Two-Stage Evolutionary Algorithm Using Clustering for Multimodal Multi-objective Optimization with Imbalance Convergence and Diversity

Guoqing Li, Wanliang Wang(✉), and Yule Wang

College of Computer Science and Technology, Zhejiang University of Technology, Hangzhou 310023, China
zjutwwl@zjut.edu.cn

Abstract. This paper proposes a two-stage multimodal multi-objective evolutionary algorithm using clustering, termed as TS_MMOEAC. In the first-stage evolution of TS_MMOEAC, the convergence-penalized density (CPD) based k-means clustering is used to divide population into multiple subpopulations. Subsequently, a local archive mechanism is adopted to maintain diverse local Pareto optimal solutions in decision space. In the second-stage evolution, an identical k-means clustering based on distance among individuals in decision space is applied to form multiple subpopulations. In this case, the convergence performance of local Pareto optimal solutions is accelerated. Meanwhile, equivalent Pareto optimal solutions with the imbalance between convergence and diversity are located by a similar local archive method with a larger clearing radius. Experimental results validate the superior performance of TS_MMOEAC, and the proposed TS_MMOEAC is capable of finding equivalent Pareto optimal solutions with the imbalance between convergence and diversity.

Keywords: Two-stage evolutionary algorithm · Clustering · Multimodal multi-objective optimization · Imbalance

1 Introduction

Multiple conflicting objectives are optimized in multi-objective optimization problems (MOPs). In the last two decades, there has been an intense research activity in studying MOPs. Particularly, Pareto-based multi-objective evolutionary algorithms (MOEAs) [1], decomposition-based multi-objective evolutionary algorithms [2], and indicator-based multi-objective evolutionary algorithms [3] are remarkable. The aim of these mentioned MOEAs algorithms and their enhanced versions is to find complete Pareto front (PF) in objective space. However, a special situation for MOPs is overlooked in which multiple Pareto optimal solutions in decision space with the same objective values in objective space. This kind of multi-objective optimization problem is defined as multimodal multi-objective optimization problems (MMOPs) [4].

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Distinct from previous MOEAs, several proposed MOEAs in recent years have begun to focus on the diversity of Pareto optimal solutions in decision space. For instance, an improving strength Pareto-based evolutionary algorithm (SPEA2) [5], an extended Omni-optimizer procedure (Omni-optimizer) [6], PCA-assisted multi-objective evolutionary algorithm [7]. However, these mentioned MOEAs are not capable of locating equivalent Pareto optimal solutions in decision space, i.e. complete Pareto set (PS). Recently, inspired by popular multi-objective evolutionary algorithms, several multimodal multi-objective evolutionary algorithms (MMEAs) are proposed for solving MMOPs. A framework of decomposition-based MMEAs [8], an indicator-based MMEA [9], and Pareto-based MMEAs [10–12] are proposed one after another. Of these, Pareto-based MMEAs are the most prominent. Niching technologies play an important role in Pareto-based MMEAs. For instance, a double-niched evolutionary algorithm (DNEA) [13], an index-ring topology particle swarm optimization algorithms (MO_Ring_PSO_SCD) [10]. These popular Pareto-based MMEAs can find equivalent Pareto optimal solutions in decision space, where the convergence and diversity of equivalent solutions in decision space are balanced. In other words, the complexity of locating equivalent Pareto optimal solutions in decision space is identical.

However, most popular Pareto-based MMEAs using Pareto-based ranking are ineffective for solving multimodal multi-objective problems with the imbalance between convergence and diversity (MMOP-ICD) in decision space. The main reason is that these solutions, which approximate one or multiple equivalent Pareto optimal solutions, are dominated by an equivalent Pareto optimal solution in evolutionary process. In this case, all equivalent Pareto optimal solutions with the imbalance between convergence and diversity are difficult to be found. To address this issue, a two-stage evolutionary algorithm using clustering (TS_MMOEAC) is developed in this paper for handling MMOP-ICD. The main contributions of this paper are as follows:

1) The proposed TS_MMOEAC is composed of the first-stage evolution and the second-stage evolution. The main purpose of the first-stage evolution using clustering is to find local optimal individuals that gradually approximate equivalent PSs with the imbalance convergence and diversity during evolutionary process.
2) To improve the convergence performance of all individuals, a unique clustering method is involved in second-stage evolution. Particularly, it accelerates the population convergence to one or several equivalent PSs with the imbalance convergence and diversity.

The remainder of this paper is structured as follows: The definition of MMOPs and MMOP-ICD, and popular MMEAs are presented in Sect. 2. The main framework of the proposed TS_MMOEAC is described in Sect. 3. Section 4 validates the superior performance of TS_MMOEAC in comparison to five competing algorithms. Section 5 concludes this paper.

2 Problem Definition and Literature Review

2.1 Problem Definition of MMOP-ICD

There are multiple equivalent Pareto optimal solutions in decision space with the same objective value in objective space for MOPs, and this kind of MOPs is defined as multimodal multi-objective optimization problems (MMOPs). A standard definition of MMOPs is given by Tanabe as follows [14]:

Definition 1. An MMOP is involved in locating all solutions which are equivalent to Pareto optimal solutions.

