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CS 331
HW 6
Professor Oliver

Q1.

A → BAB|AB|∈
B → 00|A|∈

First step,
S_0 → A
A → BAB|AB|∈
B → 00|A|∈

Second step,
S_0 → A
A → BAB|AB|BB|B
B → 00|A|∈

Third step,
A → BAB|AB|BA|BB|B
B → 00|A

Fourth step,
S_0 → BAB|AB|BA|BB|B
B → 00|BAB|AB|BA|BB|B

S_0 → BC|AB|BA|BB|00
C → AB

S_0 → BC|AB|BA|BB|00
D → 0
S_0 → BC|AB|BA|BB|DD
B → 00|BAB|AB|BA|BB|B

Hence it's in Chomsky's normal form now!

Q2.

Proof:
Let M=(Q, Σ, δ, s, F) be a DFA that accepts B, and let M'=(Q', Σ ', Γ, δ, s, F) be a PDA that accepts A. We construct a new PDA, N, with state set QxQ' in the following way, (Note that N accepts the languages A/B)

Let M be a DFA for B and M' be a Push Down Automata for A. So we construct a PDA for A/B by modifying M' so that it can run simultaneously. M's finite state control in addition to its own finite state control. So, only the original logic of M' is running. The changed machine, using no determinism, will start up M after w has read. So, the machine guesses a string x, and processes x by M and M'. It will only accept if both the machines are accepting simultaneously. In this case, we will know that M' has "guess" (term used by the professor in class) that has been reached the end of "w".
For δ' it has been defined as,

$$\delta' (q_a, \mathrm{j}, \mathrm{p}) = \begin{cases} \delta_a(q_a, j, p) & \text{if j } \epsilon\ \Sigma \\ \delta_a(q_a, \epsilon, p)\ U\ \{((q_a, s), \varepsilon\}, & \text{if j} = \epsilon \end{cases}$$

Q3.

We know that G is in CNF
So, q$\epsilon L(G)$ is n $\geq$1 for string q. We need to have 2n-1 steps for the derivation of string q
Using induction,

[Base Case]
n = 1 because the valid derivation would be starting symbol that goes to letter j if the size of the string was 1.
So, 2n-1 = 1

[Induction Hypothesis]
n = k. 2k -1 would be only a valid derivation if and only if the length is k $\geq n$ in CNF

[Induction Step]
So lets say if the length of the string is n = k + 1 and language in CNF

S → JP
J → *y
P → *z

For the starting state, |q| = yz where |y| > 0 and |z| > 0

Since we know that J has the same as length of |y| whereas for P it has the length of |z|

Therefore we can say that 2n-1 steps are necessary for the derivation of q ε L(G) in CNF