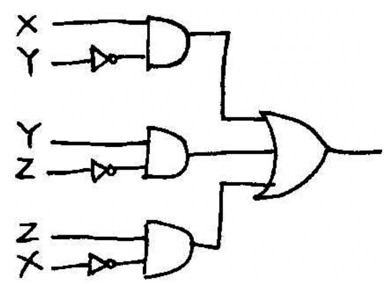
CPRE 281 – Solutions to Mock Exam #1

1. (a)

X	Y	Z	XY’+YZ’+ZX’
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

(b)

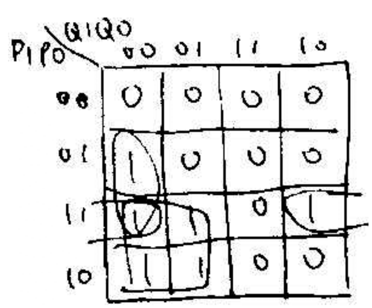


(c) F = X’.Y’.Z + X’.Y.Z’ + X’.Y.Z + X.Y’.Z’ + X.Y’.Z + X.Y.Z’
(d) F(X,Y,Z) = Σ m(1,2,3,4,5,6)
(e) F = (X+Y+Z)(X’+Y’+Z’)
(f) F(X,Y,Z) = Π M(0,7)

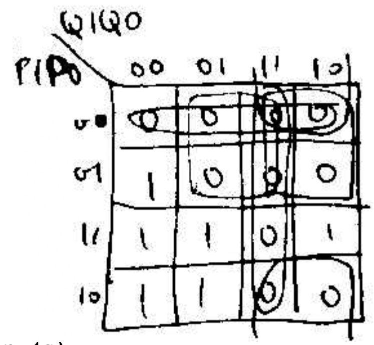
2. (a)

P1	P0	Q1	Q0	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

(b) F = P1 Q1’ + P0 Q0’ Q1’ + P1 P0 Q0’

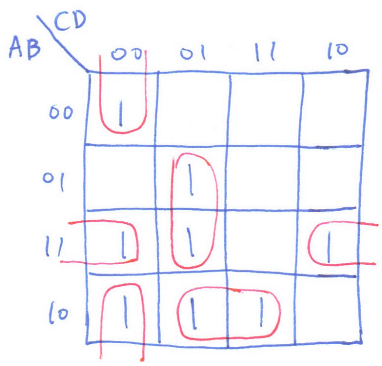


(c) F = (P1+ P0) (P1 + Q0’) (P1 + Q1’) (Q1’ + Q0’) (P0 + Q1’)



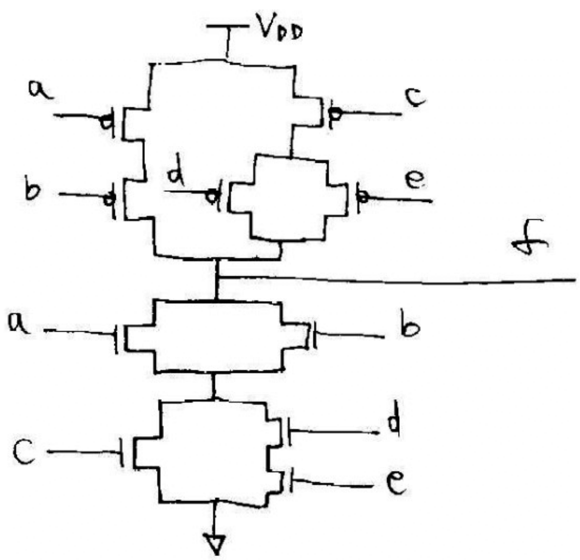
(d) Cost of (b) = 19 (2 NOT, 3 AND, 1 OR, 13 inputs)
Cost of (c) = 25 (2 NOT, 1 AND, 5 OR, 17 inputs)

3. (a) F = B’D’ + D’A + ABC’ + BDA’
(b) F = B’ + CD
(c) B’.C’.D’ = A. B’.C’.D’ + A’. B’.C’.D’
A.C’ = A.C’.B’.D’ + A.C’.B’.D + A.C’.B.D’ + A.C’.B.D



F = B’.C’.D’ + A.B.D’ + B.C’.D + A.B’.D

4. f = (a+b)’ + (c+de)’ = a’b’ + c’(d’+e’)   by DeMorgan’s Law



5. (a)

