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CS 331
HW 7
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Q1.
The CFG for the given PDA that recognizes {w∈ (a, b)* | w =wR} is

A → aAa
A → bAb
A → a|b|∈

Now, we can easily derive the string baaab from it, by this;
A -> bAb → baAab → baaab

Q2.

Assume B be the language for the entire palindrome containing equal number of 0’s and 1’s

We need to show that B is not a CFG. We can use contradiction to prove this. So, let’s assume that B is a CFG.

Since B is a CFG, using PL there is a number of P, where if s is a string in B of length at least P, then say we have a string s = abcde

1. for each i ≥ 0, $ab^i\ cd^ie\ \in A$,
2. |ad| > 0 and
3. |bcd| $\leq p$

Now, if we select the string, s = $0^n\ 1^{2n}\ 0^n$ → this shows that s is a member of B of length P
The string for S’ is not a palindrome due to the 1st condition of PL so, s’ $\notin B$
So, the language B is not following the condition 1st of the PL
Therefore, the assumption B is a CFG is wrong.
Hence, B is not a CFG.

Q3.

To get a Contradiction, assume that C is a CFL.
let P be the pumping length guaranteed by the pumping lemma for the language C,

Let's assume that
s = $1^p 3^p 2^p 4^p \in C \; with \; |s| > p$

Then the string S can be written in the form s=abcde where;

1. $ab^i cd^i e \in C$ for all i $\geq$ 0,
2. |bd| > 0
3. |bcd| $\leq$ p

We will prove that for any values we will see a contradiction using the conditions below:

Condition a: bcd contains a 1. Then $ab^2cd^2e \notin C$, this is because it wont have the same number of 1s and 2s. Because from |bcd| $\leq p$ and bcd cannot contain any 2s
Condition b: bcd contains a 2. Then $ab^2cd^2e \notin C$, this is because it wont have the same number of 1s and 2s. Because from |bcd| $\leq p$ and bcd cannot contain any 1s
Condition c: bcd contains a 3. Then $ab^2cd^2e \notin C$, this is because it wont have the same number of 3s and 4s. Because from |bcd| $\leq p$ and bcd cannot contain any 4s
Condition d: bcd contains a 4. Then $ab^2cd^2e \notin C$, this is because it won't have the same number of 3s and 4s. Because from |bcd| $\leq p$ and bcd cannot contain any 3s
but from |bd| > 0 there are all the cases. But we contradict $ab^i cd^i e \in C$ for all i $\geq$ 0.

Therefore, we see that there is a contradiction from condition 2 to 1 which then shows that, C is not a CFG.

Q4. [Extra Credit]

Proof:
Assume F for CFL
Let p be the pumping length for L
Consider, $a^p b^p c^{p^2} \in F$ so,
$a^p b^p c^{p^2}$ = uvxyz where |vy| ≥ 1 |vxy| ≤ p

Case1: v within b^p, y within c^{p^2}
Then, v = b^i & y = c^j for some i,j
So that 1≤ $i + j$ ≤p without loss assume i ≥ 1
Note: j < p
u$v^0 x y^0 \geq L$ |v| = i
|v| = j
p+p+p^2

uxz = $a^p b^{p-i} c^{p2-j} \in L$
Then, p (p-i) = $p^2 - j$
p(p-i) = $p^2 - pi$
$\leq p^2 - p$
< $p^2 - j$

Case2: if vy ∈ a*, let |vy| = k, so then $uv^2 xy^2$z = $a^{(2p)!+k} b^{p+1}$ is not L, because p+1 divides (2p)!

Case3: if vy ∈ b*, let |vy| = k, so then
$uv^2 xy^2$z = $a^{(2p)!+1} xy^{(2p)!+1}$z = $a^{(2p)!} + b^{p+1+1k(2p)!}$ will not be in F since, p+1+k(2p)! does not divide (2p)!

Now, we can tell that if v = a^i and y = b^j then 0 < I, j < p, then
$uv^2 xy^2 z = a^{(2p)!+i} b^{p+i+j}$ is not in F because it divides (2p)!

If we notice, there will be a contradiction in all so F cannot be context free.