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1 Math

1.1 卡塔蘭數 Catalan number

```
// 1, 1, 2, 5, 14, 42
ll c[N];
c[0] = 1;
for(int i=1; i<n; i++){
    c[i] = 2*(2*(i-1)+1)*c[i-1]/(i+1);
}
```

1.2 錯排公式 Derangements

```
// 1, 0, 1, 2, 9, 44, 265
ll d[n]; //d[0]=1
d[1]=0, d[2]=1;
for(int i=3; i<n; i++){
    d[i] = (i-1)*(d[i-1]+d[i-2]);
}
```

1.3 Euler function

```
ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n; // 0(sqrtN)
    for(ll i=2; i*i<=a; i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

1.4 Euler function(建表)

```
void phi_table(int n, int *phi){
    memset(phi, 0, sizeof(phi)); //初始化
    phi[1] = 1;
    for(int i=2; i<=n; i++){
        if(phi[i]==0){
            for(int j=i; j<=n; j+=i){
                if(phi[j]==0) phi[j] = j;
                phi[j] = phi[j] / i * (i - 1);
            }
        }
    }
}
```

1.5 Sieve Prime

```
const int N = 20000000; //質數表大小
bool sieve[N];
vector<int> prime;
void linear_sieve(){
    for (int i = 2; i < N; i++){
        if (!sieve[i]) prime.push_back(i);
        for (int p : prime){
            if (i * p >= N) break;
            sieve[i * p] = true;
            if (i % p == 0) break;
        }
    }
}
```

1.6 快速幂

```
/*快速幂*/
ll FastPow(ll x, ll y, ll p) {
    long long ans = 1;
    while (y) {
        if (y & 1) ans = ans * x % p; //prime
        x = x * x % p;
        y >>= 1;
    }
    return ans;
}
```

1.7 Cn 取 m 模反, Combination, inverse

```
#define MXN 1'000'005
#define N 1'000'000
long long fac[MXN], inv[MXN];
fac[0] = 1; // 0! = 1
for(long long i = 1; i <= N; i++)
    fac[i] = fac[i-1] * i % MOD; // 階乘
inv[N] = FastPow(fac[N], MOD-2); // 模逆元
for(ll i = N-1; i >= 0; i--)
    inv[i] = inv[i+1] * (i+1) % MOD;
ll c(ll n, ll m){return fac[n]*inv[m]*inv[n-m];}
```

1.8 質因數分解 Factorizationn

```
int factor[MXN];
for(ll i = 2; i <= N; i++){
    if(factor[i]) continue;
    for(ll j = i*i; j <= N; j+=i){
        factor[j] = i;
    }
}
map<int, int> factorization(int x){
    map<int, int> prime;
    while(factor[x]){
        prime[factor[x]]++;
        x /= factor[x];
    }
    prime[x]++;
    return prime;
}
```

1.9 乘法取餘 Multiplication

```
/*大數乘法取餘數*/
ll mul(ll x, ll y, ll mod) {
    ll ret = x * y - (ll)((long double)x / mod * y) *
        mod;
    // LL ret=x*y-(LL)((Long double)x*y/mod+0.5)*mod;
    return ret < 0 ? ret + mod : ret;
}
```

1.10 ax+by=gcd(a,b)

```
/*ax+by=gcd(a,b) 一組解*/
ll a, b, x, y;
ll exgcd(ll a, ll b, ll& x, ll& y) {
    if (b) {
        ll d = exgcd(b, a % b, y, x);
        return y -= a / b * x, d;
    }
    return x = 1, y = 0, a;
}
```

1.11 Miller Rabin

```
/*Miller_Rabin 質數判定*/
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
ll magic[N] = {};
bool witness(ll a, ll n, ll u, int t) {
    if (!a) return 0;
    ll x = mypow(a, u, n); //快速幂
    for (int i = 0; i < t; i++) {
        ll nx = mul(x, x, n); //快速乘
        if (nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(ll n) {
    int s = (magic number size);
    // iterate s times of witness on n
    if (n < 2) return 0;
    if (!(n & 1)) return n == 2;
    ll u = n - 1; int t = 0;
    // n-1 = u*2^t
    while (!(u & 1)) u >>= 1, t++;
    while (s--) {
        ll a = magic[s] % n;
        if (witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

1.12 快速乘法 karatsuba

```
/*karatsuba 快速乘法*/
// Get size of the numbers
int getSize(ll num){
    int count = 0;
    while (num > 0)
    {
        count++;
        num /= 10;
    }
    return count;
}
ll karatsuba(ll X, ll Y){
    // Base Case
    if (X < 10 && Y < 10)
        return X * Y;
    // determine the size of X and Y
    int size = fmax(getSize(X), getSize(Y));
    // Split X and Y
    int n = (int)ceil(size / 2.0);
    ll p = (ll)pow(10, n);
    ll a = (ll)floor(X / (double)p);
    ll b = X % p;
    ll c = (ll)floor(Y / (double)p);
    ll d = Y % p;
    // Recur until base case
    ll ac = karatsuba(a, c);
    ll bd = karatsuba(b, d);
    ll e = karatsuba(a + b, c + d) - ac - bd;
    // return the equation
    return (ll)(pow(10 * 1L, 2 * n) * ac + pow(10 * 1L,
        n) * e + bd);
}
```

1.13 josephus1

```
/*約瑟夫問題：n個人圍成一桌，數到m的人出列*/
int josephus(int n, int m) { //n人每m次
    int ans = 0;
    for (int i = 1; i <= n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

1.14 大數 Big number

```
/*大數(Big Number)*/
void add(int a[100], int b[100], int c[100]){
    int i = 0, carry = 0;
    for (i = 0; i < 100; ++i) {
        c[i] = a[i] + b[i] + carry;
        carry = c[i] / 10;
        c[i] %= 10;
    }
}

void sub(int a[100], int b[100], int c[100]){
    int i = 0, borrow = 0;
    for (i = 0; i < 100; ++i) {
        c[i] = a[i] - b[i] - borrow;
        if (c[i] < 0) {
            borrow = 1;
            c[i] += 10;
        }
        else
            borrow = 0;
    }
}

void mul(int a[100], int b[100], int c[100]){
    int i = 0, j = 0, carry = 0;
    for (i = 0; i < 100; ++i) {
        if (a[i] == 0) continue;
        for (j = 0; j < MAX; ++j)
            c[i + j] += a[i] * b[j];
    }
    for (i = 0; i < MAX; ++i) {
        carry = c[i] / 10;
        c[i] %= 10;
    }
}

void div(int a[100], int b[100], int c[100]){
    int t[100];
    for (i = 100 - 1; i >= 0; i--) {
        for (int k = 9; k > 0; k--) // 嘗試商數
        {
            mul(b + i, k, t);
            if (largerthan(a + i, t))
            {
                sub(a + i, t, c + i);
                break;
            }
        }
    }
}
}
```

1.15 矩陣快速幂

