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Hw #5

CS 230

Q1. n $\geq 1, 21|4^{n+1} + 5^{2n-1}$

Basis:

$= 4^{(1+1)} + 5^{2(1)-1}$

= $4^2 + 5^{2-1}$

= $4^2 + 5$

= 16 + 5

= 21

Hence, 21/21 = 1 ------ (1)

Induction step:

Let it be true for k = n then, 21/4^{n+1} + 5^{2n-1} ----- (2)

We need to prove, k = n + 1

21| $4^{(n+1)+1} + 5^{2(n+1)-1}$

So, $4^{n+1+1} + 5^{2n+2-1}$

= $4^{n+2} + 5^{2n+1}$

= 4.$4^{n+1} + 5^{2n+1}.25$

= 4.$4^{n+1} + (4+21)5^{2n-1}$

= 4[4^{n+1} + 5^{2n-1}] + 21. 5^{2n-1}

From eq 2, 21|4^{n+1} + 5^{2n-1} and 21|21. 5^{2n-1} Hence it is also true for k = n +1 ---- (3)

From eq 1,2 and 3 we can say that 21|4^{n+1}+ 5^{2n-1}

Q2. n $\geq 1, 5|8^n - 3^n$

Basis:

$= 8^1 - 3^1 => 8 - 3 => \frac{5}{5} = 1$

Induction step:

Let p(n) = $8^n - 3^n; n \geq 1$

Let S = { $n \geq 1 | 5 | p(n)$}

p (1) = 8^1– 3^1 = 5 and 5|p (1)

hence, 1 ∈ S

Let n ∈ $S, so\ that\ 5 \mid p\ (n)$

Then p(n) = $8^n - 3^n = 5q$

Where, q ∈ N

$\rightarrow\ 8^n - 3^n = \ 5q$ ----- (1)

p (n + 1) = $8^{n+1} - 3^{n+1} = \ 8^n . 8 - 3^{n+1}$ ---- (2)

By substituting eq 1 and 2 we get,

= [$3^n + 5q$].8 – 3^{n+1}

= 8. $3^n + 40q - 3^n . 3$

= 40q + 5. 3^n

= 5 (8q+3^n)

= 5q’

By reversing we know that q’ = 8q + 3^n ∈ N

Hence, 5|p(n+1) and hence n + 1 ∈ S

Hence, n ∈ $S\ \rightarrow n + 1\ \in S$

From Induction we know that S = N

Q3. $\forall n > 1, \sum_{k=1}^{n}(2k-1) = \ n^2$

Let the statement p(n) = 1 + 3 + 5 + …..+ 2n - 1 = n^2

Basis step:

For n = 1,

2(1)- 1 = $(1)^2$

1 = 1

The statement is true for n = 1

Induction step:

Let's assume that the statement is true for k = n

So that means, 1 + 3 + 5 + …. + 2n – 1 = n^2

If we add 2 (n + 1) - 1 = 2n + 1

1 + 3 + 5 + …... + (2n – 1) + (2n + 1) = n^2+ (2n + 1)

= $n^2 + (2n + 1)$

= $(n + 1)^2$

Hence we can tell that, p (n) is true for all k = n + 1 ---- (proved by induction)

Q4. Show that n$\geq 1, \sum_{k=1}^{n} k.k! = (n + 1)! - 1$

Let p(n) = 1.1! + 2.2! + 3.3! + …. n.n! = (n+1)! -1

If n =1:

Then, 1.1! = 1 and (n+1)! - 1

(1 + 1) = 1

= 1

That means both of the sides are equal for n = 1

Now, we let's do it for n = k + 1

L.H.S => 1.1! + 2.2! + ….. k.k! + (k+1) (k+1)!

= (k + 1)! – 1 + k + 1 (k + 1)!

= (k + 1)! [1 - $\frac{1}{(k+1)!} + k + 1$]

= (k + 1)! [k + 2 - $\frac{1}{(k+1)!}$]

= (k + 1)! [$\frac{(k + 2)\ (k + 1)! - 1}{(k+1)!}$]

= (k + 2)! – 1 => which is R.H.S

Hence Proved by induction.