Definition 2. If $\|f(x_1) - f(x_2)\| < \delta$, two solutions x_1 and x_2 are considered to be equivalent.
where $\|f(x_1) - f(x_2)\|$ is the arbitrary norm between $f(x_1)$ and $f(x_2)$, and δ is a threshold defined by the decision-maker.

In MMOPs, equivalent Pareto optimal solutions have identical convergence and diversity in decision space. Additionally, the complexity of locating equivalent Pareto optimal solutions is uniform. Given the unique scenario that equivalent Pareto optimal solutions do not have identical search complexity. Inspired by imbalanced multi-objective optimization problems [15], the MMOPs with imbalance convergence and diversity (MMOP-ICD) is proposed by Liu [16] and defined as follows:

Definition 3. An MMOP is defined as MMOP-ICD, if one or both of the following conditions are met [16]:

1) The solutions in objective space, which close to one equivalent Pareto optimal solution, are likely to dominate other solutions that approximates another equivalent Pareto optimal solution.
2) The complexity of finding an equivalent Pareto optimal solution in objective space is smaller than that of searching for another equivalent Pareto optimal solution.

2.2 Literature Review

Recently, MMOPs have attracted the attention of researchers. Several MMEAs have been developed for MMOPs. Liang [4] proposed the multimodal multi-objective problems and introduced a niching-based fast and elitist multi-objective genetic algorithms (DN-NSGAII), making a tremendous step forward for MMOPs. An index-based ring topology multimodal multi-objective particle swarm optimization algorithm (MO_Ring_PSO_SCD) is suggested by Yue [10]. In [10], the left and right index-based neighbors of each particle form a ring topology. And local version particle swarm optimization algorithm is adopted to locate equivalent Pareto optimal solutions in decision space. Additionally, several MMOPs and performance metrics are presented in [10, 11]. Inspired by MO_Ring_PSO_SCD, a clustering-based ring topology multimodal multi-objective particle swarm optimization algorithm (MMO-CLRPSO) is proposed by Zhang [12]. In MMO-CLRPSO, the population is divided into multiple subpopulations, and a ring structure is formed among subpopulations. Global version PSO is used within each subpopulation to locate equivalent Pareto optimal solutions in decision space, and local version PSO is applied in all subpopulations to improve the diversity of subpopulations. Subsequently, Lin presented a dual-clustering-based evolutionary algorithm (MMOEA/DC) [17] for MMOPs. In MMOEA/DC, one clustering method is adopted to decision space for maintaining the local PS, and the other clustering method is used to objective space to locate diverse and equivalent Pareto optimal solutions. Additionally, a clearing-based multimodal multi-objective evolutionary optimization using a layer-to-layer strategy [18] is designed for solving MMOPs with local PS.

Several decomposition-based and indicator-based multimodal multi-objective algorithms have also been presented one after another. Tanabe developed a decomposition-based MMEA, called MOEA/D-AD [19], and then proposed a decomposition-based framework to handle MMOPs [8]. Additionally, a niching indicator-based multimodal multi-objective evolutionary algorithm (NIMMO) is also developed by Tanabe [9]. Subsequently, a standard MMOPs benchmark with imbalance convergence and diversity in decision space (MMOP-ICD) is designed [16]. And an evolutionary multimodal multi-objective algorithm via a convergence-penalized density method (CPDEA) is developed for solving MMOP-ICD.

These mentioned MMEAs are capable of finding multiple equivalent PSs with the same diversity and convergence, but most of them cannot find all PSs when dealing with MMOP-ICD since that the diversity and convergence of all equivalent PSs are unbalanced. Therefore, the above-mentioned MMEAs are rarely capable of handling MMOP-ICD. To address this issue, a two-stage multimodal multi-objective evolutionary algorithm using clustering (TS_MMOEAC) is proposed. In the proposed TS_MMOEAC, a two-stage evolutionary strategy is developed to find equivalent Pareto optimal solutions in decision space. The population is divided into multiple subpopulations via the CPD-based k-means clustering method in the first-stage evolution for locating Pareto optimal solutions with high quality, and these high-quality solutions will converge to multiple equivalent PSs with the imbalance in the second-stage evolution. Sustainably, the population is classified into several subpopulations using distance-based k-means clustering for accelerating subpopulations convergence and finding equivalent Pareto optimal solutions with the imbalance in the second-stage evolution. The main framework of TS_MMOEAC is described in Sect. 3.

3 Proposed TS_MMOEAC

3.1 The Main Framework of TS_MMOEAC

In Algorithm 1, the population P with N individuals is initialized in line 1. If the used fitness evaluation number Fes is less than half of the maximum fitness evaluation numbers $MaxFes$, the first-stage evolution (Algorithm 2) is performed in line 3. Otherwise, the second-stage evolution (Algorithm 4) is implemented in line 5. Of these, the local archive mechanism (Algorithm 3) is implemented in both the first-stage and the second-stage evolution. Finally, output the population P. The main framework of TS_MMOEAC is shown in Fig. 1.