```
LL len, mod;
vector<vector<LL>> operator*(vector<vector<LL>> x,
vector<vector<LL>> y){
    vector<vector<LL>> ret(len, vector<LL>(len, 0));
    for(int i=0; i<len; i++){
        for(int j=0; j<len; j++){
            for(int k=0; k<len; k++){
                ret[i][j]=(ret[i][j]+x[i][k]*y[k][j])%
                mod;
            }
        }
    }
    return ret;
}
```

```
}
struct Martix_fast_pow{ //O(Len^3 Lg k)
    LL init(int _len, LL m=9223372036854775783LL){
        len=_len, mod=m;
    } // mfp.solve(k,{0, 1}, {1, 1}) k'th fib {值, 係數} // 0-base
    LL solve(LL n, vector<vector<LL>> poly){
        if(n<len) return poly[n][0];
        vector<vector<LL>> mar(len, vector<LL>(len, 0)), x
            (len, vector<LL>(len, 0));
        for(int i=0; i<len; i++) mar[i][i]=1;
        for(int i=0; i+1<len; i++) x[i][i+1]=1;
        for(int i=0; i<len; i++) x[len-1][i]=poly[i][1];
        while(n){
            if(n&1) mar=mar*x;
            n>>=1, x=x*x;
        }
        LL ans=0;
        for(int i=0; i<len; i++) ans=(ans+mar[len-1][i]
            *poly[i][0]%mod)%mod;
        return ans;
    }
}mfp;
```

1.16 GaussElimination

```
/*GaussElimination*/
// by bcw_codebook
const int MAXN = 300;
const double EPS = 1e-8;
int n;
double A[MAXN][MAXN];
void Gauss() {
    for(int i = 0; i < n; i++) {
        bool ok = 0;
        for(int j = i; j < n; j++) {
            if(fabs(A[j][i]) > EPS) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if(!ok) continue;
        double fs = A[i][i];
        for(int j = i+1; j < n; j++) {
            double r = A[j][i] / fs;
            for(int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}
}
```

1.17 Epsilon

```
/*精準度(Epsilon)*/
float eps = 1e-8;
bool Equal(float a, float b)
    return fabs(a - b) < eps
bool NEqual(float a, float b)
    return fabs(a - b) > eps
bool Less(float a, float b)
    return (a - b) < -eps
bool Greater(float a, float b)
    return (a - b) > eps
```

1.18 Floor-Ceil

```
/*floor向下取整, ceil向上取整*/
int floor(int a, int b){return a/b - (a%b and a<0^b<0);}
int ceil (int a, int b){return a/b + (a%b and a<0^b>0);}
```

2 Geometry

2.1 Struct

```

struct Pt{
    ld x, y;
};
struct Line{
    Pt st, ed;
};
struct Circle{
    Pt o; // 圓心
    ld r; // 半徑
};
struct poly{
    int n; // n 邊形
    vector<Pt> pts;
};
struct Pt {
    ll x, y; // LL or ld
    Pt(){ }
    Pt(ll _x, ll _y){
        x=_x, y=_y;
    }
    Pt operator+(const Pt &a) {
        return Pt(x+a.x, y+a.y);
    }
    Pt operator-(const Pt &a) {
        return Pt(x-a.x, y-a.y);
    }
    Pt operator*(const ld &a) {
        return Pt(x*a, y*a);
    }
    Pt operator/(const ld &a) {
        return Pt(x/a, y/a);
    }
    ll operator*(const Pt &a){ //內積
        return x*a.x + y*a.y;
    }
    ll operator^(const Pt &a){ //外積
        return x*a.y - y*a.x;
    }
    bool operator<(const Pt &a) const {
        return x < a.x || (x == a.x && y < a.y);
    }
    friend int cross(const Pt& o, const Pt& a, const Pt& b)
    { //向量外積
        Pt lhs = o-a, rhs = o-b;
        return lhs.x*rhs.y - lhs.y*rhs.x;
    }
    friend bool operator ==(const Pt& lhs, const Pt& rhs){
        return (rhs.x==lhs.x && rhs.y==lhs.y);
    }
};

```

2.2 Function

```

//是否三點共線
bool collinearity(const Pt& a, const Pt& b, const Pt& c)
{
    return (b-a)^(c-a) < EPS; //外積為0: ==0
}
//判斷點是否在線段上
bool inline(const Pt& p, const Line& li){
    return collinearity(li.st, li.ed, p) && (li.st-p)*(li.ed-p) < EPS; //內積為負: <=0
}
//平行四邊形面積
ll areaPt(Pt i, Pt j, Pt k){
    return abs(cross(i, j, k));
}
//兩點距離
ld distance(Pt i, Pt j){

```

```

    Pt vt = i - j;
    return sqrt((ld)(vt.x)*(vt.x)+(ld)(vt.y)*(vt.y));
}
ld distance(int i, int j){
    Pt vt = p[i]-p[j];
    return sqrt((ld)(vt.x)*(vt.x)+(ld)(vt.y)*(vt.y));
}
// 輸出
round(double x); //四捨五入至整數
cout << fixed << setprecision(11); //精度
void print(ll x){ //兩倍面積輸出判斷
    if(x&1)
        cout << (x>>1) << ".5\n";
    else
        cout << (x>>1) << "\n";
}

```

2.3 凸包 Convex hull

```

vector<Pt> convex_hull(vector<Pt> hull){ //凸包
    sort(hull.begin(), hull.end());
    int top=0;
    vector<Pt> stk;
    for(int i=0; i<hull.size(); i++){
        while(top>=2&&cross(stk[top-2], stk[top-1], hull[i])<=0) //或<0
            stk.pop_back(), top--;
        stk.push_back(hull[i]);
        top++;
    }
    for(int i=hull.size()-2, t=top+1; i>=0; i--){
        while(top>=t&&cross(stk[top-2], stk[top-1], hull[i])<=0) //或<0
            stk.pop_back(), top--;
        stk.push_back(hull[i]);
        top++;
    }
    stk.pop_back();
    return stk;
}

```

2.4 旋轉卡尺

```

// 最遠兩點配對
double FarthestPair(vector<Pt> arr){
    double ret = 0;
    for (int i = 0, j = i + 1; i < arr.size(); i++){
        while (distance(i, j) < distance(i, (j + 1) % arr.size())){
            j = (j + 1) % arr.size();
        }
        ret = max(ret, distance(i, j));
    }
    return ret;
}
// 點是否凸包內
bool in_hull(vector<Pt> &hull, Pt p){
    int flag = 0;
    for (int i = 0; i < hull.size(); i++){
        if (cross(p, hull[i], hull[(i + 1) % hull.size()]) > 0){
            //與凸包上前後兩點作外積，如果夾角在0~180度內，則在內側。
            flag++;
        }
    }
    if (flag == hull.size())
        return true;
    else
        return false;
}

```

