
1 0 0	1	4
1 0 1	1	5
1 1 0	0	6
1 1 1	1	7

(a) (3 points) Write the **canonical sum-of-products** expression. Please do **not** use shorthand notation.

$$F = X'YZ' + X'YZ + XY'Z' + XY'Z + XYZ$$

(b) (3 points) Write the **canonical product-of-sums** expression. Please do **not** use shorthand notation.

$$F = (X+Y+Z)\cdot(X+Y+Z')\cdot(X'+Y'+Z)$$

(c) (3 points) Write the **canonical sum-of-products** expression in shorthand notation.

$$F(X,Y,Z) = \Sigma m(2,3,4,5,7)$$

(d) (3 points) Write the **canonical product-of-sums** expression in shorthand notation.

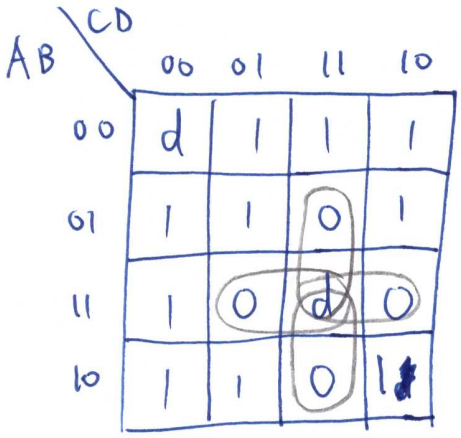
$$F(X,Y,Z) = \Pi M(0,1,6)$$

3. (Total 17 points) Four people participate in a lucky draw. Each one of them draws a ball from a box containing 3 red balls and 3 blue balls. The balls will not be put back to the box once they are drawn. They win as a team if and only if at most two people get a red ball. Let the color of the four balls drawn be represented by logic variables A, B, C, and D. A variable is set to 1 if the ball is red, and 0 if the ball is blue. Let F be a function describing whether or not they win. F=1 if they win, or 0 if they lose.

(a) (8 points) Complete the truth table for F.
(Use "d" to indicate "Don't care conditions".)

A B C D	F
0 0 0 0	d
0 0 0 1	1
0 0 1 0	1
0 0 1 1	1
0 1 0 0	1
0 1 0 1	1
0 1 1 0	1
0 1 1 1	0
1 0 0 0	1
1 0 0 1	1
1 0 1 0	1
1 0 1 1	0
1 1 0 0	1
1 1 0 1	0
1 1 1 0	0
1 1 1 1	d

(b) (5 points) Draw a Karnaugh map for F.

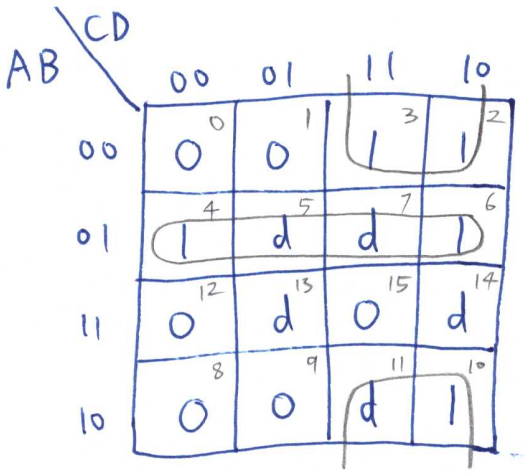


(c) (4 points) Write down the **simplest POS** logic expression for F that you can get.

$$F = (A'+B'+C')(A'+B'+D')(A'+C'+D')(B'+C'+D')$$

4. (Total 18 points) Use K-map to get the **simplest SOP** expression of function F.