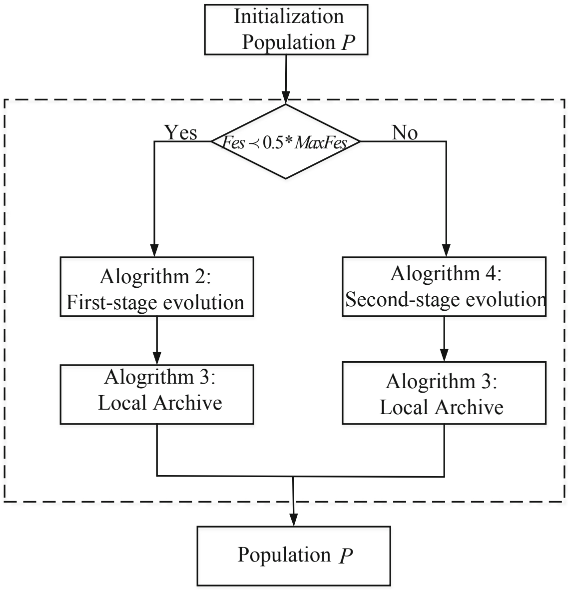

Fig. 1. The main framework of TS_MMOEAC.

Algorithm 1. TS_MMOEAC

Input: N (population size), $MaxFes$ (the maximum fitness evaluation numbers)
Output: P
Initialize the population P with N individuals;
while $Fes < MaxFes$
    $P \leftarrow$ The first-stage evolution; //Algorithm 2
else
    $P \leftarrow$ The second-stage evolution; //Algorithm 4
end
return P ;

3.2 First-Stage Evolution Strategy

In Algorithm 2, the CPD value of each individual x in the population P is calculated according to the convergence-penalized density method [16] at first. Then, a popular clustering method, k-means clustering, is used to divide the CPD values set into Num clusters $f_{CPD} = \{cluster_1, cluster_2, ..., cluster_i, ..., cluster_{Num}\}$. Then, the population P is divided into Num subpopulations, $P = \{subpop_1, subpop_2, ..., subpop_i, ...subpop_{Num}\}$, according to Num clusters f_{CPD}. The roulette selection operation is utilized to select parents in the first-stage evolution. For each individual $subpop_{i,j}$ in the subpopulation $subpop_i$, the CPD-based roulette selection method is applied to select the parents of $subpop_{i,j}$, and then a reproduction operator is used to generate the offspring $offspring_{i,j}$ of $subpop_{i,j}$. Subsequently, recombining all individuals in each subpopulation to produce the offspring subpopulations $O = \{offspring_1, offspring_2, ..., offspring_i, ..., offspring_{Num}\}$. Finally, the local archive mechanism is used to maintain the outstanding and diverse local optimal individual between the population P and the offspring O.

Algorithm 2 The first-stage evolution

Input P
Output P

1. Calculate the $f_{CPD}(x)$ value for each individual x in P;
2. $f_{CPD} = \{cluster_1, cluster_2, ..., cluster_i, ..., cluster_{Num}\}$ ← The f_{CPD} is divided into Num clusters using the k-means clustering method;
3. $P = \{subpop_1, subpop_2, ..., subpop_i, ...subpop_{Num}\}$ ← The population P is divided into Num subpopulation according to f_{CPD};
4. **for** each subpopulation $subpop_i$
5. $offspring_i$ ← Selecting parents and reproduction $subpop_i$ using roulette selection operation;
6. **end**
7. $O = \{offspring_1, offspring_2, ..., offspring_i, ..., offspring_{Num}\}$;
8. $P = LocalArchive(\{P, O\})$;
9. **return** P

3.3 Local Archive Strategy

For MMOP-ICD, one equivalent Pareto optimal solution is easily located. However, other equivalent solutions are difficult to find since there is an imbalanced diversity and convergence among several unique equivalent solutions. The main reason is that popular MMEAs using dominance relations choose superior solutions and remove poor (local) solutions, but these poor solutions have a greater potential to become equivalent PS. Therefore, popular MMEAs are not capable of locating all equivalent PSs in handling MMOP-ICD. To tackle this issue, inspired by [18, 20], a local archive strategy is presented to maintain diverse local optimal individuals in the first-stage evolution. And these individuals in the local archives have a greater tendency to become equivalent Pareto optimal solutions with imbalanced diversity and convergence in comparison to other equivalent Pareto optimal solutions. The local archive mechanism is described as follows.

First, local Pareto optimal individuals are found in the local archive mechanism. In this step, the mean value of the sum of the distances between all individuals and their Ns nearest neighbors is calculated and denoted as MS. To remove poor local individuals from the population P and the offspring O, the parameter $\theta_r = MS/D$ is viewed as the clearing radius, where D is the decision variable number. For each individual x, any individual x' is within the radius θ_r of the individual x and is dominated by x, and x' is removed. These superior local Pareto optimal solutions are maintained by performing the above operation for each individual x. These poor individuals are removed from the set of the population P and the offspring O, and survived in the archive A. Given this scenario that the number of superior local Pareto optimal individuals is less than the population size N, it is essential to select several superior individuals from A to P. Distinct from the first step, the number of the nearest neighbors is set to $Ns = 0.25 * N_A$, where N_A is the size of the archive A.