```
// 線找點最短距離
ld rot(vector<Pt> hull){
    ld mn = 2.0 * r;
    int n = hull.size();
    hull.pb(hull[0]);
    for (int i = 0, j = i + 2; i < n; i++){
        //平行四邊形面積 除以 底 = 高
        ld area = abs(cross(hull[i], hull[i + 1], hull[j]));
        ;
        ld base = abs(dis(hull[i], hull[i + 1]));
        while (area <= abs(cross(hull[i], hull[i + 1], hull
            [(j + 1) % n]))){
            j = (j + 1) % n;
            area = abs(cross(hull[i], hull[i + 1], hull[j]));
        }
        mn = min(mn, area / base);
    }
    return mn;
}

// 最大三角形
ld solve(vector<Pt> arr){
    ld ret = 0;
    for (int i = 0; i < arr.size(); i++)
        for (int j = i + 1; j < arr.size(); j++)
            for (int k = j + 1; k < arr.size(); k++)
                ret = max(ret, area(i, j, k));
    return ret;
}

// 最大四邊形
ll solve(){//兩倍面積
    ll ret = 0;
    for (int i = 0; i < hull.size(); i++){
        int k = (i + 1) % hull.size(), k2 = (i + 3) % hull.
            size();
        for (int j = i + 2; j <= i + hull.size() - 2; j++){
            while (area(i, j % hull.size(), k) < area(i, j %
                hull.size(), (k + 1) % hull.size())){
                k = (k + 1) % hull.size();
            }
            while (area(i, j % hull.size(), k2) < area(i, j %
                hull.size(), (k2 + 1) % hull.size())){
                k2 = (k2 + 1) % hull.size();
            }
            ret = max(ret, area(i, j % hull.size(), k) + area
                (i, j % hull.size(), k2));
        }
    }
    return ret;
}
```

3 Algorithm

3.1 Binary search

```
void binary_search(ll n, ll target){
    ll L=0, R=n;
    while(L<R){
        ll mid=(L+R)>>1;
        if(check(mid)) R=mid;
        else L=mid+1;
    }
    cout << L << "\n";
}
```

3.2 DFS

```
/*n queen*/
//k為第幾行，a[k]為第幾列，n個皇后
int a[100], n, count;
void DFS(int k) {
    if (k > n) {//當k=n+1時找到解
```

```
        count++;
        print("answer");
    }
    else {
        for (int i = 1; i <= n; i++) {
            a[k] = i; //存入皇后
            if (check(a, k))DFS(k + 1);//放入，求下一行
        }
    }
}

/*Traveling Knight Problem*/
#define X 5 //棋盤
#define Y 5
int dir[8][2] =
{{1,2},{2,1},{1,-2},{-2,1},{-1,2},{2,-1},{-1,-2},{-2,-1}};

int board[X][Y]={0},tot=0,_x,_y;
void dfs(int x,int y,int t) {
    if (t>X*Y) {
        print("answer");
        return;
    }
    for (int i=0;i<8;i++) {
        int xx=x+dir[i][0];
        int yy=y+dir[i][1];
        if ((xx>=X)|| (xx<0)|| (yy>=Y)|| (yy<0)|| (board[xx][yy]
            ))) continue;
        board[xx][yy]=t;
        dfs(xx,yy,t+1);
        board[xx][yy]=0; //回溯
    }
}

signed main(){
    board[_x][_y]=1; //starts
    dfs(_x,_y,2);
}
```

3.3 Brute Force

```
#define MAXN 1<<18+5 //雙倍空間
/*折半枚舉 與 二進制枚舉*/
int main() {
    int n, m, i, temp;
    ll mod, mod_max = 0;
    vector<ll> arr, ans(MAXN,0), ans2(MAXN,0);
    cin >> n >> m;
    for(i=0;i<n;i++){
        cin >> temp;
        arr.push_back(temp%m);
    }
    //折半枚舉
    for(int i=0;i<(1<<(n/2));i++){ //2^(n/2)
        for(int j=0;j<n/2;j++){
            if(i>>j&1) //二進制枚舉(選或不選)
                ans[i] = (ans[i] + arr[j]) % m; //前半
                枚舉
        }
    }
    for(int i=0;i<(1<<(n-n/2));i++){ //2^(n-n/2)
        for(int j=0;j<(n-n/2);j++){
            if(i>>j&1) ans2[i] = (ans2[i] + arr[n/2+j])
                % m; //後半枚舉
        }
    }
    //二分維護
    temp = 1<<(n-n/2);
    sort(ans2.begin(), ans2.begin() + temp);
    for(auto i:ans){
        mod_max = max(mod_max, i + *(upper_bound(ans2.
            begin(), ans2.begin() + temp, m-1-i)-1));
        //mod最大為m-1，配對另一半最優解
    }
    cout << mod_max << "\n";
}
```

3.4 Divide and Conquer

```

/*最近點對*/
double dist(pair<double, double> a, pair<double, double>
    > b)
    return sqrt(pow((a.first - b.first), 2) + pow((a.
        second - b.second), 2));
double closest(int l, int r) {
    if (l >= r)
        return 10000;
    int mid = (l + r) / 2;
    double radl = closest(l, mid);
    double radr = closest(mid + 1, r);
    double range = min(radl, radr), middle = (point[mid].
        first + point[mid + 1].first) / 2, minimum;
    minimum = range;
    for (int i = mid + 1; i <= r && point[i].first <
        middle + range; i++) {
        for (int j = mid; j >= l && point[j].first > middle
            - range; j--) {
            minimum = min(minimum, dist(point[i], point[j]));
        }
    }
    return minimum;
}
void quicksort(int arr[], int left, int right){
    if (left >= right) return;
    int i = left, j = right;
    int k = (left + right) / 2;
    int pivot = arr[k];
    while (true){
        while (arr[i] < pivot) i++;
        while (arr[j] > pivot) j--;
        if (i >= j) return;
        swap(arr[i], arr[j]);
        i++; j--;
    }
    quicksort(arr, left, j);
    quicksort(arr, i, right);
}
void Merge(vector<int> &Arr, int front, int mid, int
    end) {
    vector<int> Left(Arr.begin() + front, Arr.begin() +
        mid + 1);
    vector<int> Right(Arr.begin() + mid + 1, Arr.begin
        () + end + 1);
    int idxLeft = 0, idxRight = 0;
    Left.insert(Left.end(), INT_MAX);
    Right.insert(Right.end(), INT_MAX);
    for (int i = front; i <= end; i++) {
        if (Left[idxLeft] < Right[idxRight]) {
            Arr[i] = Left[idxLeft];
            idxLeft++;
        } else {
            Arr[i] = Right[idxRight];
            idxRight++;
        }
    }
}
void MergeSort(vector<int> &Arr, int front, int end) {
    if (front >= end)
        return;
    int mid = front + (end - front) / 2;
    MergeSort(Arr, front, mid);
    MergeSort(Arr, mid + 1, end);
    Merge(Arr, front, mid, end);
}

```

4 Data Structure

4.1 SQRT

