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Hw #2

CS 230

Q18. $(\bigcap_{n=1}^{\infty} An) \cup (\bigcap_{m=1}^{\infty} Bm) = \bigcap_{n=1}^{\infty} \bigcap_{m=1}^{\infty} (An \cup Bm)$

Answer: Let x ∈ $(\bigcap_{n=1}^{\infty} An) \cup (\bigcap_{m=1}^{\infty} Bm)$

↔ x ∈ $(\bigcap_{n=1}^{\infty} An)$ $or\ x \in (\bigcap_{m=1}^{\infty} Bm)$

↔ $x \in An\ \forall n = 1\ to\ \infty\ or\ x \in Bm\ \forall m = 1\ to\ \infty$

↔ $x \in An\ or\ x \in Bm\ \forall n\ m = 1\ to\ \infty$

$$\leftrightarrow x \in \bigcap_{n=1}^{\infty} \bigcap_{m=1}^{\infty} (An \cup Bm)$$

Therefore, $(\bigcap_{n=1}^{\infty} An) \cup (\bigcap_{m=1}^{\infty} Bm) = \bigcap_{n=1}^{\infty} \bigcap_{m=1}^{\infty} (An \cup Bm)$

Hence proved.

Q20. X – $(\bigcap_{n=1}^{\infty} An)$ = $\bigcup_{n=1}^{\infty} (X - An)$

Answer: Let x ∈ $X - (\bigcap_{n=1}^{\infty} An)$

↔ x ∈ $X\ and\ x \notin \bigcap_{n=1}^{\infty} An$

↔ $x \in X\ and\ x \notin$ An for at least one m

↔ $x \in (X - am) for\ at\ least\ one\ m$

$$\leftrightarrow x \in \bigcup_{n-1}^{\infty} (X - An)$$

Therefore, X – $(\bigcap_{n=1}^{\infty} An)$ = $\bigcup_{n=1}^{\infty} (X - An)$

Hence proved.

Q21. a

$$\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0$$

$$so,\left\{[1,1],\left[\frac{1}{2},1\right],\left[\frac{1}{3},1\right],\dots,[0,1]\right\}$$

$$\forall_x[x\in\mathbb{R}|0\le x\le 1]$$

$$So,(0,1]$$

$b)$

$$\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0$$

$$so,\left\{(1,1],\left(\frac{1}{2},1\right],\left(\frac{1}{3},1\right],\dots,(0,1]\right\}$$

$$\forall_x[x\in\mathbb{R}|0< x\le 1]$$

$$So,(0,1]$$

$c)$

$$\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0$$

$$so,\left\{[1,1),\left[\frac{1}{2},1\right),\left[\frac{1}{3},1\right),\dots,[0,1)\right\}$$

$$\forall_x[x\in\mathbb{R}|0\le x< 1]$$

$$So,(0,1)$$

$d)$

$$\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0$$

$$so,\left\{(1,1),\left(\frac{1}{2},1\right),\left(\frac{1}{3},1\right),\dots,(0,1)\right\}$$

$$\forall_x[x\in\mathbb{R}|0< x< 1\}$$

$$So,(0,1)$$

$e)$

$$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

$$so, \left\{(0,1), \left(0, \frac{1}{2}\right), \left(0, \frac{1}{3}\right), \dots, (0,0)\right\}$$

$So, \{0\}$

$f)$

$$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

$$so, \left\{[0,1), \left[0, \frac{1}{2}\right), \left[0, \frac{1}{3}\right), \dots, [0,0)\right\}$$

$So, \{0\}$

$g)$

= (0,n] $\cap \left[0, \frac{1}{2}\right] \cap \left(0, \frac{1}{3}\right) \cap$

= $so, \{\emptyset\}$

$h)$

$= (-\frac{1}{2}, \frac{1}{2}) \cap (-\frac{1}{3}, \frac{2}{3}) \cap (-\frac{1}{4} \cap \frac{3}{4}) \cap$

= So, $[0, \frac{1}{2})$

Q33.