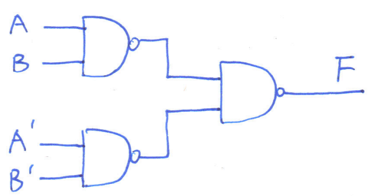
X	Y	LHS
0	0	0
0	1	0
1	0	1
1	1	1

(b) (a+b)(a’b’+a+b’)

(a+b)(a’b’+a+b’)	= (a+b)a’b’ + (a+b)(a+b’)	12a
	= (a+b)(a+b)’ + (a+b)(a+b’)	15b
	= 0 + (a+b)(a+b’)	8a
	= (a+b)(a+b’) + 0	10b
	= (a+b)(a+b’)	6b
	= a	14b

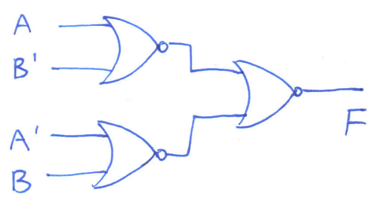
(c) The dual theorem is: ab + (a’+b’)ab’ = a
It can be proved similarly as part (b) but using the dual version of an axiom / theorem / property in every step (i.e., if you use 12a in part (b), use 12b here).

6. (a)



(b) 4x3 = 12

(c) F(A,B) = Σ m(0,3) = Π M(1,2) = (A+B’).(A’+B)

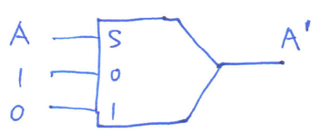


(d) 4x3 = 12

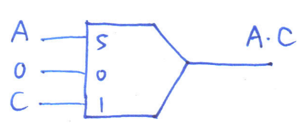
(e) 8

7. Notice that F = A’.B + A.C

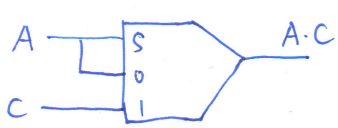
(a)



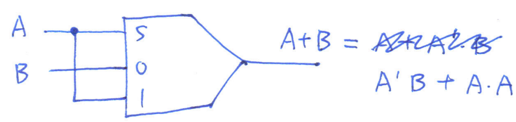
(b)



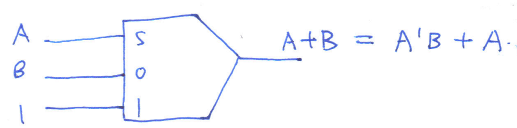
OR



(c)



OR



(d) Multiplexer

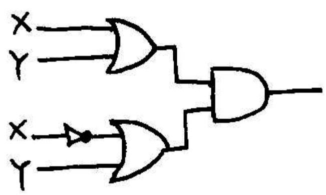
Solutions to Extra Exercises

1. (a) 51
   (b) 1010001_2
   (c) $1EE_{16}$
   (d) 110100011_2
   (e) 7756_8
   (f) 222_6

2. (a)

X	Y	(X+Y)(X'+Y)
0	0	0
0	1	1
1	0	0
1	1	1

   (b)



   (g) F = X'.Y + X.Y
   (h) F = (X+Y)(X'+Y)

3. (a) F = P1' P0 Q1' Q0' + P1 P0' Q1' Q0' + P1 P0' Q1' Q0
   + P1 P0 Q1' Q0' + P1 P0 Q1' Q0 + P1 P0 Q1 Q0'
   (b) F = Σ m(4,8,9,12,13,14)
   (c) F = (P1+P0+Q1+Q0) (P1+P0+Q1+Q0') (P1+P0+Q1'+Q0) (P1+P0+Q1'+Q0')
   (P1+P0'+Q1+Q0') (P1+P0'+Q1'+Q0) (P1+P0'+Q1'+Q0') (P1'+P0+Q1'+Q0)
   (P1'+P0+Q1'+Q0') (P1'+P0'+Q1'+Q0')
   (d) F = Π M(0,1,2,3,5,6,7,10,11,15)

4. Define:
   - D = 1 when doors are closed
   - S = 1 when seat belts are buckled
   - B = 1 when parking brake is off
   - F = 1 when warning light is on

   Therefore F = D S' + SB' + BD'
   (or F = B'D + S'B + D'S)

D	S	B	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The system is not a reasonable one. The warning light will turn on even for some safe situations (e.g., when D=1, S=1, B=0).