(a) (9 points) F(A,B,C,D) = Σ m(2,3,4,6,10) + D(5,7,11,13,14)

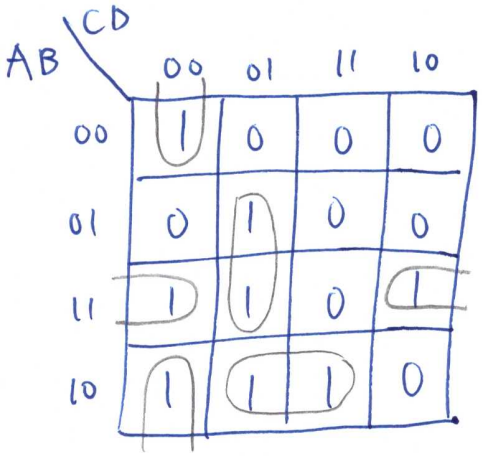


$F = A'B + B'C$

(b) (9 points) F(A,B,C,D) = B'.C'.D' + A.C' + A'.B.C'.D + A.B'.C.D. + A.B.C.D'

$= AC' \cdot B'D' + AC' \cdot B'D + AC' \cdot BD' + AC' \cdot BD$

$= B'C'D' \cdot A' + B'C'D' \cdot A$



$F = C'D'B' + D'AB + C'DB + DAB'$

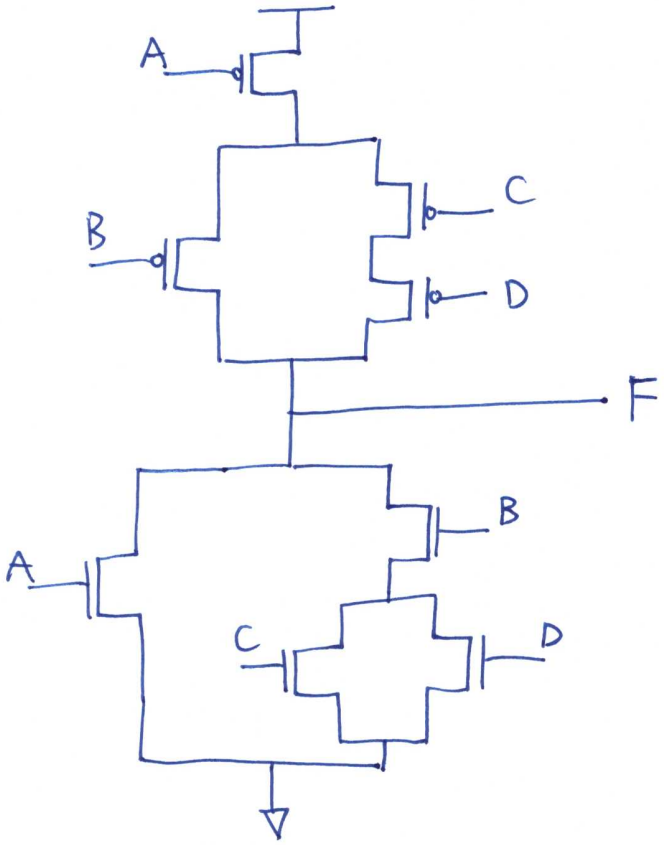
5. (Total 10 points) Let F = A'.(B'+C'.D'). ← PUN

(a) (2 points) Convert F' into a form that combines A, B, C and D with AND and OR functions only.

$$F' = [A' \cdot (B' + C' \cdot D')]'$$

$$= A + B \cdot (C + D) \quad \leftarrow \text{PDN}$$

(b) (8 points) Draw a CMOS complex gate to implement the function F using 4 PMOS and 4 NMOS transistors. Please label the signals clearly.



6. (Total 12 points) Consider the logic expression F = (A + 0).(A + B).(A' + B').

(a) (2 points) Please write the **dual expression** of F.

$$F^D = A \cdot 1 + A \cdot B + A' \cdot B'$$

(b) (4 points) Complete the truth table for F.

A	B	F
0	0	0
0	1	0
1	0	1
1	1	0

(c) (6 points) Use Boolean algebra to simplify F as much as you can. Please show **all steps** and write down the rule number for each step.

