CS 312 Work Assignment 5

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1. (a) 4
   $5*6$
   $8/2$

   (b) $\vdash S_1 : c_1$

   $$\frac{\vdash S_1 : c_1 \qquad \vdash S_2 : c_2}{\vdash S_1 * S_2 : c_1 * c_2}$$

   $\vdash S_2 : c_2$ is not zero
   $$\frac{\vdash S_1 : c_1}{\vdash S_1/S_2 : c_1/c_2}$$

   $$\frac{\vdash e_2[e_1/x] : e}{\vdash \text{let x = } e_1 \text{ in } e_2\text{: e}}$$

   (c) $$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : nonzero}$$

   $$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

   $$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : zero}$$

   $$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

   $$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : zero}$$

   $$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : nonzero}$$

   $\Gamma \vdash S_1 : t_1$
   $t = t_1$
   $$\frac{\Gamma[x \leftarrow t] \vdash S_2 : t_2}{\Gamma \vdash let x = S_1 in S_2 : T_2}$$

   (d) **Base:**

   $\underline{nonzero\ i}$

$\Gamma \vdash i : nonzero$

$$\frac{zero\ i}{\Gamma \vdash i : zero}$$

Inductive Case 1:

$$\frac{\vdash S_1 : c_1 \qquad \vdash S_2 : c_2}{\vdash S_1 * S_2 \,:\, c_1 * c_2}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : nonzero}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

Since we know c_1 is a non zero or a zero, and c_2 is a non zero or a zero we know $c_1 * c_2$ is also non zero or zero.

Inductive Case 2:

$$\frac{\vdash S_1 : c_1 \qquad \vdash S_2 : c_2}{\vdash S_1/S_2 \,:\, c_1/c_2}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : nonzero}$$

Since we know c_1 is a zero or a non zero, and we know c_2 is a non zero, c_1/c_2 must be a non zero.

Inductive Case 3:

$$\frac{\vdash e_2[e_1/x] : e}{\vdash \text{let x = } e_1 \text{ in } e_2\text{: e}}$$

$$\begin{array}{l} \Gamma \vdash S_1 : t_1 \\ t = t_1 \\ \Gamma[x \leftarrow t] \vdash S_2 : t_2 \\ \hline \end{array}$$

$\Gamma \vdash let x = S_1 in S_2 : T_2$

Since we know c_1 is zero or non zero, we know c_2 must also be zero or non$_zero$.

(e) **Base:**

$$\frac{nonzero\ i}{\Gamma \vdash i : nonzero}$$

$$\frac{zero\ i}{\Gamma \vdash i : zero}$$

Clearly, if i types as zero or non zero, the corresponding operational semantics rule applies
Inductive Case 1:

$$\frac{\vdash S_1 : c_1 \qquad \vdash S_2 : c_2}{\vdash S_1 * S_2 : c_1 * c_2}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : nonzero}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1 * S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : zero}{\Gamma \vdash S_1 * S_2 : zero}$$

We know from the inductive hypothesis that the evaluation of S_1 and S_2 will never get stuck. We also know from preservation that the expressions S_1 and S_2 must evaluate to zero or non zero, therefore the operational semantics rule for multiplication will always apply since the hypotheses only require that c_1 and c_2 are zero or non zero.

Inductive Case 2:

$$\frac{\vdash S_1 : c_1 \qquad \vdash S_2 : c_2}{\vdash S_1/S_2 : c_1/c_2}$$

$$\frac{\Gamma \vdash S_1 : zero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : zero}$$

$$\frac{\Gamma \vdash S_1 : nonzero \qquad \Gamma \vdash S_2 : nonzero}{\Gamma \vdash S_1/S_2 : nonzero}$$

We know from the inductive hypothesis that the evaluation of S_1 and S_2 will never get stuck. We also know from preservation that the expression S_1 must evauate to zero or non zero and S_2 must evaluate to non zero, therefore the operational semantics rule for division will always apply since the hypotheses only require that c_1 is a non zero or zero and c_2 is a non zero.

Inductive Case 3:

$$\frac{\vdash e_2[e_1/x] : e}{\vdash \text{let x} = e_1 \text{ in } e_2\text{: e}}$$

$$\frac{\Gamma \vdash S_1 : t_1 \quad t = t_1 \quad \Gamma[x \leftarrow t] \vdash S_2 : t_2}{\Gamma \vdash letx = S_1 inS_2 : T_2}$$

Here, we know from the inductive hypothesis that S_1 is of type t_1 and E[x $\leftarrow t$]S$_2$: t_2 will not get stuck since they are well-typed. Therefore, this rule will also always apply.

2. (a) These are large step Operational Semantics.

   (b) TU 'd' $\rightarrow' D'$
   $TU'a' \rightarrow' A'$
   $TL'B' \rightarrow' b'$

(c) 1. A char is defined as a char.
2. AL returns a lowercase char if it's the correct case.
3. AU returns an uppercase char if it's the correct case.
4. TU converts a char to uppercase and returns it.
5. TL converts a char to lowercase and returns it.

(d) A runtime error will occur if AL is called on an uppercase char. A runtime error will also occur if AU is called on a lowercase char.

(e) A type system to prevent runtime errors for this language would consist of the types lowercase and uppercase.
The concretization of the type lowercase is: $\gamma(lowercase)$ = 'a'-'b'.
The concretization of the type uppercase is: $\gamma(uppercase)$ = 'A'-'B'.

(f) $$\frac{\text{lowercase c}}{\Gamma \vdash c : c}$$

$$\frac{\text{lowercase c}}{\Gamma \vdash c : c}$$

$$\frac{\Gamma \vdash P : lowercase}{\Gamma \vdash AL\ P : lowercase}$$

$$\frac{\Gamma \vdash P : uppercase}{\Gamma \vdash AU\ P : uppercase}$$

$$\frac{\Gamma \vdash P : lowercase}{\Gamma \vdash TL\ P : lowercase}$$

$$\frac{\Gamma \vdash P : uppercase}{\Gamma \vdash TL\ P : lowercase}$$

$$\frac{\Gamma \vdash P : uppercase}{\Gamma \vdash TU\ P : uppercase}$$

$$\frac{\Gamma \vdash P : lowercase}{\Gamma \vdash TU\ P : uppercase}$$