Homework: Lambda Calculus & Racket

Due-date: Feb 09 at 11:59pm; Submit online on Blackboard LS
(**Q1 solution in pdf format and Q2 solution as one Racket file**)

Homework must be individual's original work. Collaborations and discussions of any form with any students or other faculty members are not allowed. If you have any questions and/or concerns, post them on Piazza and/or ask our TA or me.

Learning Outcomes

- Application of knowledge of computing and mathematics
- Ability to understand the implications of mathematical formalisms in computer science
- Knowledge and application of Functional Programming
- Ability to follow requirement specification

Questions

1. Given

   0: $\lambda f.\lambda x.x$

   $succ$: $\lambda n.\lambda f.\lambda x.(f\ ((n\ f)\ x))$

   n: if n is a natural number then its semantics is the result of n applications of $succ$ on 0.

   $true$: $\lambda x.\lambda y.x$

   $false$: $\lambda x.\lambda y.y$

   $second$: $\lambda x.\lambda y.\lambda z.y$

   g: $\lambda n.((n\ second)\ false)$

   What is the result of

   (a) $(g\ n)$ when n is 0.

   (b) $(g\ n)$ when n results from some application $succ$ on 0.

   (c) What mathematical/logical operation is computed by g?

(5)

$$
(a) \qquad \begin{aligned}
(g\ 0) &= (\lambda n.((n\ second)\ false)\ 0) \\
&= ((0\ second)\ false) \\
&= ((\lambda f.\lambda x.x\ second)\ false) \\
&= (\lambda x.x\ false) \\
&= false
\end{aligned}
$$

$$
\begin{aligned}
(g\ n) &= (\lambda n.((n\ second)\ false)\ n) \\
&= ((n\ second)\ false) \\
(b) \qquad &= (((succ\ (\ldots(succ\ 0)\ldots))second)\ false) \\
&= (second\ (second\ (\ldots(second\ false)\ldots))) \\
&= true \text{ as } (second\ \texttt{anything}) = \lambda y.\lambda z.y
\end{aligned}
$$

(c) Representing a function g returns true when (non-negative integer) n is greater than 0.

2. Consider the following λ-expression:

$$Y :\ \lambda t.(\lambda x.(t\ (x\ x))\ \ \lambda x.(t\ (x\ x)))$$

Prove/disprove that $(Y\ t)$ after application of several β-reductions result in $(t\ (Y\ t))$. (5)

$$
\begin{aligned}
(Y\ t) &= (\underbrace{\lambda t.(\lambda x.(t\ (x\ x))\ \ \lambda x.(t\ (x\ x)))}\ \ \underbrace{\ t\ }) \\
&= (\underbrace{\lambda x.(t\ (x\ x))}\ \ \underbrace{\lambda x.(t\ (x\ x))}) \\
&= (t\ (\lambda x.(t\ (x\ x))\ \lambda x.(t\ (x\ x)))) \\
&= (t\ (Y\ t))
\end{aligned}
$$