5. F = A + BC + BD

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	1	1	1	1
10	1	1	1	1

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

6. (a) True. By rule 8(b).
   (b) False. ab + a'b' = 0 when a=0 and b=1
   (c) False. ab'c + a'bc' = 0 when a = b = c = 1
   (d) True. Can be proved by constructing a truth table.

X	Y	(ab+a'b')'	a'b + ab'
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

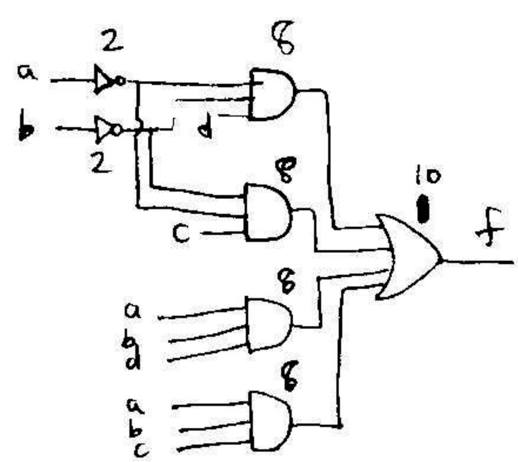
7. (a) (a+b)a'b' = (a+b)(a+b)' 15a
   = 0 8a
   (b) (a'(a+a'b) = a'(a+b) 16a
   = a'((a')'+b) 9
   = a'b 16b
   (c) (a+a'bc)(a+c)' = (a+bc)(a+c)' 16a
   = (a+bc)a'c' 15b
   = a'(a+bc)c' 10a
   = a'((a')'+bc)c' 9
   = a'bcc' 16b
   = a'b0 8a
   = 0 5a

8.

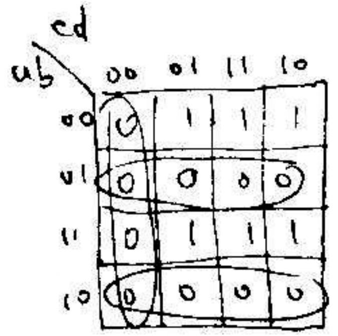
a \ bc	00	01	11	10
0	0	0	0	0
1	0	0	0	0

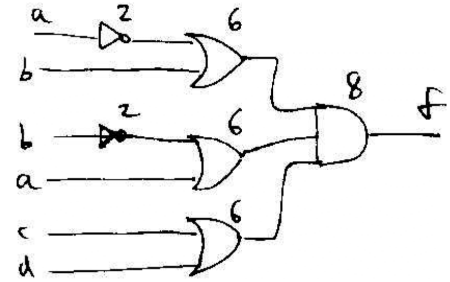
9. (a) f = a'b'd + a'b'c + abd + abc
   Cost = 46

