Jay Patel
CS 331
HW #4
Professor Oliver

Q1.

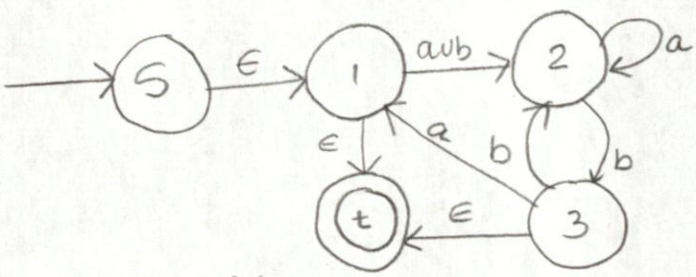
Using induction on k.

[Base Case]
Lets prove that there are k = 3 states. We can tell that G has total of 3 states so then G' will have 2 states.
If we take out one state from G then G' will consist of one transition from q_{start} to q_{accept} this will accept all the strings that are noticed in the language. Since both accept the same language we can tell that they are equivalent.

[Induction Step]
If the proof is true for k-1 states, every time we try to remove one state, we take out and replace with a regular expression that is similar to it which will then allow G' to accept the corresponding language. We can tell that G' will remain the same if and only if R contains the transition strings in states like $q_1, q_2, q_3, q_4 \cdots q_k$
So, if G accepts a state then G' will also accept the same state as well as it will accept the regular expression if and only if k $\geq$ 3

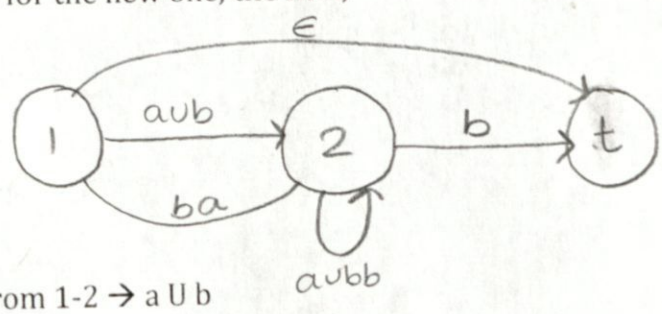
Q2.

Convert DFA
M to an equivalent G'NFA



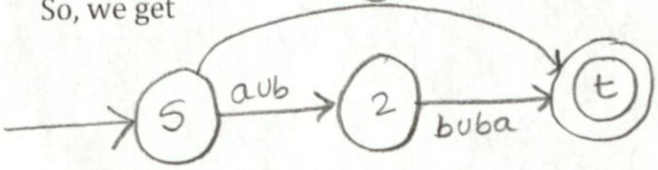
From 2-3-1 → ba
From 2-3-2 → bb
From 2-3-t → b ∈ = b

So for the new one, the DFA,



From 1-2 → a U b
From 1-t → ∈
From 2-1 → 2
From 2-1 → t

So, we get



From combining we get,
$\in U\ (a\ U\ b)\big(a\ U\ bb\ U\ ba\ (a\ U\ b)\big)^\wedge * (b\ U\ ba)$
So, the regular expression of the language,
L (M) M is $\in U\ (a\ U\ b)(a\ U\ bb\ U\ ba)(a\ U\ b))^\wedge *\ (b\ U\ ba)$
For every path that starts in 1 and ends in either 3 or 1 are accept states of DFA

Q3.

Proposition 1:
DFA M = $(Q, \Sigma, \delta, q_0, F)$ and any q $\in Q$ & $w \in \Sigma *, 1^{\hat{}}\delta m(q, \omega) = 1$
Applying induction on |w|

[Base Case]
Prove when w $\in \Sigma 0$
Thus by definition w → m, q e → m
q_0 If and only if $q_0 = q$
So, $1\text{^}\delta m\ (q, w) = |\{q\}| = 1$

[Induction Step]
Every q $\in Q$ *and* $w \in \Sigma *$ such that |w| < i, $1\text{^}\delta m(q, w)| = 1$
ai $\in \Sigma$
Take u = $a_i \ldots a_{i-1}\ q\ w \dashrightarrow m\ q0$
If there are $r_0, r_1 \ldots . r_i$ and $\delta\ (r_j, r_{j+1})$
Consider, r_{i-1} such that q u → m
r_{i-1} and $\delta(r_{i-1}) = q_0$

By using induction hypotheses,
$|\ \text{^}\ \delta m\ (q, u)| = 1$
There is a unique r_{i-1} such that q u → m r_{i-1}
So, $|\text{^}\delta m(q, w)| = 1$

Q4.

a. L = $\{www|w\in \{0,1\}*\}$ *is non regular*}
At first, we assume that L is regular
Choose x = $0^p10^p10^p$ where P is the number of states.
According to pumping lemma x can be written as u,v,w with $|v| \geq 1$ so that u v^m w is also in L so that $|uv| \leq P$
So uv must have 0's
Hence we can tell that it does not satisfy any of the pumping lemma condition

b. L = $\{0^n1^m0^n|m.n \geq 0\}$
Choose x = 0^p10^p
According to the pumping lemma x $\in$ L and |x| > p, so we can write it as x = xyz
Using pumping lemma's 2nd condition we can tell that x and y must have 0s
Using pumping lemma's 1st condition we get y = 0^k for some k > 0
Using pumping lemma's 3rd condition we can take the string L

Hence, xy^0z *should be in L.* But it isn't so there is a contradiction, therefore L is not correct.

c. L = $\{w|w \in \{0,1\}*$ *is not a palindrome*}
Choose x = $0^p10^p10^p1$
After breaking the string in three different groups and associate them in xy and z we get x = xyz

1. $xy^1z \in L$ for I ≥ 0
2. $|y| \geq 0$
3. $|xy| \leq p$

using pumping lemma's 1st condition, we can tell that, all 0s contains x by p
using pumping lemma's 3rd condition, and y are made up only 0's by p
Hence Z has all the 0's with 10^p10^p1
using pumping lemma's 2nd condition, y has at least one 0.
using the first condition we get that there is contradiction. Hence we can tell that L is not regular.