NAME **MAT 227 Differential Equations** **Test 4A**

5 problems- 20 points each **Show all work for full credit!**

1 Suppose $f(t)=\begin{cases}1 & if\ \ 0\le t<\pi \\ e^t & if\ \ \pi\le t<2\pi \\ \sin t & if\ \ t\ge 2\pi\end{cases}$. Use the definition to find the Laplace transform.(You may use the table of integrals from your book, be sure to reference any formula used by the formula number.)

2 Use the Laplace transform and partial fractions (show all steps) to solve the differential equation with initial conditions:

$$y''+2y'+5y=15 \qquad y(0)=2, y'(0)=1$$

3 Use Laplace transforms to solve the initial value problem.

$$y''+x+2y=0$$
$$x''+x+2y=0$$
$$x(0)=0, y(0)=0$$
$$x'(0)=1, y'(0)=1$$

4 Use the Translation (Shifting) Theorem (Theorem 1 in section 4.3) to find the Laplace transform of $f(t)=\cosh kt\cos kt$.

(Recall: $\cosh kt=(e^{kt}+e^{-kt})/2$). Also, show your answer is algebraically equivalent to $F(s)=\dfrac{s^3}{s^4+4k^4}$.

5 Use the Differential of Transforms Theorem (Theorem 2 in section 4.4) to find the inverse Laplace transform of $F(s)=\ln\left(\dfrac{s-2}{s+4}\right)$