```

// build O(n)
// update O(vn)
// query O(vn)
//分塊結構
//假設要求區間總和
struct blk{
    vector<int> local;    //每塊的全部元素
    int global;          //儲存每塊的總和
    int tag;              //儲存整塊一起更新的值
    blk(){                //初始化
        local.clear();    //清空區間元素
        tag = global = 0; //將區間總和先設為0
    }
};
vector<blk> b;
void build(){
    int len=sqrt(n),num=(n+len-1)/len;
    for(int i=0;i<n;i++){    //第i個元素分在第 i/len 塊
        cin>>x;
        //存入區間中
        b[i/len].local.push_back(x);
        //更新區間總和
        b[i/len].global += x;
    }
}
void update(int ql,int qr,int v){
    int blk_l=ql/len,blk_r=qr/len,ret=0;
    if(blk_l == blk_r){
        //如果都在同一塊直接一個一個跑過去就好
        for(int i=ql;i<=qr;i++)
            b[blk_l].local[i%len]+=v;
        b[blk_l].global+=(qr-ql+1)*v;
        return;
    }
    for(int i=ql;i<(blk_l+1)*len;i++){ //最左的那一塊
        b[blk_l].local[i%len]+=v;
        b[blk_l].global+=v;
    }
    for(int i=blk_l+1;i<blk_r;i++){ //中間每塊
        b[i].tag+=v;
        b[i].global+=v*len;
    }
    for(int i=blk_r*len;i<=qr;i++){ //最右的那一塊
        b[blk_r].local[i%len]+=v;
        b[blk_r].global+=v;
    }
}
int query(int ql,int qr){
    int blk_l=ql/len,blk_r=qr/len,ret=0;
    if(blk_l == blk_r){
        //如果都在同一塊直接一個一個跑過去就好
        for(int i=ql;i<=qr;i++)
            ret+=b[blk_l].local[i%len]+b[blk_l].tag;
        return ret;
    }
    for(int i=ql;i<(blk_l+1)*len;i++) //最左的那一塊
        ret+=b[blk_l].local[i%len]+b[blk_l].tag;
    for(int i=blk_l+1;i<blk_r;i++) //中間每塊的總和
        ret+=b[i].global;
    for(int i=blk_r*len;i<=qr;i++) //最右的那一塊
        ret+=b[blk_r].local[i%len]+b[blk_r].tag;
    return ret;
}

```


4.2 Mo's Algorithm

```
// n為序列總長度，q為詢問比數，p為移動一格的複雜度
// O(p(q+n)√n)
int n,k = sqrt(n);    //每塊大小為k
struct query{
    int l,r,id;        //詢問的左界右界 以及 第幾筆詢問
    friend bool operator<(const query& lhs,const query& rhs){
        return lhs.l/k==rhs.l/k ? lhs.r<rhs.r : lhs.l<rhs.l;
    }    //先判斷是不是在同一塊
        //不同塊的話就比較塊的順序，否則比較右界r
};
int num = 0;
int cnt[1000005], ans[30005];
vector<query> q;
void add(int x){    //新增元素到區間內
    ++cnt[x];
    if(cnt[x] == 1)    ++num;
}
void sub(int x){    //從區間內移除元素
    --cnt[x];
    if(cnt[x] == 0)    --num;
}
void solve(){
    sort(q.begin(),q.end());
    for(int i=0,l=-1,r=0;i<n;i++){
        while(l>q[i].l) add(--l);
        while(r<q[i].r) add(++r);    //記得要先做新增元素的
        while(l<q[i].l) sub(l++);    //再做移除元素的
        while(r>q[i].r) sub(r--);
        ans[q[i].id] = num;    //移到區間後儲存答案
    }
}
```

4.3 Treap

```
struct Treap{
    int key,pri,sz;    //key,priority,size
    Treap *l, *r;    //左右子樹
    Treap(){
        Treap(int _key){
            key = _key;
            pri = rand();    //隨機的數維持樹的平衡
            sz = 1;
            l = r = nullptr;
        }
    };
    Treap *root;
    int Size(Treap* x){ return x ? x->sz : 0 ; }
    void pull(Treap *x)
    { x->sz = Size(x->l) + Size(x->r) + 1; }
    Treap* merge(Treap *a,Treap *b){
        //其中一個子樹為空則回傳另一個
        if(!a || !b)    return a ? a : b;
        if(a->pri > b->pri){    //如果a的pri比較大則a比較上面
            a->r = merge(a->r,b);    //將a的右子樹跟b合併
            pull(a);
            return a;
        }
        else{    //如果b的pri比較大則b比較上面
            b->l = merge(a,b->l);    //將b的左子樹跟a合併
            pull(b);
            return b;
        }
    }
    void splitByKth(Treap *x,int k,Treap*& a,Treap*& b){
        if(!x){ a = b = nullptr; }
        else if(Size(x->l) + 1 <= k){
            a = x;
            splitByKth(x->r, k - Size(x->l) - 1, a->r, b);
            pull(a);
        }
    }
}
```

```
else{
    b = x;
    splitByKth(x->l, k, a, b->l);
    pull(b);
}
}
void splitByKey(Treap *x,int k,Treap*& a,Treap*& b){
    if(!x){ a = b = nullptr; }
    else if(x->key<=k){
        a = x;
        splitByKey(x->r, k, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitByKey(x->l, k, a, b->l);
        pull(b);
    }
}
void insert(int val){    //新增一個值為val的元素
    Treap *x = new Treap(val);    //設一個treap節點
    Treap *l,*r;
    splitByKey(root, val, l, r);    //找到新節點要放的位置
    root = merge(merge(l,x),r);    //合併到原本的treap裡
}
void erase(int val){    //移除所有值為val的元素
    Treap *l,*mid,*r;
    splitByKey(root, val, l, r);    //把小於等於val的丟到l
    splitByKey(l, val-1, l, mid);
    //小於val的丟到l,等於val的就會在mid裡
    root = merge(l,r);    //將除了val以外的值合併
}
int findVal(int val){    //小於等於val的size
    int size = -1;
    Treap *l, *r;
    splitByKey(root, val, l, r);    //把小於等於val的丟到l
    size = Size(l);
    root = merge(l,r);
    return size;
}
void interval(Treap *o, int l, int r) {    // [l,r]區間
    Treap *a, *b, *c;
    splitByKey(o, l - 1, a, b), splitByKey(b, r, b, c);
    // operate
    o = merge(a, merge(b, c));
}
void inOrderTraverse(Treap* o, int print) {    // 中序
    if (o != NULL){
        push(o);
        inOrderTraverse(o->l, print);
        // print
        if(print) cout << o->val << " ";
        inOrderTraverse(o->r, print);
    }
}
// Kth(k)：查找第k小的元素
int kth(Treap* o, int k){
    if(o == NULL || k > o->sz || k <= 0)    return 0;
    int s = (o->l == NULL ? 0 : o->l->sz);
    if(k == s + 1)    return o->key;
    else if(k <= s)    return kth(o->l, k);
    else    return kth(o->r, k - s - 1);
}
// Rank(x)：x的名次，即x是第幾小的元素
int rank(Node* o, int x){
    if(o == NULL)    return 0;
    int res = 0;
    int s = (o->l == NULL ? 0 : o->l->sz);
    if(x <= o->key){
        res += rank(o->l, x);
        res += x == o->key;
    }
    else{
        res += s + 1;
        res += rank(o->r, x);
    }
    return res;
}
```

4.4 BIT

