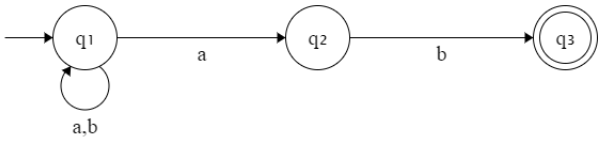
Homework 3
Com S 331, Spring 2017
Due date: **Monday, February 6, 2017**
Please submit the homework via BlackBoard **before the class that day**.

Note: All submissions should be .pdf or .doc(x) format. However, state diagrams can be drawn with hand and presented in the final manuscript as images. We recommend to use Latex for typing homeworks. You **do not** need to formally prove the correctness of your constructions unless a question specifically asks to do so.

Total points available: 100

0. Read pages 64–76 up to Section 1.4 in the class-book (Sipser, 3rd edition).

1. **(20 points)** Use induction over the size of strings to prove that the following NFA over the alphabet $\Sigma = \{a, b\}$



recognizes the regular language $\Sigma^* ab$ (that is, it accepts all $w \in \Sigma^*$ that end with ab).

Hint: use the following statement as induction hypothesis

(1) For each string $w \in \Sigma^* a$ (i.e., strings that end with an a) all existing computation paths terminate either in state q_1 or in state q_2 (note that we consider only those computation paths that do not "die" in the middle of the computation).

(2) For each string $w \in \Sigma^* ab$ all existing computation paths terminate either in state q_1 or in state q_3.

(3) For every other string (which is neither in $\Sigma^* a$ nor in $\Sigma^* ab$, i.e., it belongs to $\Sigma^* bb \cup b \cup \varepsilon$), all existing computation paths terminate in state q_1.

Note that since state q_3 is the only accept state, this statement implies that the language of the above NFA is $\Sigma^* ab$ (see item (2)). Therefore, by proving this statement we also prove that $\Sigma^* ab$ is the language of the above automaton. In the base case you need to show that the statement holds for strings of size 0 and 1. As for the induction step, you need to

show that the above induction hypothesis holds for all strings of fixed size $n \geq 2$, assuming that it holds for all strings of size smaller than n.

For example, for each string $w \in \Sigma^* a$ you need to show that there exists a computation path that terminates in q_1 and there exists another computation path that terminates in q_2. To do that you can consider the prefix of w of size $n-1$, for which the induction hypothesis holds.

2. **(16 points)** Solve problem **1.20** from the class-book.

3. **(34 points)** Solve problem **1.43** from the class-book.

4. **(30 points)** Convert the DFA (over $\Sigma = \{a, b\}$) presented on the figure below to a corresponding regular expression (see Lemma 1.60 in the class-book). Show the process of converting the DFA to a two-state GNFA step-by-step (i.e., show the initial GNFA with 5 states, then an equivalent GNFA with 4 states, and so on up to 2 states).