Next, the population P is sorted based on the Pareto-dominance ranking, and the population $P = \{F_1, F_2, ..., F_m\}$ is divided into m layers. A sharing method is applied to obtain diverse global and local solutions in both decision space and objective space. For each layer F_i, $i = 1, 2, ..., m$, if the individual numbers in the F_i layer are more than N_F, where $N_F = 0.5 * N$ is the maximum individual numbers in each layer, several individuals with poor sharing values in the F_i layer are removed. The sharing value of each individual $F_{i,q}$, $i = 1, 2, .., M$, $q = 1, 2, ..., |F_i|$ in F_i is calculated by Eq. (1).

$$f_{DS}(F_{i,q}) = \sum_{k=1}^{K} sh_{dec}(F_{i,q}, F_{i,k}) + sh_{obj}(F_{i,q}, F_{i,k}) \tag{1}$$

where $F_{i,k}$ is the k-th nearest neighbors of $F_{i,q}$, $sh_{dec}(F_{i,q}, F_{i,k})$ and $sh_{obj}(F_{i,q}, F_{i,k})$ are the sharing values of the individual $F_{i,q}$ in decision space and objective space, respectively. The $sh_{dec}(F_{i,q}, F_{i,k})$ and $sh_{obj}(F_{i,q}, F_{i,k})$ are calculated as follows [20].

$$\begin{aligned} sh_{dec}(F_{i,q}, F_{i,k}) &= \max\{0, 1 - d_{dec}(q, k)/\sigma_{dec}\}, \\ sh_{obj}(F_{i,q}, F_{i,k}) &= \max\{0, 1 - d_{obj}(q, k)/\sigma_{obj}\}. \end{aligned} \tag{2}$$

where $d_{dec}(q, k)$ and $d_{obj}(q, k)$ are the Euclidean distance between $F_{i,q}$ and $F_{i,k}$ in both decision space and objective space, respectively. σ_{dec} and σ_{obj} are the niche radius in both decision space and objective space, respectively.

f_{DS} evaluates the density of the individuals in F_i between decision space and objective space, simultaneously. The larger f_{DS}, the worse the diversity. Removing the individual with the largest f_{DS} value from F_i to maintain the diverse individuals in both decision space and objective space. In the sharing method, the individual number in each layer $F_i, i = 1, 2, ..., q$ is no more than N_F. Add each layer F_i to P' until the number of P' is greater than N. The pseudo-code for the local archive mechanism is presented in Algorithm 3.

Algorithm 3 *LocalArchive*(P)

Input P
Output P

$MS \leftarrow$ the mean value of the sum of the distances between all individuals and its Ns nearest neighbors is calculated;
$\theta_r = MS / D$, $A = \varnothing$, $P' = \varnothing$;
for each individual x in P
if anyone individual x' in P is within the radius θ_r of x and is dominated by x
$P = \{P \setminus x'\}, A = \{A \cup x'\}$;
end
end
if $|P| < N$
$P \leftarrow$ select several superior individuals from A to P;
else
$P = \{F_1, F_2, \ldots, F_q\} \leftarrow sort(P)$;
for each F_i in P
$f_{DS} \leftarrow$ The sharing value of each individual $F_{i,q}$ in F_i is calculated;
while $|F_i| > N_F$
Remove the individual with the largest f_{DS} value from F_i;
end
$P' = \{P' \cup F_i\}$;
if $|P'| \geq N$
break;
end
end
end
$P = P'$;
return P;

3.4 Second-Stage Evolution Strategy

When the computing resources used by the first-stage evolution are greater than half of the total computing resources ($0.5 * MaxFes$), the second-stage evolution of the proposed TS-MMOEAC is performed and shown in Algorithm 4. The second-stage evolution is similar to the first stage, and there are three main distinctions. The first is that the clustering method in the second-stage evolution is slightly different. Instead of the CPD-based k-means clustering, the distance-based k-means clustering is used to divide the population into multiple subpopulations. The individuals nearby are clustered into a subpopulation in decision space, and the distance-based k-means clustering used in the second-stage evolution accelerates the convergence of subpopulations. The second distinction is that the tournament selection method for each subpopulation is used to choose the parents based on f_{DS} values in the second-stage evolution. The main reason is that each subpopulation is more likely to approach these solutions with the smallest f_{DS} values and locate diverse Pareto optimal solutions. The third difference is that the θ_r value is larger for dominating poor local Pareto optimal solutions and maintaining global Pareto optimal solutions in P, where θ_r is equal to MS in the local archive in Algorithm 3. The main purpose is to preserve elite local optimal solutions and move them towards equivalent Pareto optimal solutions with imbalanced diversity and convergence.

Algorithm 4 The second-stage evolution

Input P
Output P

1. $P = \{subpop_1, subpop_2, ..., subpop_i, ...subpop_{Num}\} \leftarrow$ The population P is divided into Num subpopulations according to the Euclidean distance.
2. **for** each subpopulation $subpop_i$
3. $\quad offspring_i \leftarrow$ Selecting parents and reproduction $subpop_i$ using tournament selection operation;
4. **end**
5. $O = \{offspring_1, offspring_2, ..., offspring_i, ..., offspring_{Num}\}$;
6. $P = \{P, O\}, P = LocalArchive(P)$;
7. **return** P

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4 Experimental Results and Discussion

4.1 Experimental Setup

To validate the performance of the proposed TS_MMOEAC for solving MMOP-ICD, several state-of-the-art algorithms are selected as the competing algorithms of TS_MMOEAC, including DNEA [13], MO_Ring_PSO_SCD [10], TriMOEATAR [21], MMOEA/DC [17], and CPDEA [16]. Among them, MO_Ring_PSO_SCD, TriMOEATAR, MMOEA/DC, and CPDEA are popular MMEAs that are published in IEEE Transactions on Evolutionary Computation in recent years. The proposed TS_MMOEAC and its competing algorithms are implemented in the standard MMOP-ICD benchmark. The MMOP-ICD benchmark consists of 4 types of imbalanced distance minimization problems, and it involves 12 problems, including IDMP-M2-T1, IDMP-M2-T2, IDMP-M2-T3, IDMP-M2-T4, IDMP-M3-T1, IDMP-M3-T2, IDMP-M3-T3, IDMP-M3-T4, IDMP-M4-T1, IDMP-M4-T2, IDMP-M4-T3, and IDMP-M4-T4. Please refer to the literature [16] for further information on MMOP-ICD.

