Cpr E 281 HW05 SOLUTION

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Karnaugh Maps
Assigned Date: Fifth Week
Due Date: First class of 6th week

P1.

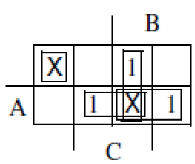
(a) Three 2-input ANDs, one 3-input OR.
So # of transistors = 3x6 + 8 = 26

(b) Three NOTs, four 3-input ANDs, one 4-input OR.
So # of transistors = 3x2 + 4x8 + 10 = 48

(c) One NOT, two 2-input ANDs, one 2-input OR.
So # of transistors = 1x2 + 2x6 + 6 = 20

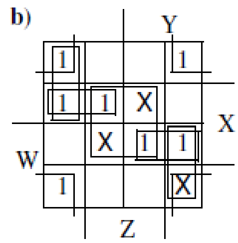
P2.

a)



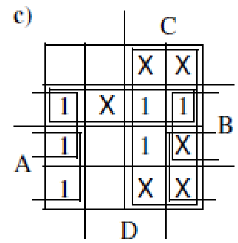
F=AB+AC+BC

b)



F=X'Z'+W'XY'+WXY

c)



F=C+AD'+(BD' or A'B)

P3. (a) Implement first the complement of *f* as

$f' = x1.x3 + x2.x4$

$= (x1 \uparrow x3) \uparrow (x2 \uparrow x4)$

Then $f = f' \uparrow f'$. Four 2-input NAND gates are used.

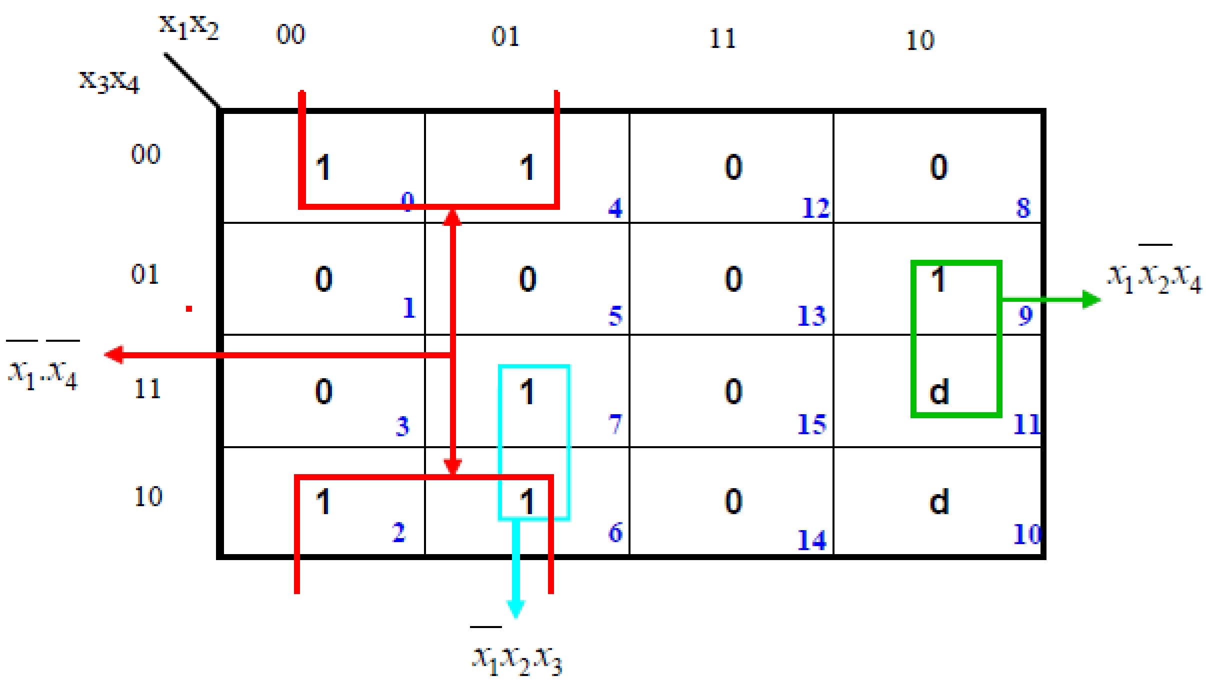
(b) Implement first the complement of *f* as

$f' = (x1' + x4)(x2' + x3')$

$= (x1' \downarrow x4) \downarrow (x2' \downarrow x3')$

Then $f = f' \downarrow f'$. Four 2-input NOR gates are used.

