Homework 5
Com S 331, Spring 2017
Due date: **Wednesday, March 1, 2017**
Please submit the homework via BlackBoard **before the class that day**.

Note: All submissions should be **typed** and in .pdf or .doc(x) format. However, state diagrams can be drawn with hand and presented in the final manuscript as images. We recommend to use Latex for typing homeworks. You **do not** need to formally prove the correctness of your constructions unless a question specifically asks to do so. However, in most cases you need to present a reasonable justification of correctness.

Total points available: 75

0. Read pages 101–111 up to the end of Section 2.1 in the class-book (Sipser, **3rd** edition).

1. **(25 points)** Consider the following construction that converts a DFA, $M = (Q, \Sigma, \delta, q_0, F)$, to a context-free grammar, $G = (V, \Sigma, R, S)$:

   - $V = \{V_i | q_i \in Q\}$. That is, we make a separate variable V_i for each state q_i in Q.
   - For each transition $\delta(q_i, a) = q_j$ we add a rule $\boldsymbol{V_i \to aV_j}$ to R. Additionally, we add a rule $\boldsymbol{v_i \to \varepsilon}$ for all $q_i \in F$. That is, R contains $|\Sigma| \cdot |Q| + |F|$ rules.
   - $S = V_0$, where V_0 corresponds to the start state of M, q_0.

   Provide a formal proof showing that $L(M) = L(G)$ for any DFA M.

2. **(20 points)** Solve Exercise **2.9** from the class-book.

3. **(30 points)** Give context-free grammars generating the following languages **and briefly justify correctness of your grammars**:

   (a) The complement of the language $\{a^n b^n \mid n \geq 0\}$.

   (b) $\{x_1 \# x_2 \# \ldots \# x_k \mid k \geq 1, x_i \in \{a, b\}^*,$ and for some $1 \leq i, j \leq k$, $x_i = x_j^{\mathcal{R}}\}$. Note that $x^{\mathcal{R}}$ denotes the *reversed* string x. Also note that i can be equal to j.