ab \ cd	00	01	11	10
00	0	1	1	1
01	0	0	0	0
11	0	1	1	1
10	0	0	0	0

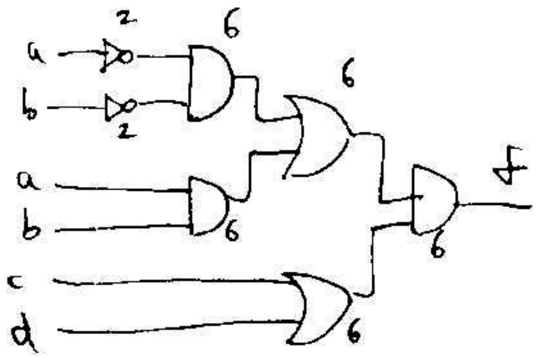


(b) f = (c+d)(a+b')(a'+b)
Cost = 30

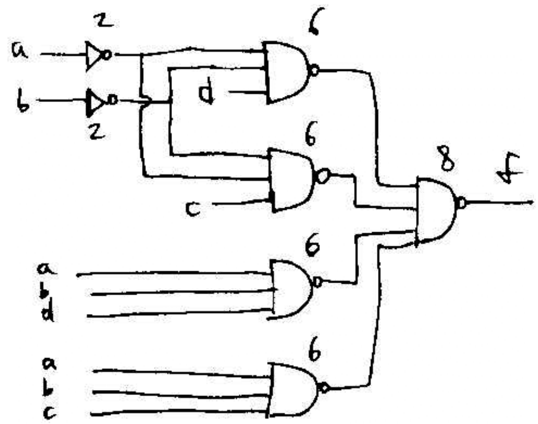




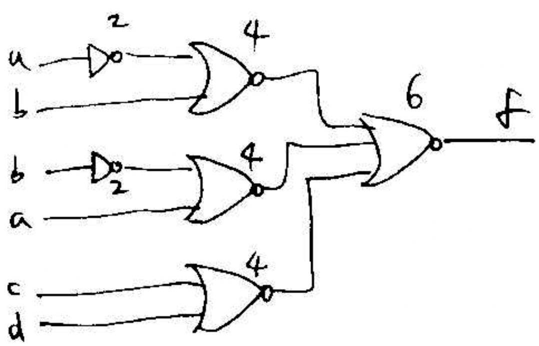
(c) Cost = 34



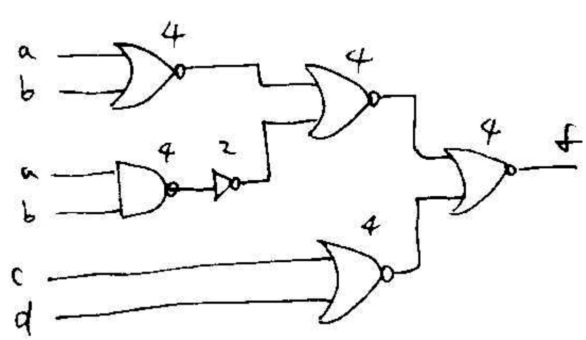
(d) Cost = 36



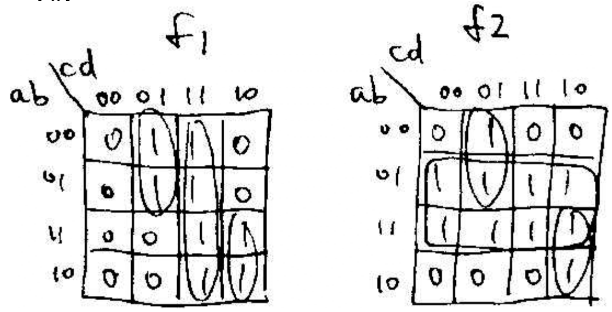
(e) Cost = 22

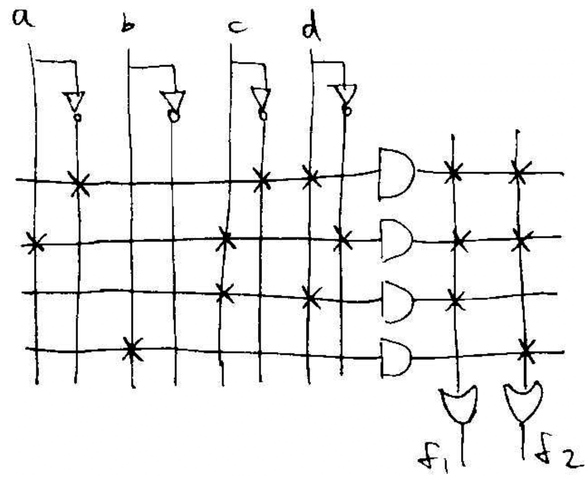


(f) Cost = 22



10.



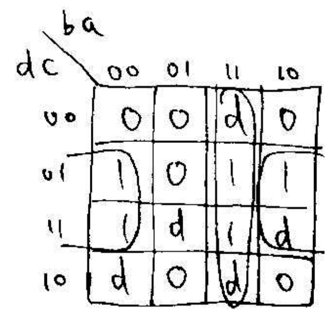


Since we can only have 4 product terms,
we must have some sharing:

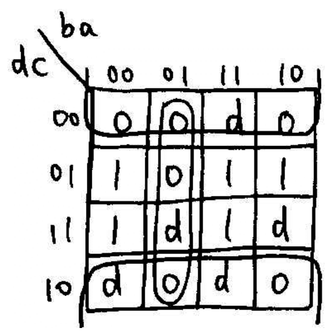
f = **a'c'd** + cd + **acd'**

g = **a'c'd** + b + **acd'**

11. (a) f = ba + ca'



(b) f = (b+a')c



12. (a) 7 2-LUTs are enough.

F1 = ab + a'b'
F2 = F1 + c
F3 = de
F4 = d'f
F5 = F3 + F4
F6 = F2.F5
F7 = F6.g

(b) 3 3-LUTs are enough.

F1 = ab + a'b' + c
F2 = de + d'f
F3 = F1.F2.g