1a: 0.0 = 0
1b: 1+1 = 1
2a: 1.1 = 1
2b: 0+0 = 0
3a: 0.1 = 1.0 = 0
3b: 1+0 = 0+1 = 1
4a: If x=0, then x' = 1
4b: If x=1, then x' = 0
5a: x.0 = 0
5b: x+1 = 1
6a: x.1 = x
6b: x+0 = x
7a: x.x = x
7b: x+x = x
8a: x.x' = 0
8b: x+x' = 1
9: (x')' = x
10a: x.y = y.x
10b: x+y = y+x
11a: x.(y.z) = (x.y).z
11b: x+(y+z) = (x+y)+z
12a: x.(y+z) = x.y + x.z
12b: x+y.z = (x+y).(x+z)
13a: x+x.y = x
13b: x.(x+y) = x
14a: x.y+x.y' = x
14b: (x+y).(x+y') = x
15a: (x.y)' = x'+y'
15b: (x+y)' = x'.y'
16a: x+x'.y = x+y
16b: x.(x'+y) = x.y

$$F = (A+0)\cdot(A+B)\cdot(A'+B')$$

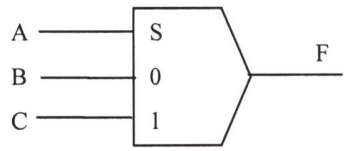
$$= A \cdot (A+B)\cdot(A'+B') \quad 6b$$

$$= A \cdot (A'+B') \quad 13b$$

$$= A \cdot B' \quad 16b$$

7. (Total 19 points) A company MacroHard introduces a new type of gate called BILL (**B**rilliant **I**ndustrial **L**ow-cost **L**ogic) gate with three inputs A, B and C and one output F. Its functionality is specified by the following truth table. Its circuit symbol is also shown below. Since the inputs of the BILL gate is not symmetric, please label the inputs of the gate by "S", "0" and "1" as in the circuit symbol shown.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

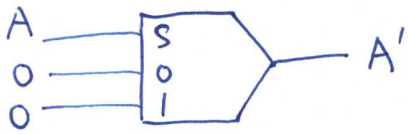


The company claims that the BILL gate can be used to replace several other types of gates. We will verify their claim in this question.

(a) (4 points) Please write a simplified SOP expression for F from the truth table. You do not need to show how to derive the expression.

$$F = A' \cdot B' + A \cdot C$$

(b) (4 points) Draw a circuit to show how to implement a NOT gate using one BILL gate.



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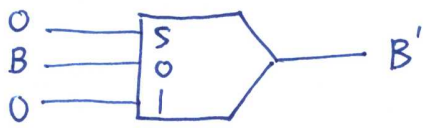

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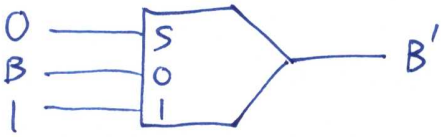
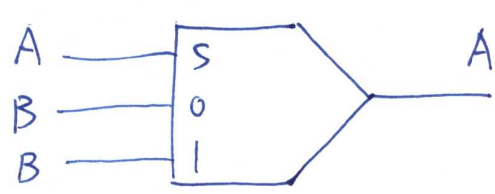


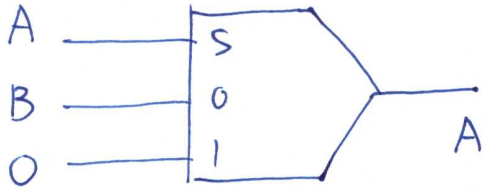
(Question 7 cont'd)

(c) (5 points) Draw a circuit to show how to implement a two-input XNOR gate using one BILL gate.



$A'B' + AB = \overline{A \oplus B}$

(d) (6 points) Draw a circuit to show how to implement a two-input NOR gate using one BILL gate.



$A'B' + A \cdot 0$

$= A'B'$ 5a, 6b

$= (A+B)'$ 15b

(END OF EXAM)