```

/*BIT 樹狀數組(動態前綴和)*/
//BIT and Array start at 1
#define MAXN 100005 //最大區間<MAXN
vector<int> arr(MAXN); //原始陣列
vector<int> bit(MAXN); //BIT數組

//前綴和查詢
ll query(int i) { //index
    ll ret = 0;
    while(i > 0) ret += bit[i], i -= i & -i; // 1-base
    return ret;
}

//單點增值
void modify(int i, int val) { //index,value
    while(i <= MAXN) bit[i] += val, i += i & -i; // i+
    Lowbit(i)
}

```

4.5 Segment tree

```

/*Segment tree 線段樹(區間問題)*/
//segment tree and Array start at 1
// [l,r] 最大區間設為[1,n]
// [ql,qr] 目標區間
// pos,val 修改位置,修改值
#define MAXN 100005*4 //tree大小為4n
#define cl(x) (x*2) //左子節點index
#define cr(x) (x*2+1) //右子節點index
#define NO_TAG 0 //懶惰記號
vector<int> tag(MAXN);
vector<int> arr(MAXN);
vector<int> tree(MAXN);

void build(int i,int l,int r){
    if(l == r){ //遞迴到區間大小為1
        tree[i] = arr[l];
        return;
    }
    int mid=(l+r)/2; //往兩邊遞迴
    build(cl(i),l,mid);
    build(cr(i),mid+1,r);
    tree[i] = max(tree[cl(i)], tree[cr(i)]); //<-
    //可修改條件
    //將節點的值設成左右子節點的最大值
}

int query(int i,int l,int r,int ql,int qr){
    if(ql <= l && r <= qr){
        //若當前區間在詢問區間內,直接回傳區間最大值
        return tree[i];
    }
    int mid=(l+r)/2, ret=0; //<-可修改條件
    if(ql<=mid) //如果左子區間在詢問區間內
        ret = max(ret, query(cl(i),l,mid,ql,qr)); //<-
    //可修改條件
    if(qr> mid) //如果右子區間在詢問區間內
        ret = max(ret, query(cr(i),mid+1,r,ql,qr)); //<-
    //可修改條件
    return ret;
}

```

```

/*單點修改*/
void update(int i,int l,int r,int pos,int val){
    if(l == r){ //修改 a[pos] 的值为 val
        tree[i] = val;
        return;
    }
    int mid=(l+r)/2;
    if(pos <= mid) //如果修改位置在左子節點,往左遞迴
        update(cl(i),l,mid,pos,val);
    else //否則往右遞迴
        update(cr(i),mid+1,r,pos,val);
    tree[i] = max(tree[cl(i)], tree[cr(i)]); //<-
    //可修改條件
}

/*區間修改*/
//將區間 [l, r] 的值都加 v
void push(int i,int l,int r){
    if(tag[i] != NO_TAG){ //判斷是否有打標記,NO_TAG=0
        tree[i] += tag[i]; //有的話就更新當前節點的值
        if(l != r){ //如果有左右子節點把標記往下打
            tag[cl(i)] += tag[i];
            tag[cr(i)] += tag[i];
        }
        tag[i] = NO_TAG; //更新後把標記消掉
    }
}

void pull(int i,int l,int r){
    int mid = (l+r)/2;
    push(cl(i),l,mid); push(cr(i),mid+1,r);
    tree[i] = max(tree[cl(i)], tree[cr(i)]);
}

void update(int i,int l,int r,int ql,int qr,int v){
    push(i,l,r);
    if(ql<=l && r<=qr){
        tag[i] += v; //將區間 [l, r] 的值都加 v
        return;
    }
    int mid=(l+r)/2;
    if(ql<=mid) update(cl(i),l,mid,ql,qr,v);
    if(qr> mid) update(cr(i),mid+1,r,ql,qr,v);
    pull(i,l,r);
}

/*動態開點*/
struct node{
    node *l, *r;
    int val,tag;
};

void update(node *x, int l, int r, int ql, int qr, int v){
    push(x, l, r);
    if(ql <= l && r <= qr){
        x->tag += v;
        return;
    }
    int mid=(l+r)>>1;
    if(ql <= mid){
        if(x->l == nullptr) //判斷是否有節點
            x->l = new node();
        update(x->l, l, mid, ql, qr, v);
    }
    if(qr > mid){
        if(x->r == nullptr) //判斷是否有節點
            x->r = new node();
        update(x->r, mid+1, r, ql, qr, v);
    }
    pull(x, l, r);
}

```


4.6 Persistent Segment tree and DSU