Three performance metrics, IGDX [10], IGDF [11], and IGDM [16], are applied to test the superior performance of TS_MMOEAC. The IGDX implies the distance between obtained PS and true PS in decision space. IGDF measures the distance between obtained PF and true PF in objective space. IGDM indicates the diversity and the convergence performance of obtained Pareto optimal solutions in both decision space and objective space, simultaneously. The smaller IGDX, IGDF, and IGDM are desired.

4.2 Experimental Result

In this experiment, the population size and the fitness evaluations are set to $30 * 2^{D-1}$ and $9000 * 2^{D-1}$, respectively, where D is the decision variable number. The parameters involved in all competing algorithms are maintained with the original papers. In the proposed TS_MMOEAC, these parameters, *Num*, *Ns*, and K, are involved. *Ns* and K is set to 50 and 5 according to [18, 20], respectively. Additionally, the parameter *Num* is discussed in Sect. 4.3. To guarantee a fair experimental comparison between TS_MMOEAC and its competing algorithms, the experiment is performed 30 times independently. The IGDX, IGDF, and IGDM performance metrics of TS_MMOEAC and its compared algorithms are present in Tables 1, 2, and 3, respectively. Additionally, the Wilcoxon rank-sum test with a significant level $\alpha = 0.05$ is used to show a significant difference between the proposed TS_MMOEAC and its competing algorithms, where the symbol "+", "−", and "∼" indicates that the proposed TS_MMOEAC is significantly worse than, better than, and similar to its competing algorithms.

It is observed from Table 1 that TS_MMOEAC performs the best result on 10 out of 12 test problems, including IDMP-M2-T1, IDMP-M2-T2, IDMP-M2-T3, IDMP-M2-T4, IDMP-M3-T1, IDMP-M3-T2, IDMP-M3-T3, IDMP-M4-T2, IDMP-M4-T3, and IDMP-M4-T4. And TS_MMOEAC is superior to DNEA, MO_Ring_PSO_SCD, and TriMOEATAR in all problems. The IGDX values of TS_MMOEAC are also significantly superior to MMOEA/DC in the benchmark problems except for IDMP-M4-T1. TS_MMOEAC performs slightly worse than CPDEA in IDMP-M3-T4 and IDMP-M4-T1, but the performance of TS_MMOEAC is superior to CPDEA. The Pareto optimal solutions obtained by DNEA, MMOEA/DC, CPDEA, and TS_MMOEAC on two problems in decision space are clearly shown in Fig. 2 and Fig. 3, respectively. It is clear that DNEA and MMOEA/DC only find one or multiple Pareto optimal solutions, but cannot locate all equivalent solutions in the decision space. CPDEA is similar to TS_MMOEAC in that they are capable of finding all equivalent solutions in decision space. Therefore, we can infer that the proposed TS_MMOEAC is superior to its competing algorithms in decision space, and it is capable of locating equivalent solutions with imbalance convergence and diversity in decision space.

It can be seen from Table 2 that the IGDF performance of DNEA outperforms the proposed TS_MMOEAC and other competing algorithms in objective space. The main reason is that DNEA is originated from MOEAs which use a niching technique to search for Pareto optimal solutions in both decision space and objective space. But the IGDF value of TS_MMOEAC is superior to MO_Ring_PSO_SCD and TriMOEATAR in all problems and outperforms MMMOEA/DC and CPDEA in most cases. It is implied that the convergence of Pareto optimal solutions obtained by TS_MMOEAC in objective space is better than other competing algorithms except for DNEA.

The performance of the proposed TS_MMOEAC in both objective space and decision space is evaluated by IGDM, simultaneously. Since the IGDF values of TS_MMOEAC