a. (A1 $\times B) \cup (A2 \times B) = (A1 \cup A2) \times B$
   X $\in (A1 \times B) \cup (A2 \times B) = S$
   Hence to prove:
   X $\in (A1 \times B) \cup (A2 \times B) = S$
   X $\in A1 \times B$ $or$ $A2 \times B$
   $\rightarrow x = (a1, b) or\ x = (a2, b) (a1 \in A1, a2 \in A2)$
   a1, a2 ∈ A1 ∪ A2     x = (a, b) where a∈A1∪A2

   x ∈ (A1 ∪ A2) ×B
   x ∈ (A1 ∪ A2)×B
   x = (a , b) where a ∈ A1 ∪ A2
   → x = (a1, b) or x = ( a2, b) (a1 ∈ A1, a2 ∈ A2)
   x ∈ A1 × B or x ∈ A2 × B
   x∈ ( A1 × B ) ∪ ( A2 × B) = S

S = T -> from above.
Hence proved,
$(A1 \times B) \cup (A2 \times B) = (A1 \cup A2) \times B$

b.

$(A1 \times B) - (A2 \times B) = (A1 - A2) \times B$
$x \in (A1 \times B) - (A2 \times B) = S$
Hence to prove,
$x \in (A1 - A2) \times B = T$
So,
$x \in (A1 \times B)$ and $(A2 \times B) = S$
$x \in A1 \times B$ and $x \notin A2 \times B$
$\rightarrow x = (a1, b)$ & $x \neq (a2, b)$ $(a1 \in A1, a2 \in A2)$
$X = (a, b)$ where $a \in A1 - A2$
$x \in (A1 - A2) \times B$
Now
$x \in (A1 - A2) \times B$
$X = (a, b)$ where $a \in A1 - A2$
$x = (a1, b)$ & $x \neq (a2, b)(a1 \in A1, a2 \in A2)$
$x \in A1 \times B$ & $x \notin A2 \times B$
$x \in (A1 \times B) - (A2 \times B) = S$
From above,
S = T
Hence,$(A1\times B) - (A2\times B)=(A1-A2)\times B$

c.

$(A1 \times B1) \cap (A2 \times B2) = (A1 \cap A2) \times (B1 \cap B2)$
So,
$X \in (A1 \times B1) \cap (A2 \times B2) = S$
Hence to Prove:
$x \in (A1 \cap A2) \times (B1 \cap B2) = T$
Solution: $x \in (A1 \times B1) \cap (A2 \times B2) = S$
$x \in (A1 \times B1)$ and $x \in (A2 \times B2)$
$\rightarrow x = (a1, b1)$ and $x = (a2, b2)$ $(a1 \in A1, a2 \in A2, b1 \in B1, b2 \in B2)$
$a1, a2 \in A1 \cap A2$ and $b1, b2 \in B1 \cap B2$
$X = (a,b)$ where $a \in A1 \cap A2$, $b \in B1 \cap B2$
$X \in (A1 \cap A2) \times (B1 \cap B2)$
$x \in (A1 \cap A2) \times (B1 \cap B2)$
$X = (a,b)$ where $a \in A1 \cap A2$, $b \in B1 \cap B2$
$\rightarrow$ x (a1, b1) and x =(a2, b2) $(a1 \in A1, a2 \in A2, b1 \in B1, b2 \in B2)$
$x \in (A1 \times B1)$ and $x \in (A2 \times B2)$
$x \in (A1 \times B1) \cap (A2 \times B2) = S$
S = T -> From above
$(A1 \times B1) \cap (A2 \times B2) = (A1 \cap A2) \times (B1 \cap B2)$

d. X $\times Y - A \times B = (X - A) \times Y \cup X \times (Y - B)$

→ a,b ∈ $X \times Y - A \times B$

→ $a, b \in (X \times Y)$ $a, b \notin$ (A $\times B$)

→ (a ∈ $X$ ^$b \in Y$) ^ (a ∉ A ∨ b ∉ B)

→ ($a$ ∈ X ^b ∈ Y ^a ∉ A) V (a ∈ X ^b Y ^b ∉ B )

→ (($a$ ∈ $X$ $^a$∉ A) ^$b$ ∈ $Y$)$V$($a$ ∈ X ^(b ∈ Y ^ b ∉ B ))