Jay Patel

Exam 1

Section 01

Math 227 - 1G

Jay Patel

Q1] Distance, = 2 Km

Velocity = 5m/s

acceleration = $a(t) = a_0 + .1 * t^{0.75}$

time = 45seconds

So,

$$\frac{d^2x}{dt^2} = a_0 + 0.1t^{0.5}$$

$$v(t)\ \frac{dx}{dt} = a_0 t + \frac{0.1\ t^{1.5}}{1.5} + v_0$$

velocity = 5m

$$v(t) = a_0 t + \frac{t^{1.5}}{15} + 5$$

$$x(t) = a_0 t^2 + \frac{t^{2.5}}{37.5} + 5t + x_0$$

$t = 0, x = 0$ so $x_0 = 0$

$t = 45$ $x = 2km = 2000m$

$$2000 = a_0(45)^2 + \frac{45^{2.5}}{37.5} + 5(45)$$

$a_0 = 0.7$

to find v

$$v(t) = a_0 t + \frac{t^{1.5}}{15} + 5$$

Jay Patel

Exam 1

$$V = 0.7(45) + \frac{45^{1.5}}{15} + 5$$

FA = $\boxed{V = 56.62 \text{ m/s}}$

Q2] $(2+x)^2 \frac{dy}{dx} = (2+x)^2$

$$\frac{dy}{dx} = \frac{(2+x)^2}{(2+x)^2}$$

$$\frac{dy}{dx} = 1$$

$$dy = dx$$

Integrate on both sides,

FA = $\boxed{y = x + C}$

Jay Patel

Exam 1

Q3] $(6x-y)dx = (4y-x)dy$

Given $(6x-y)dx = (4y-x)dy$

So $(6x-y)dx + (x-4y)dx = 0$ — (1)

Similar to $Adx + Bdy = 0$

$A = (6x-y)$, $B = x-4y$

we have, $\frac{\partial A}{\partial y} = -1$ and $\frac{\partial B}{\partial x} = 1$

So, $\frac{\partial A}{\partial y} \neq \frac{\partial B}{\partial x}$

Therefore, they aren't the same.

As far as equation 1 is concerned

$Ax + By = (6x-y)x + (x-4y)y$

$= 6x^2 - xy + xy - 4y^2$

$= 6x^2 - 4y^2 \neq 0$

multiplying 1 with $\frac{1}{6x^2-4y^2}$ we get,

$$\frac{6x-y}{6x^2-4y^2}\,dx + \frac{x-4y}{6x^2-4y^2}\,dy = 0 \quad \text{— (2)}$$

So $A_1 = \frac{6x-y}{6x^2-4y^2}$ $\qquad B_1 = \frac{x-4y}{6x^2-4y^2}$

Jay Patel

Exam 1

$$\frac{\partial A_1}{\partial y} = \frac{(6x^2-4y^2)(-1)-(6x-y)(-8y)}{(6x^2-4y^2)^2}$$

$$= \frac{-6x^2-4y^2+48xy}{(6x^2-4y^2)^2}$$

Similar for $\frac{\partial B_1}{\partial x} = \frac{(6x^2-4y^2)(1)-(x-4y)(12x)}{(6x^2-4y^2)^2}$

$$= \frac{-6x^2-4y^2+48xy}{(6x^2-4y^2)^2} \Rightarrow \text{Similar to } (2)$$

So,

$$\int_{y\,const} A_1\,dx + \int (\text{terms of } B_1)\,dy = c$$

$$= \int_{y\,cons} \frac{6x-y}{6x^2-4y^2}\,dx + \int_{(y\,con)} \frac{-y}{6x^2-4y^2}\,dx + 0 = c.$$

$$= \frac{1}{2}\int_{y\,const} \frac{12x}{6x^2-4y^2}\,dx - \frac{y}{6}\int_{y\,const} \frac{1}{x^2-\left(\frac{2y}{\sqrt{6}}\right)^2}\,dx = c$$

Jay Patel

Exam 1

$$= \frac{1}{2}\log|6x^2 - 4y^2| - \frac{y}{6}\ \frac{1}{2\left(\frac{2y}{\sqrt{6}}\right)}\log\left|\frac{x - (2y/\sqrt{6})}{x - \left(\frac{2y}{\sqrt{6}}\right)}\right|$$

$$= \frac{1}{2}\log|6x^2 - 4y^2| - \frac{1}{4\sqrt{6}}\log\left|\frac{\sqrt{6}x - 2y}{\sqrt{6}x - 2y}\right| = C$$

So, similar to (2) and answer

Q4] $Cy' - Ay = B + be^{-2ax}$

Divide by C on both sides

$$\frac{dy(x)}{dx} - \frac{Ay(x)}{c} = \frac{B + be^{-2ax}}{c}$$

Let $\mu(x) = \exp\left(\int -\frac{A}{c}\,dx\right) = e^{-Ax/c}$

multiply both the sides

$$e^{-Ax/c}\frac{dy(x)}{dx} - \left(\frac{Ae^{-Ax/c})y(x)}{c}\right.$$

$$= \frac{e^{-Ax/c}(B + be^{-2ax})}{c}$$

Substitute $\frac{-Ae^{-Ax/c}}{c} = \frac{d}{dx}\left(e^{-Ax/c}\right)$

Jay Patel

Exam 1

$$e^{-Ax/c} \frac{dy(x)}{dx} + \frac{d}{dx}\left(e^{-Ax/c}\right)$$

$$y(x) = \frac{e^{-Ax/c}(B + be^{-2ex})}{c}$$

Applying reverse product rule

$$\frac{d}{dx}\left(e^{-Ax/c}\, y(x)\right) = \frac{e^{-Ax/c}(B + be^{-2ax})}{c}$$

By integrating on both sides we get

$$\int \frac{d}{dx}\left(e^{-Ax/c}\, y(x)\right) dx = \int \frac{e^{-Ax/c}(B + be^{-2ax})}{c}\, dx$$

∴ sub the integrals

$$e^{-Ax/c}\, y(x) = -\frac{e^{\frac{-(A+2ac)x}{c}}\left(2aBce^{2ax} + A(b + Be^{2ax})\right)}{A(A+2ac)}$$

$+c_1$ (constant)

Dividing $u(x) = e^{-Ax/c}$ we get,

$$FA = \boxed{y(x) = e^{Ax/c}\left(-\frac{e^{\frac{-(A+2ac)x}{c}}\left(2aBce^{2ax} + A(b + Be^{2ax})\right)}{A(A+2ac)} + c_1\right)}$$