```
#define push_back emplace_back
struct node{
    ll val;
    node *l, *r;
    node(){val = 0;}
};
ll n,idx=0;
vector<node> *version;
//用一個vector紀錄全部版本的根節點
node mem[MAXN*25];
node *newNode(){
    return &mem[idx++];
}
node *build(int l, int r){
    node *x = newNode();
    if(l == r) return x;
    int mid = (l+r)>>1;
    x->l = build(l, mid);
    x->r = build(mid+1, r);
    return x;
}
node *update_version(node *pre, ll l, ll r, ll pos, ll v){
    node *x = newNode(); //當前位置建立新節點
    if(l == r){
        x->val = v;
        return x;
    }
    int mid = (l+r)>>1;
    if(pos <= mid){ //更新左邊
        //左邊節點連向新節點
        x->l = update_version(pre->l, l, mid, pos, v);
        x->r = pre->r; //右邊連到原本的右邊
    }
    else{ //更新右邊
        //右邊節點連向新節點
        x->l = pre->l; //左邊連到原本的左邊
        x->r = update_version(pre->r, mid+1, r, pos, v);
    }
    x->val = min(x->l->val, x->r->val); //<-修改
    return x;
}
ll query(node *x,int ql,int qr,int v){ //bin search
    if(ql == qr) return qr;
    int mid=(ql+qr)>>1;
    if(x->l->val<v) // 如果左子區間在詢問區間內
        return query(x->l,ql,mid,v);
    else// 如果右子區間在詢問區間內
        return query(x->r,mid+1,qr,v);
}
void add_version(int x,int v){ //修改位置 x 的值為 v
    version.push_back(update_version(version.back(), 0, n-1, x, v));
    //前一個版本
}
int find(int x) {
    int fa = query(version.back(), 0, n - 1, x);
    if (fa == x) return x;
    return find(fa);
}
void merge(int a, int b) {
    int fa = find(a), fb = find(b);
    if (sz[fa] < sz[fb])
        swap(fa, fb);
    sz[fa] += sz[fb];
    add_version(fb, fa);
}
signed main(){
    io
    ll q,temp,i,l,r;
    cin >> n >> q;
    version.push_back(build(0,n-1));
    for(i=1;i<=n;i++){
```

```
        cin >> temp;
        add_version(temp,i);
    }
    for(i=0;i<q;i++){
        cin >> l >> r;
        cout << query(version[r],0,n-1,l) << "\n";
    }
}
```

4.7 Monotonic Queue(單調隊列)

```
vector<int> a; //array
deque<pair<int,int>> mx,mn; //index, value
vector<pair<int,int>> dp(MAXN); //min,max
pair<int,int> calc(){
    pair<int,int> temp;
    temp.first = mn.front().second; temp.second = mx.front().second;
    return temp;
}
if(k>n)k=n; // k = windows size, n = array size
for(i=0;i<n;i++){ // dp 求最大值
    while(!mn.empty() && mn.front().first <= i-k)
        mn.pop_front(); //判斷dq裡的東西有沒有過期
    while(!mn.empty() && mn.back().second >= a[i])
        mn.pop_back();
    while(!mx.empty() && mx.front().first <= i-k)
        mx.pop_front(); //判斷dq裡的東西有沒有過期
    while(!mx.empty() && mx.back().second <= a[i])
        mx.pop_back();
    mn.push_back(make_pair(i,a[i]));
    mx.push_back(make_pair(i,a[i]));
    if(i>=k-1) dp[i] = calc();
}
```

5 Graph(path)

5.1 Disjoint Set(Union-Find)

```
int f[N]; // 宣告父節點陣列 f
int sz[N]; // 子樹大小
void init(int n) {
    for (int i = 0; i < n; i++){
        f[i] = i;
        sz[i] = 1;
    }
}
int find(int x) {
    return f[x] == x ? x : f[x] = find(f[x]);
}
void merge(int x, int y) {
    x = find(x), y = find(y);
    if (x != y){
        sz[x] += sz[y];
        f[y] = x;
    }
}
```

5.2 Kruskal' s algorithm 最小生成樹

```
/*Kruskal' s algorithm 最小生成樹*/
//搭配 Disjoint Set(Union-Find)
struct Edge {
    int u, v, w; // 點 u 連到點 v 並且邊權為 w
    friend bool operator<(const Edge& lhs, const Edge& rhs) {
        return lhs.w > rhs.w; //兩條邊比較大小用邊權比較
    }
};
```

```

priority_queue<Edge> graph();// 宣告邊型態的陣列 graph
int kruskal(int m){
    int tot = 0;
    for (int i = 0; i < m; i++) {
        if (find(graph.top().u) != find(graph.top().v))
            { // 如果兩點未聯通
                merge(graph.top().u, graph.top().v);
                // 將兩點設成同一個集合
                tot += graph.top().w; // 權重加進答案
            }
        graph.pop();
    }
    return tot;
}
}
signed main() {
    cin >> n >> m; //node, edge
    init(n);
    for (int i = 0; i < m; i++) {
        cin >> u >> v >> w;
        graph.push(Edge{u,v,w});
    }
    cout << kruskal(m) << "\n";
}

```

5.3 Minimum Steiner Tree.cpp 最小斯坦因樹

```

// Minimum Steiner Tree 重要點的mst
// O(V^3T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0; i < n; i ++ ){
            for( int j = 0; j < n; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
        void add_edge( int ui, int vi, int wi ){
            dst[ ui ][ vi ] = min( dst[ ui ][ vi ], wi );
            dst[ vi ][ ui ] = min( dst[ vi ][ ui ], wi );
        }
        void shortest_path(){ // using spfa may faster
            for( int k = 0; k < n; k ++ )
                for( int i = 0; i < n; i ++ )
                    for( int j = 0; j < n; j ++ )
                        dst[ i ][ j ] = min( dst[ i ][ j ],
                                                dst[ i ][ k ] + dst[ k ][ j ] );
        }
        // call shortest_path before solve
        int solve( const vector<int>& ter ){
            int t = (int)ter.size();
            for( int i = 0; i < ( 1 << t ); i ++ )
                for( int j = 0; j < n; j ++ )
                    dp[ i ][ j ] = INF;
            for( int i = 0; i < n; i ++ )
                dp[ 0 ][ i ] = 0;
            for( int msk = 1; msk < ( 1 << t ); msk ++ ){
                if( msk == ( msk & (-msk) ) ){
                    int who = __lg( msk );
                    for( int i = 0; i < n; i ++ )
                        dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                    continue;
                }
                // Transfer starts at i=0, These points will be
                // used as relay points
                for( int i = 0; i < n; i ++ )
                    for( int submsk = ( msk - 1 ) & msk; submsk; submsk = ( submsk - 1 ) & msk )
                        dp[ msk ][ i ] = min( dp[ msk ][ i ],
                                                dp[ submsk ][ i ] +
                                                dp[ msk ^ submsk ][ i ] );
                for( int i = 0; i < n; i ++ ){
                    tdst[ i ] = INF;

```

```

                    for( int j = 0; j < n; j ++ )
                        tdst[ i ] = min( tdst[ i ],
                                                dp[ msk ][ j ] + dst[ j ][ i ] );
                }
                for( int i = 0; i < n; i ++ )
                    dp[ msk ][ i ] = tdst[ i ];
            }
            int ans = INF;
            for( int i = 0; i < n; i ++ )
                ans = min( ans, dp[ ( 1 << t ) - 1 ][ i ] );
            return ans;
        }
    } solver;
}

```

5.4 Dijkstra' s algorithm