Table 1. The IGDX performance of TS_MMOEAC and its competing algorithms

Problem	DNEA	MO_Ring_ PSO_SCD	Tri MOEATAR	MMOEA/DC	CPDEA	TS_ MMOEAC
IDMP-M2-T1	6.06E−01 (2.05E−01)−	2.21E−01 (3.03E−01)−	6.30E−01 (1.66E−01)−	6.92E−02 (2.05E−01)−	2.02E−03 (1.04E−04)−	**1.60E−03 (1.12E−04)**
IDMP-M2-T2	5.61E−01 (2.55E−01)−	6.74E−01 (5.15E−04)−	6.52E−01 (1.18E−01)−	9.17E−02 (2.32E−01)−	2.09E−03 (1.11E−04)−	**1.70E−03 (7.88E−05)**
IDMP-M2-T3	3.42E−01 (3.40E−01)−	1.45E−01 (2.02E−01)−	5.10E−01 (2.79E−01)−	1.14E−01 (2.55E−01)−	2.94E−03 (3.65E−04)−	**2.04E−03 (6.41E−05)**
IDMP-M2-T4	6.51E−01 (1.23E−01)−	1.30E+00 (3.12E−03)−	6.73E−01 (6.09E−05)−	4.26E−01 (3.28E−01)−	1.96E−03 (8.26E−05)−	**1.61E−03 (1.25E−04)**
IDMP-M3-T1	6.65E−01 (3.03E−01)−	1.26E−01 (1.87E−01)−	8.61E−01 (2.04E−01)−	3.75E−02 (7.40E−02)−	1.13E−02 (1.92E−04)−	**1.07E−02 (2.82E−04)**
IDMP-M3-T2	5.98E−01 (3.01E−01)−	8.34E−02 (1.05E−01)−	8.09E−01 (2.29E−01)−	5.36E−02 (9.19E−02)−	1.13E−02 (1.63E−04)−	**1.07E−02 (5.36E−04)**
IDMP-M3-T3	5.17E−01 (2.60E−01)−	1.67E−02 (1.93E−03)−	6.63E−01 (2.12E−01)−	1.04E−01 (1.54E−01)−	1.22E−02 (2.31E−04)−	**1.06E−02 (1.80E−04)**
IDMP-M3-T4	7.55E−01 (2.76E−01)−	5.29E−02 (8.66E−02)−	8.75E−01 (1.99E−01)−	2.64E−01 (2.49E−01)−	**1.12E−02 (1.59E−04)+**	3.50E−02 (7.37E−02)
IDMP-M4-T1	1.17E+00 (3.88E−02)−	5.98E−01 (3.90E−01)−	1.19E+00 (8.38E−04)−	1.54E−01 (2.11E−01)+	**8.64E−03 (8.03E−05)+**	4.32E−01 (3.58E−01)
IDMP-M4-T2	1.03E+00 (2.03E−01)−	1.26E−01 (1.31E−01)−	1.06E+00 (1.68E−01)−	1.55E−01 (2.31E−01)−	8.71E−03 (1.40E−04)−	**8.07E−03 (3.46E−04)**
IDMP-M4-T3	8.28E−01 (3.25E−01)−	1.51E−02 (1.33E−03)−	9.16E−01 (2.26E−01)−	6.67E−02 (1.40E−01)−	1.85E−02 (4.86E−02)−	**8.52E−03 (1.02E−04)**
IDMP-M4-T4	1.06E+00 (2.34E−01)−	1.74E−02 (3.42E−03)−	1.04E+00 (2.29E−01)−	1.82E−01 (1.71E−01)−	3.53E−02 (8.11E−02)−	**1.72E−02 (4.86E−02)**
±/~	0/12/0	0/12/0	0/12/0	1/11/0	2/10/0	

Table 2. The IGDF performance of TS_MMOEAC and its competing algorithms

Problem	DNEA	MO_Ring _PSO_SCD	Tri MOEATAR	MMOEA/DC	CPDEA	TS_ MMOEAC
IDMP-M2-T1	**1.23e−3 (1.55e−5)+**	2.55E−03 (2.19E−04)−	1.76e−3 (1.04e−4) −	1.59e−3 (5.26e−4)−	1.55e−3 (9.70e−5) −	1.33e−3 (6.78e−5)
IDMP-M2-T2	**1.24e−3 (1.42e−5)+**	2.15E−03 (2.28E−04)−	1.78e−3 (1.69e−4) −	1.42e−3 (7.23e−5)−	1.47e−3 (4.59e−5)−	1.28e−3 (5.37e−5)
IDMP-M2-T3	**1.29e−3 (5.24e−5)~**	2.20E−03 (2.17E−04)−	1.14e−2 (1.98e−2) −	1.62e−3 (9.28e−5)−	1.56e−3 (5.33e−5)−	1.30e−3 (3.15e−5)
IDMP-M2-T4	**1.23e-3 (1.16e-5)+**	2.12E+02 (5.13E−02)−	1.74e−3 (1.75e−4)−	1.47e−3 (8.68e−5)−	1.51e−3 (6.07e−5)−	1.33e−3 (1.10e−4)
IDMP-M3-T1	**6.15e−3 (2.18e−4)+**	1.00E−02 (5.63E−04)−	1.31e−2 (6.65e−4)−	8.17e−3 (4.44e−4)−	6.75e−3 (1.54e−4)−	6.61e−3 (2.42e−4)
IDMP-M3-T2	**6.14e−3 (2.00e−4)+**	8.34E−03 (2.82E−04)−	1.30e−2 (7.49e−4)−	7.82e−3 (3.04e−4)−	6.70e−3 (1.70e−4)~	6.66e−3 (2.23e−4)

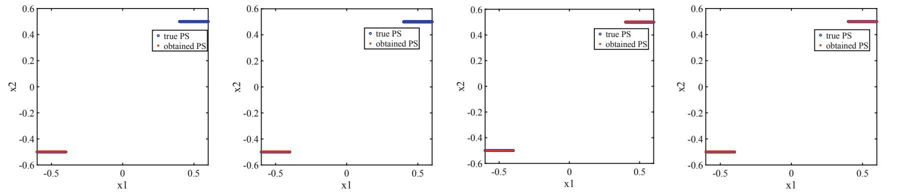
(*continued*)

