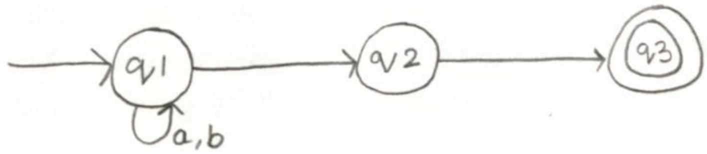
Jay Patel
CS 331
Homework #3
Professor Oliver

Q1.



Lets say group I = $\{Q_1\}$
For group II = $\{Q_1, Q_2\}$
For group III = $\{Q_1, Q_3\}$

Lets prove that, L (M) =R where R = $\Sigma^* ab$ & $\Sigma = \{a, b\}$
[Base Case]
$\varepsilon \rightarrow$ Group I because E $(\delta\ (Q_1, \in)) = \{Q_1\}$
|W| = 0 b $\rightarrow$ Group II because E $(\delta\ (Q_1, b))$ = $\{Q_1\}$
OR
|W| = 1 a $\rightarrow$ Group III because E $(\delta\ (Q_1, a))$ = $\{Q_1, Q_2\}$
So there is one computational path to Q_2

[Induction Step]
Case1: Suppose computation on w for k -1 has left it in group I. $\{Q_1\}$ to process "a"
E $(\delta\ (Q_1, a)) = \{Q_1, Q_2\}$ so then it terminate in group II $\{Q_1, Q_2\}$

Similarly we do for group II. $\{Q_1, Q_2\}$ For "a" then E $(\delta\ (II, a)) =$ E $(\delta\ (Q_1, a))$ or E $(\delta\ (Q_2, a))$ = $\{Q_1, Q_2\} \cup \emptyset$ that will then end up in group II as well

Similarly we do for group III. $\{Q_1, Q_3\}$ for "a" like other cases then we get
E $(\delta\ (III, a)) = E\ (\delta\ (Q_1, a)) \cup E\ (\delta\ (Q_3, a)) = \{Q_1, Q_3\} \cup \emptyset = \{Q_1, Q_2\}$ which also ends up in group II

Case2:
For group I:
E $(\delta\ (E(\delta(I, a)), b)) = E\ (\delta\ (II, b))$ = E $(\delta\ (q_1, b))\ U\ E\ (\delta\ (q_2, b)) = \{q_1\}\ U\ \{q_3\} =$
$\{q_1, q_3\} \rightarrow Group\ III$

For group II:

E $(\delta\ (E(\delta(II, a))b)) = E\ (\delta\ (E(\delta(q_1, a) U\ \delta(q_2, a))b)) = E\ (\delta(q_1, q_2) U, b))$

= E $(\delta(q_1, b) U \delta\ (q_2, b)) = E\ (\{q_1\}\ U\ \{q_3\}) = \{q_1, q_3\} \rightarrow$ Group III

For group III:

E $(\delta\ (q_3, \propto)) = \emptyset$

Taking from q_1 we can tell that,

E $(\delta\ (E(\delta(q_1, a)), b)) = E\ (\delta\ (II, b)) = \{q_1, q_3\} \rightarrow$ Group III

So, everything ends up in Group III

Q2.

a) $a^* b^*$
   Member: $\in, aaaabbbb, ab$
   Non-Member: ba, bbaa

b) a (ba)*b
   Member: ababab, ababa
   Non-Member: ba, babab

c) a*∪ b^*
   Member: $\in, aa, bb$
   Non-Member: ab, ba

d) (aaa)*
   Member: aaa, aaaaaa
   Non-Member: aa,a

e) $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$
   Member: aaaba, abbaa
   Non-Member: baa, aaa

f) aba ∪ bab
   Member: aba, bab
   Non-Member: ababab, bababa

g) (∈∪ a) b
   Member: ab, b
   Non- Member: bb,a

h) $(a \cup ba \cup bb)\Sigma^*$

Member: bababa, bbbb

Non-Member: $\in, b$

Q3.

Let DROP-OUT (A) be the language containing all strings that can be obtained by remaining one symbol.