```

/*Dijkstra's algorithm 單源最短路徑*/
#define MAX_V 100
#define INF 10000
struct Edge {
    int idx,w;
};
bool operator>(const Edge& a, const Edge& b) {
    return a.w > b.w;
}
int dist[MAX_V];
vector<vector<Edge> > adj(MAX_V);
void dijkstra(int vn, int s) {
    vector<bool> vis(vn, false);
    fill(dist, dist + vn, INF); dist[s] = 0;
    priority_queue<Edge, vector<Edge>, greater<Edge> >
        pq;
    Edge node;
    node.idx = s; node.w = 0;
    pq.emplace(node);
    while (!pq.empty()) {
        int u = pq.top().idx; pq.pop();
        if (vis[u])continue;
        vis[u] = true;
        for (auto v : adj[u]) {
            if (dist[v.idx] > dist[u] + v.w) {
                dist[v.idx] = dist[u] + v.w;
                node.w = dist[v.idx];
                node.idx = v.idx;
                pq.emplace(node);
            }
        }
    }
}
signed main() {
    //從start連接到end的最短路徑
    cin >> start >> end;
    dijkstra(n, start);
    if(dist[end]==INF) cout << "NO\n";
    else cout << dist[end] << "\n";
}

```

5.5 Floyd-Warshall

```

/*Floyd-Warshall 全點對最短路徑*/
void floyd(){
    //將每個點對距離設為INF
    memset(dist,0x3f3f3f3f,sizeof(dist));
    //dist[u][v]為點u到點v的最短路徑
    for(int i=0;i<n;i++) dist[i][i]=0;
    for(int i=0;i<m;i++) cin>>u>>v>>w,dist[u][v]=w;
    for(int i=0;i<n;i++) //窮舉中繼點
        for(int j=0;j<n;j++) //j,k窮舉點對
            for(int k=0;k<n;k++)
                dist[j][k]=min(dist[j][k],dist[j][i]+
                                dist[i][k]);
}

```

5.6 SPFA

```

/*SPFA 單源最短路徑(negative cycle)*/
struct Edge {
    int idx, w;
};
vector<Edge> adj[MAX_V]; //adjacency List
vector<bool> inp(MAX_V);
int dist[MAX_V];
//return true if negative cycle exists
bool spfa(int vn, int s) {
    fill(dist, dist + vn, INF); dist[s] = 0;
    vector<int> cnt(vn, 0);
    vector<bool> inq(vn, 0);
    queue<int> q; q.push(s); inq[s] = true;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        inq[u] = false;
        for (auto v : adj[u]) {
            if (dist[v.idx] > dist[u] + v.w) {
                if (++cnt[v.idx] >= vn) return true;
                dist[v.idx] = dist[u] + v.w;
                if (!inq[v.idx]) inq[v.idx] = true, q.push(v.idx);
            }
        }
    }
    return false;
}

```

6 Graph(Tree)

6.1 DFS and BFS

```

void dfs(int x){
    vis[x]=1;
    for(int i:adj[x])
        if(!vis[i]) dfs(i);
}
void bfs(int s){
    queue<int> q;
    q.push(s);
    vis[s]=1;
    while(!q.empty()){
        int x=q.front();q.pop();
        for(int i:adj[x])
            if(!vis[i])
                q.push(i),vis[i]=1;
    }
}

```

6.2 Topological Sort

```

// O(N+M)
for(int i=0;i<m;i++){
    cin >> u >> v; //點 u 連到點 v
    adj[u].push_back(v);
    ++deg[v];
}
// can solve DAG
void bfs(int n){ // Topological Sort
    queue<int> q;
    for(int i=0;i<n;i++)
        if(!deg[i]) q.push(i); //入度0先出
    while(!q.empty()){
        int u = q.front(); q.pop();
        for(int i:adj[u]){
            --deg[i];
            deg_sum[i] += deg_sum[u];
            if(deg[i] == 0) q.push(i);
        }
    }
}

```

6.3 Eulerian Path and Circuit

```

// O(M)
// 歐拉迴路 歐拉路徑
// 無向圖/所有點的度數為偶數/度數為奇數的點數量不超過2
// 有向圖/所有點入度等於出度/全部點的入度出度一樣
//或剛好一個點出度-1=入度 另一點入度-1=出度, 其他點入度等於出度
vector<int> path;
void dfs(int x){
    while(!edge[x].empty()){
        int u = edge[x].back();
        edge[x].pop_back();
        dfs(u);
    }
    path.push_back(x);}
int main(){
    build_graph();
    dfs(st); // 如果剛好一個點出度-1=入度 則為起點
    reverse(path.begin(),path.end());}

```

6.4 LCA

```

// pre O(NlgN)
// query O(LgN)
// 最近共同祖先
// 兩點間距離 / 兩點間最大邊 / 兩點間重合長度
// 時間戳記, 判斷祖先關係
int ti = 0; // 當前時間
int tin[MAXN+5], tout[MAXN+5];
int dis[MAXN+5]; // 計算距離深度
int query[MAXN+5][lgN+5]; // 點N的2^LgN祖先的最大邊
void dfs(int x, int f, int deep){
    fa[x] = f;
    tin[x] = ti++;
    dis[x] = deep;
    for(auto i:edge[x]){
        if(i.v == f){
            //query[x][0] = i.w;
            continue;
        }
        dfs(i.v, x, deep+i.w);
    }
    tout[x] = ti++;
}
bool isAncestor(int u, int v){
    return tin[u]<=tin[v] && tout[u] >= tout[v];
}
// LCA
int n, lgn;
int anc[MAXN+5][lgN+5]; // 點N的2^LgN祖先
int getLca(int u, int v){
    if(isAncestor(u, v)) return u;
    if(isAncestor(v, u)) return v;
    for(int i=lgn;i>=0;i--){
        // 判斷 2^LgN, 2^(LgN-1), ..., 2^1, 2^0 倍祖先
        if(!isAncestor(anc[u][i], v))
            u = anc[u][i]; // 則往上移動
    }
    return anc[u][0];
}
// 找出路徑最大邊
int max_cost(int u, int v){
    int max_cost = 0;
    if(u == v) return max_cost;
    for(int i=lgn;i>=0;i--){
        if(!isAncestor(anc[u][i], v)){
            max_cost = max(max_cost, query[u][i]);
            u = anc[u][i]; // 則往上移動
        }
    }
    return max(max_cost, query[u][0]);
}
// max(max_cost(u, nodeLca), max_cost(v, nodeLca))

```

```
// 兩點距離
int dist(int u, int v){
    //depth[X] + depth[Y] - 2 * depth[ancestor]
    return dis[u] + dis[v] - 2 * dis[find(v)];
}

// init 建表
for(s=1; s<=n; s++) anc[s][0] = fa[s];
for(i=1; i<=lgn; i++)
    for(s=1; s<=n; s++)
        anc[s][i] = anc[anc[s][i-1]][i-1];
// query[s][i] =
// max(query[s][i-1], query[anc[s][i-1]][i-1]);
```

6.5 DSU on Tree