Table 2. (*continued*)

Problem	DNEA	MO_Ring _PSO_SCD	Tri MOEATAR	MMOEA/DC	CPDEA	TS_ MMOEAC
IDMP-M3-T3	**6.35e−3 (2.58e−4)+**	8.76E−03 (4.12E−04)−	1.28e−2 (8.02e−4)−	8.58e−3 (5.07e−4)−	7.21e−3 (1.95e−4)−	6.79e−3 (1.87e−4)
IDMP-M3-T4	**6.13e−3 (2.21e−4)+**	8.41E−03 (3.35E−04)−	1.31e−2 (7.28e−4)−	7.53e−3 (7.85e−4)−	6.76e−3 (1.63e−4)−	6.60e−3 (2.06e−4)
IDMP-M4-T1	**5.78e−3 (2.02e−4)+**	1.71E−02 (8.22E−04)−	2.64e−2 (1.44e−3)−	7.47e−3 (3.55e−4)−	6.62e−3 (2.14e−4)−	6.20e−3 (4.17e−4)
IDMP-M4-T2	**5.79e−3 (2.13e−4)+**	1.07E−02 (4.12E−04)−	2.62e−2 (1.51e−3)−	6.99e−3 (3.13e−4)−	6.31e−3 (2.33e−4)~	6.28e−3 (2.52e−4)
IDMP-M4-T3	**6.00e−3 (2.96e−4)+**	1.08E−02 (3.70E−04)−	2.74e−2 (7.57e−3)−	8.20e−3 (3.15e−4)−	6.66e−3 (1.73e−4)−	6.47e−3 (2.02e−4)
IDMP-M4-T4	**5.86e−3 (2.29e−4)+**	1.08E−02 (4.82E−04)−	2.62e−2 (1.32e−3)−	6.96e−3 (4.07e−4)−	6.23e−3 (2.49e−4)~	6.25e−3 (2.40e−4)
±/~	11/0/1	0/12/0	0/12/0	0/12/0	0/9/3	

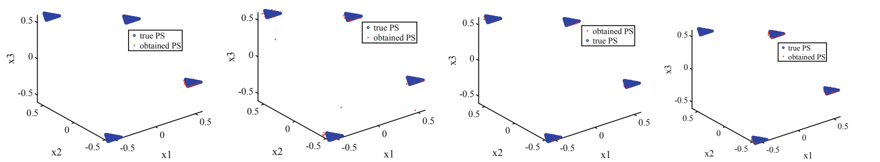
Table 3. The IGDM performance of TS_MMOEAC and its competing algorithms

Problem	DNEA	MO_Ring_ PSO_SCD	Tri MOEATAR	MMOEA /DC	CPDEA	TS_ MMOEAC
IDMP-M2-T1	4.54E−01 (1.50E−01)−	2.33E−01 (1.97E−01)−	4.80E−01 (9.14E−02)−	6.40E−02 (1.50E−01)−	1.44E−02 (7.82E−04)−	**1.14E−02 (6.13E−04)**
IDMP-M2-T2	4.21E−01 (1.86E−01)−	5.05E−01 (5.69E−04)−	4.94E−01 (5.35E−02)−	7.96E−02 (1.69E−01)−	1.42E−02 (6.68E−04)−	**1.18E−02 (5.49E−04)**
IDMP-M2-T3	2.77E−01 (2.32E−01)−	1.75E−01 (1.42E−01)−	4.69E−01 (9.21E−02)−	9.70E−02 (1.85E−01)−	1.75E−02 (2.30E−03)−	**1.21E−02 (3.38E−04)**
IDMP-M2-T4	4.86E−01 (8.94E−02)−	1.00E+00 (0.00E+00)−	5.04E−01 (4.37E−04)−	3.24E−01 (2.40E−01)−	1.39E−02 (6.23E−04)−	**1.16E−02 (5.64E−04)**
IDMP-M3-T1	5.59E−01 (1.78E−01)−	2.45E−01 (1.23E−01)−	7.02E−01 (9.87E−02)−	1.18E−01 (6.53E−02)−	8.08E−02 (1.59E−03)−	**7.66E−02 (1.81E−03)**
IDMP-M3-T2	5.20E−01 (1.95E−01)−	2.09E−01 (8.08E−02)−	6.83E−01 (1.29E−01)−	1.30E−01 (8.13E−02)−	8.02E−02 (1.24E−03)−	**7.64E−02 (2.88E−03)**
IDMP-M3-T3	4.82E−01 (1.61E−01)−	1.16E−01 (1.49E−02)−	6.31E−01 (1.10E−01)−	1.78E−01 (1.32E−01)−	8.40E−02 (1.66E−03)−	**7.35E−02 (1.24E−03)**
IDMP-M3-T4	6.20E−01 (1.77E−01)−	1.73E−01 (7.37E−02)−	7.13E−01 (1.03E−01)−	2.96E−01 (1.83E−01)−	**8.03E−02 (1.05E−03)+**	9.95E−02 (6.78E−02)
IDMP-M4-T1	7.57E−01 (5.31E−05)−	5.62E−01 (1.52E−01)−	7.83E−01 (1.82E−03)−	1.86E−01 (1.53E−01)~	**6.32E−02 (7.71E−04)+**	3.59E−01 (2.21E−01)
IDMP-M4-T2	7.13E−01 (9.00E−02)−	2.52E−01 (1.02E−01)−	7.59E−01 (5.60E−02)−	1.84E−01 (1.65E−01)−	6.21E−02 (1.12E−03)−	**5.88E−02 (1.39E−03)**
IDMP-M4-T3	6.38E−01 (1.68E−01)−	1.06E−01 (9.69E−03)−	7.38E−01 (7.45E−02)−	1.22E−01 (1.08E−01)−	7.29E−02 (4.10E−02)−	**5.76E−02 (6.93E−04)**
IDMP-M4-T4	7.27E−01 (1.30E−01)−	1.28E−01 (2.46E−02)−	7.58E−01 (1.14E−01)−	2.29E−01 (1.46E−01)−	8.47E−02 (6.95E−02)−	**6.72E−02 (4.19E−02)**
±/~	0/12/0	0/12/0	0/12/0	0/11/1	2/10/0	