P4. The Karnaugh Map for getting the minimum cost SOP form for the function F is:

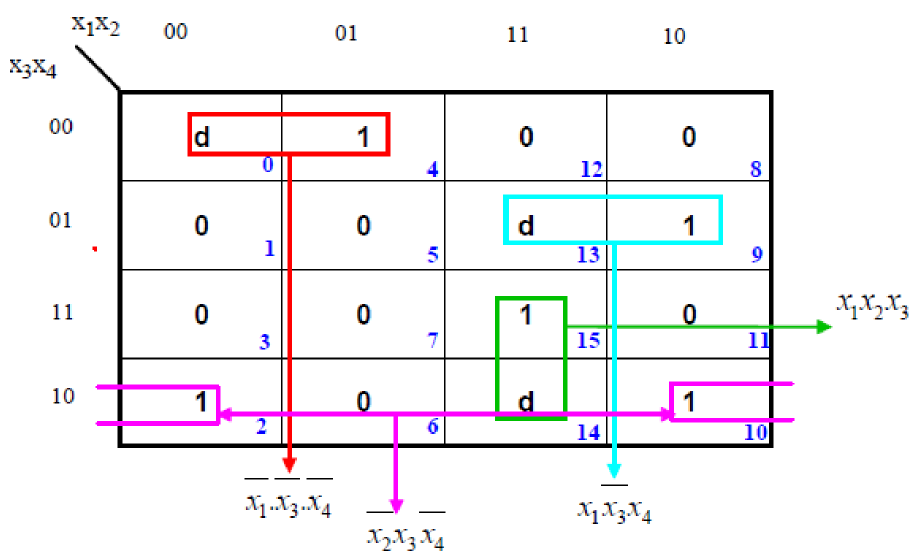


From the Karnaugh map the minimum cost SOP expression is

$$f(x_1, x_2, x_3, x_4) = \bar{x}_1 . \bar{x}_4 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_4$$

The realization of function requires,
Two 3-input AND gate.
One 2-input AND gate.
One 3-input OR gate.
The number of gates needed to implementation the function is:
The cost of f=total no. of gates+ total no of inputs=(2+1+1)+{(2)(3)+(2)(1)+(3)(1)}=15

The Karnaugh Map for getting the minimum cost SOP form for the function G is:



From the Karnaugh map the minimum cost SOP expression is

$$g(x_1,x_2,x_3,x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_3\bar{x}_4 + x_1\bar{x}_3x_4 + x_1x_2x_3$$

The realization of this function requires,

Four 3-input AND gate.

One 4-input OR gate.

The number of gates needed to implementation the function is:

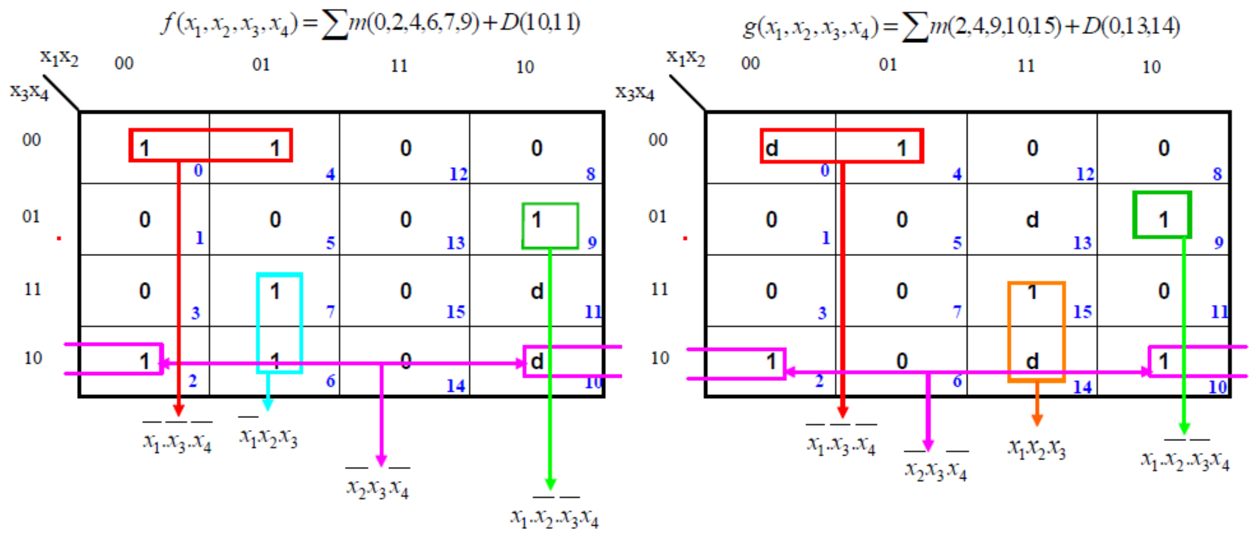
The cost of g=total no. of gates+ total no of inputs=(4+1)+{(3)(4)+(4)(1)}=21

