4 problems- 25 points each

Show all work for full credit!

- 1 Solve by using power series: $2 y'-y = \sinh(x)$. Compute the first 6 coefficients $(a_0 a_5)$.
- 2 Solve the differential equation below with initial conditions. Compute the first 6 coefficients $(a_0 a_5)$. Find the general pattern.

$$(1-2x)y''-y'+xy=0$$

$$y(0) = 0, y'(0) = 1$$

- 3 Consider differential equation: $x^3(x^2-1)^2(x^2+1)y''+(x-1)x$ y'+y=0. Determine whether x=0 is a regular singular point. Determine whether x=1 is a regular singular point. Are there any regular singular points that are complex numbers? Justify conclusions.
- 4 Use the Frobenius method to solve: xy'' y' + 2y = 0. Find index r and recurrence relation. Compute the first 5 terms $(a_0 a_4)$ using the recurrence relation for each solution and index r.