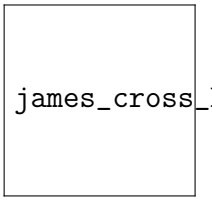

Solving the 2-D Poisson equation using relaxation

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The objective of assignment 9 was to solve for the electric potential in a grounded 10 cm x 10 cm metal box with two charges inside. This was completed numerically by applying a relaxation method to the Poisson equation in a program written in FORTRAN 2008.

The Poisson equation (in Gaussian/CGS units) with boundary conditions are below, where U is the electric potential and q is the charge density.

$$\begin{aligned} \nabla^2 U(x,y) &= 4\pi q && (1)\\ U(0\text{cm},y) &= 0 && (2)\\ U(10\text{cm},y) &= 0 && (3)\\ U(x,0\text{cm}) &= 0 && (4)\\ U(x,10\text{cm}) &= 0 && (5)\\ & && (6) \end{aligned}$$

The box was discretized into 100 cells on each side with (i,j) representing the i-th cell along the x-coordinate and j-th cell along the y- coordinate. The charge distribution was $+4$ in the $(75,75)$ cell and -4 in the (25,25) cell.

The iterative formula for the the discretized electric potential at the i-th cell along x and the j-th cell along y, is given by the following.

$$U_{i,j}^{\text{new}} = \frac{U_{i+1,j}^{\text{old}} + U_{i-1,j}^{\text{old}} + U_{i,j+1}^{\text{old}} + U_{i,j-1}^{\text{old}} - 4\pi q_{i,j}}{4} \tag{7}$$

The result of the relaxation method is shown below.