The total cost of realization of function f and g is=15+21=36

In order to find the combined cost of implementing both the function f and g, we draw the Karnaugh map of f and g and groped the elements such that both the function have as many common groping as they possibly can have. Hence the resultant modified Karnaugh maps:

Cpr E 281 HW05 SOLUTION
ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Karnaugh Maps
Assigned Date: Fifth Week
Due Date: First class of 6th week



Modified minimum cost SOP forms are

$$f(x_1,x_2,x_3,x_4)=\boxed{\bar{x}_1\bar{x}_3\bar{x}_4+\bar{x}_2x_3\bar{x}_4+x_1\bar{x}_2\bar{x}_3x_4}+\bar{x}_1x_2x_3$$
$$g(x_1,x_2,x_3,x_4)=\boxed{\bar{x}_1\bar{x}_3\bar{x}_4+\bar{x}_2x_3\bar{x}_4+x_1\bar{x}_2\bar{x}_3x_4}+x_1x_2x_3$$

Hence the first three terms are shared between the two functions.
The implementation of both the functions require:
Four 3-input AND gate
One 4-input AND gate
Two 4-input OR gate
Now the combine cost of the system is
The cost of system=total no. of gates+ total no of inputs=(4+1+2)+{(3)(4)+(4)(1)+(4)(2)}=31

P5. There are 11 gates and 30 inputs, so the total cost is 41.
After

$$f = \bar{x}_1\,\bar{x}_3\,\bar{x}_4 + x_1x_4 + x_3x_4 + \bar{x}_1\,\bar{x}_2\,x_3 + x_1x_2\bar{x}_3\,\bar{x}_4$$
$$g = \bar{x}_1\,\bar{x}_4 + \bar{x}_1\,x_3 + x_3\bar{x}_4 + \bar{x}_2\,x_4 + x_1x_2\,\bar{x}_3\,\bar{x}_4$$

K-map of f, g shows below:

X3X4 \ X1 X2	00	01	11	10
00	1	1	1	0
01	0	0	1	1
11	1	1	1	1
10	1	0	0	0

X3X4 \ X1 X2	00	01	11	10
00	1	1	1	0
01	1	0	0	1
11	1	1	0	1
10	1	1	1	1

Then based on the optimum combined circuit:

$$f = x_2\,\overline{x_3}\,\overline{x_4} + \overline{x_1}\,\overline{x_2}\,\overline{x_4} + \overline{x_1}\,x_3\,x_4 + x_1\,x_4$$

$$g = x_2\,\overline{x_3}\,\overline{x_4} + \overline{x_1}\,\overline{x_2}\,\overline{x_4} + \overline{x_1}\,x_3\,x_4 + \overline{x_2}x_4 + x_3\overline{x_4}$$

So f and g share the first three product terms.

The cost 8 gates, 24 inputs. Total cost of redesigned solution: 32.

Note that other solutions are possible depending on which 1's combination are taken (but they must have at least 3 common factors in k-maps for f and g expressions)

P6. The SOP joint minimization:

$$F = W + X\,\overline{Y} + X\,\overline{Z} + \overline{Y}\,\overline{Z}$$

$$G = W + X\,\overline{Y} + X\,\overline{Z} + \overline{X}\,Y$$

Note that the POS joint minimization has the same cost of 22:

$$F = (\overline{X} + \overline{Y} + \overline{Z})(W + X + \overline{Z})(X + \overline{Y})$$

$$G = (\overline{X} + \overline{Y} + \overline{Z})(W + X + Y)$$