(a) DNEA on IDMP-M2-T4 (b) MMOEA/DC on IDMP-M2-T4 (c) CPDEA on IDMP-M2-T4 (d) TS_MMOEAC on IDMP-M2-T4

Fig. 2. Obtained PS on IDMP-M2-T4 by four algorithms.



(a) DNEA on IDMP-M3-T4 (b) MMOEA/DC on IDMP-M3-T4 (c) CPDEA on IDMP-M3-T4 (d) TS_MMOEAC on IDMP-M3-T4

Fig. 3. Obtained PS on IDMP-M3-T4 by four algorithms.

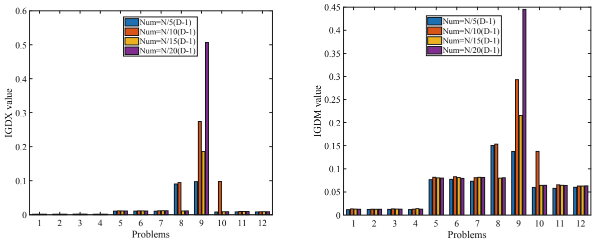
and its competing algorithms are approximate in objective space, and the IGDM metric is similar to IGDX for six algorithms in Table 3. It is clear that TS_MMOEAC also obtains 10 best values, and CPDEA gains 2 best results in 12 test problems in Table 3. Additionally, the IGDM value of the proposed TS_MMOEAC is significantly better than five competing algorithms.

From the above analysis, it is deduced that the proposed TS_MMOEAC is capable of handling such multimodal multi-objective optimization problems with imbalance convergence and diversity in decision space and is superior to currently popular MMEAs.

4.3 The Influence of Subpopulation Numbers

An experiment, TS_MMOEAC with different subpopulation numbers, is designed to investigate the influence of subpopulation numbers on the performance of TS_MMOEAC. In this experiment, the subpopulation numbers are set to $Num = N/5(D-1)$, $Num = N/10(D-1)$, $Num = N/15(D-1)$, and $Num = N/20(D-1)$, respectively, where N and D are population size and decision variable numbers, respectively. The IGDX and IGDM performance of TS_MMOEAC are shown in Fig. 4. In Fig. 4, the numbers on the horizontal axis denote the following problems: 1 = IDMP-M2-T1, 2 = IDMP-M2-T2, 3 = IDMP-M2-T3, 4 = IDMP-M2-T4, 5 = IDMP-M3-T1, 6 = IDMP-M3-T2, 7 = IDMP-M3-T3, 8 = IDMP-M3-T4, 9 = IDMP-M4-T1, 10 = IDMP-M4-T2, 11 = IDMP-M4-T3 and 12 = IDMP-M4-T4.

It is observed that the IGDX and IGDF performance of TS_MMOEAC is similar on 12 problems except for IDMP-M3-T4, IDMP-M4-T1, and IDMP-M4-T2. It implies that the influence of the subpopulation numbers on the performance of TS_MMOEAC is negligible. A larger subpopulation number is desirable for improving the convergence performance of TS_MMOEAC in the second-stage evolution. Therefore, the number of subpopulations is set to $Num = N/5(D-1)$ in the proposed TS_MMOEAC.



(a). IGDX values (b).IGDM values

Fig. 4. The influence of subpopulation numbers for TS_MMOEAC.

5 Conclusion

In this paper, we suggest a two-stage multimodal multi-objective evolutionary algorithm using clustering (TS_MMOEAC) for solving the multimodal multiobjective problem with the imbalance between convergence and diversity. The first-stage evolution using CPD-based clustering in TS_MMOEAC is to locate diverse local optimal solutions in decision space. Because these superior local optimal solutions have the potential to become equivalent Pareto optimal solutions with the imbalance between convergence and diversity in comparison to other equivalent Pareto solutions in the second-stage evolution. The second-stage evolution uses distance-based clustering to find all equivalent Pareto optimal solutions. The imbalance Pareto optimal solutions are more likely to be found in all subpopulations. Additionally, the convergence of the Pareto optimal solutions is remarkably boosted in the second-stage evolution. The experimental results validate the effectiveness of the two-stage evolutionary algorithm for solving the imbalanced MMOPs.

The two-stage evolutionary algorithm is capable of locating equivalent Pareto solutions with the imbalance between convergence and diversity, and our future work is to develop the proposed TS_MMOEAC to address real-world MMOPs.

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