Lets assume L (D) -> A for DFA

$M = (Q, \Sigma, \delta, Q_0, F)$

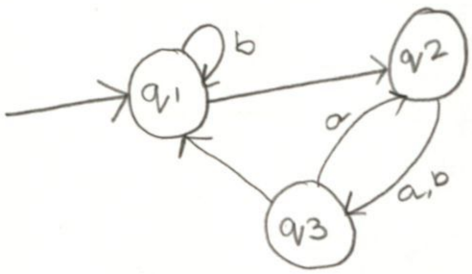
$\exists Q_1 \in Q | \delta (Q_0, x) = Q_i \wedge \exists Q_{i+1} | \delta(q_i, 4) = q_{i+1}$

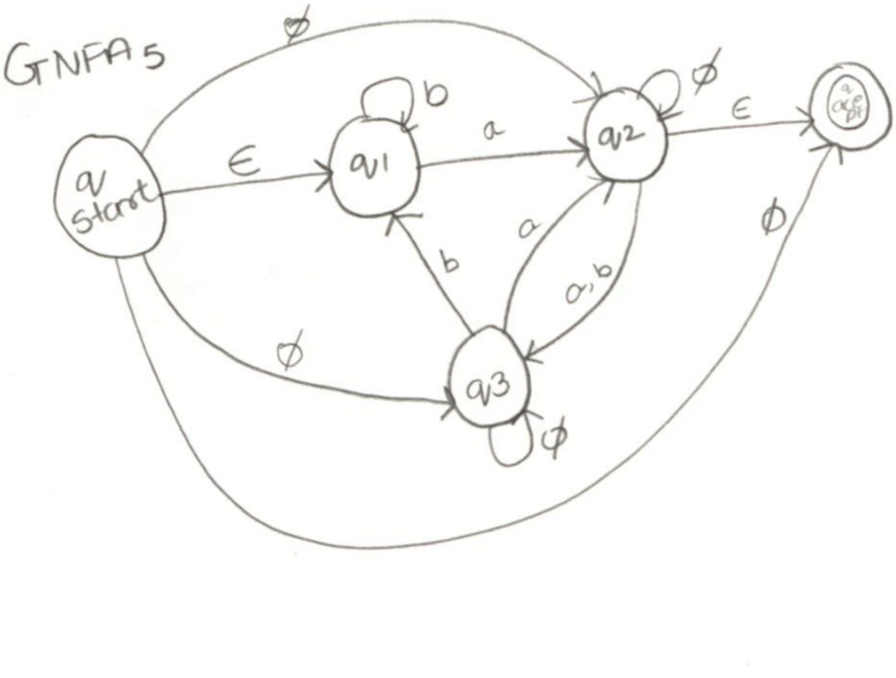
$Q' = Q \{ q_i, q_{i+1}\} + \{q_i\}$

$$\delta'(q_i, \propto) =: \begin{cases} \delta(q_i, \propto) \cup \delta(q_{i+1}, \propto), q_k = q_n \\ \delta(q_k, \propto), q_k \neq q_n \end{cases}$$

$N = (Q, \Sigma, \delta', q_0, F)$

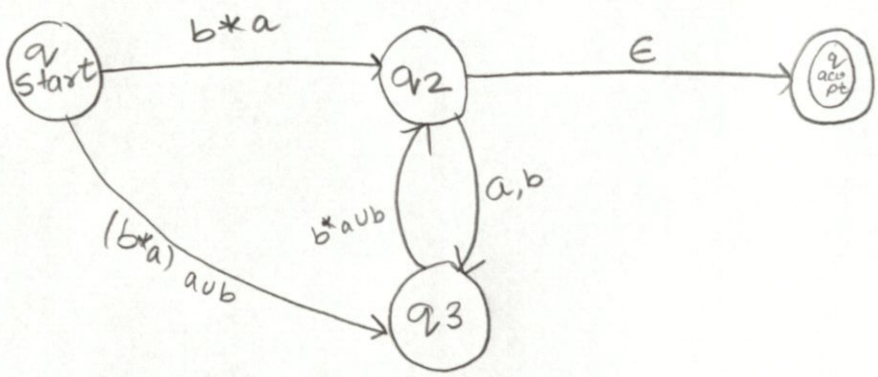
Q4. DFA



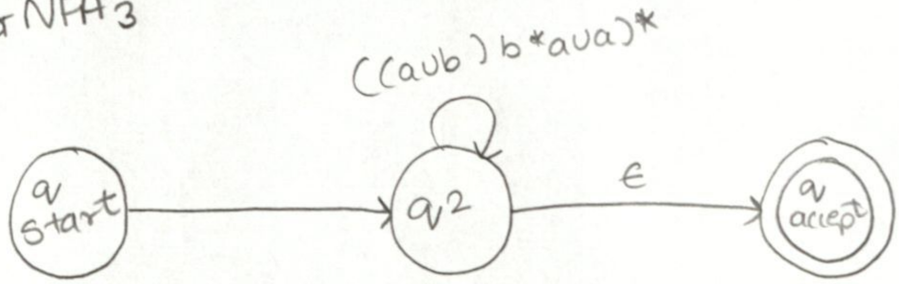


Q4. Continue...

GNFA$_4$



GNFA$_3$



(b*a) ∪ (((b*a) a∪b) b*a∪a)

GNFA$_2$