```
void add(int v, int p, int x){
    cnt[ col[v] ] += x;
    // now you can insert test
    for(auto u : g[v])
        if(u != p && !big[u])
            add(u, v, x);
}

void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p && sz[u] > mx)
            mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            // run a dfs on small childs and clear them
            // from cnt
            dfs(u, v, 0);

    if(bigChild != -1)
        // bigChild marked as big and not cleared from cnt
        dfs(bigChild, v, 1), big[bigChild] = 1;
    add(v, p, 1);
    ans[v] = sum;
    //now cnt[c] is the number of vertices in subtree
    // of vertex v that has color c. You can answer
    // the queries easily.
    if(bigChild != -1)
        big[bigChild] = 0;
    if(keep == 0){
        add(v, p, -1);
        // now you can init to 0
    }
}
```

6.6 樹上差分

```
#define MAX 3e5+5
int n;
vector<vector<int>>>edge(MAX), fa(MAX, vector<int>(21, 0));
vector<int>a(MAX), dep(MAX), cnt(MAX, 0);
void dfs(int rt, int f) {
    fa[rt][0] = f;
    dep[rt] = dep[f] + 1;
    for (int i = 1; i <= 20; i++) {
        fa[rt][i] = fa[fa[rt][i-1]][i-1];
    }
    for (auto i : edge[rt]) {
        if (i == f) continue;
        dfs(i, rt);
    }
}

int lca(int a, int b) {
    if (dep[a] < dep[b]) {
        swap(a, b);
    }
    for (int i = 20; i >= 0; i--) {
        if (dep[fa[a][i]] >= dep[b]) {
            a = fa[a][i]; //上跳
        }
    }
    if (a == b)
        return a;
    for (int i = 20; i >= 0; i--) {
        if (fa[a][i] != fa[b][i]) {
            a = fa[a][i];
            b = fa[b][i];
        }
    }
    return fa[a][0];
}

void dfssum(int rt, int f) {
    for (auto i : edge[rt]) {
        if (i == f) continue;
        dfssum(i, rt);
        cnt[rt] += cnt[i];
    }
}

void solve() {
    int u, v, cmnlca;
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    for (int i = 0; i < n - 1; i++) {
        cin >> u >> v;
        edge[u].push_back(v);
        edge[v].push_back(u);
    }
    dfs(1, 0);
    for (int i = 0; i < n - 1; i++) {
        cmnlca = lca(a[i], a[i + 1]);
        cnt[fa[cmnlca][0]]--; //父節點 -v
        cnt[cmnlca]--; //Lca -v
        cnt[a[i]]++; //兩端點 +v
        cnt[a[i + 1]]++;
    }
    dfssum(1, 0);
    for (int i = 1; i <= n; i++) { //多加的減回去
        cnt[a[i]]--;
    }
    for (int i = 1; i <= n; i++) {
        cout << cnt[i] << "\n";
    }
}

int main() {
    cin >> n;
    solve();
}
```

6.7 HLD with Segment tree

```

#define MXN 10005
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
#define INF 1e9+5
int n;
int sz[MXN], fa[MXN], heavy[MXN], dep[MXN];
int root[MXN]; //鍊的根節點
int len[MXN]; //鍊長度
struct Edge {int u, v;};
struct node {int v, w;};
vector<Edge> edge;
vector<node> graph[MXN];
vector<int> tree[MXN]; // 第i個節點為根的線段樹
vector<int> val[MXN]; // 第i個節點為根的序列
// 子樹大小
void dfs_sz(int u, int f, int d){
    sz[u] = 1, fa[u] = f, dep[u] = d;
    for(auto v : graph[u]){
        if(v.v != f){
            dfs_sz(v.v, u, d+1);
            sz[u] += sz[v.v];
            if(sz[v.v] > sz[heavy[u]]) heavy[u] = v.v;
            //重兒子
        }
    }
}
// 樹鍊剖分
void dfs_hld(int u, int f){
    for(auto v : graph[u]){
        if(v.v != f){
            if(v.v == heavy[u]) root[v.v] = root[u];
            //重兒子的根，重鍊的頭
            else root[v.v] = v.v; //輕兒子的根
            val[root[v.v]].push_back(v.w); //點權
            dfs_hld(v.v, u);
        }
    }
    len[root[u]]++; //鍊長度
}
// LCA
int getLca(int x, int y){
    while(root[x] != root[y]){
        if(dep[root[x]] > dep[root[y]])
            x = fa[root[x]]; //跳鍊
        else
            y = fa[root[y]];
    }
    return (dep[x] <= dep[y] ? x : y);
}
// 線段樹
void build(int ver, int i, int l, int r){
    if(l == r){
        tree[ver][i] = val[ver][l];
        return;
    }
    int mid = (l+r)>>1;
    build(ver, cl(i), l, mid);
    build(ver, cr(i), mid+1, r);
    tree[ver][i] = max(tree[ver][cl(i)], tree[ver][cr(i)]); //最大邊
}
void update(int ver, int i, int l, int r, int pos, int val){
    if(l == r){ // 修改 a[pos] 的值為 val
        tree[ver][i] = val; return;
    }
    int mid = (l+r)>>1;
    if(pos <= mid) update(ver, cl(i), l, mid, pos, val);
    else update(ver, cr(i), mid+1, r, pos, val);
    tree[ver][i] = max(tree[ver][cl(i)], tree[ver][cr(i)]);
}
// i 為當前節點index, l, r當前區間左右界, ql, qr詢問左右界

```

```

int query(int ver, int i, int l, int r, int ql, int qr){
    if(ql <= l && r <= qr){
        return tree[ver][i];
    }
    int mid = (l+r)>>1, ret = -INF;
    if(ql <= mid) ret = max(ret, query(ver, cl(i), l, mid, ql, qr));
    if(qr > mid) ret = max(ret, query(ver, cr(i), mid+1, r, ql, qr));
    return ret;
}
void init(){
    edge.clear(); edge.resize(n-1);
    for(int i=1; i<=n; i++){
        graph[i].clear();
        tree[i].clear();
        val[i].clear();
        heavy[i] = len[i] = 0;
    }
}
signed main(){
    int i, t, a, b, w, ti;
    string op;
    cin >> n;
    init();
    for(i=0; i+1<n; i++){
        cin >> a >> b >> w;
        graph[a].push_back(node{b, w});
        graph[b].push_back(node{a, w});
        edge[i] = Edge{a, b};
    }
    val[1].push_back(-INF);
    root[1] = 1;
    dfs_sz(1, 1, 0);
    dfs_hld(1, 1);
    // build tree
    for(i=1; i<=n; i++){ // 第i個節點為根的線段樹
        if(root[i] == i){
            tree[i].resize(len[i]*4, 0);
            build(i, 1, 0, len[i]-1);
        }
    }
    // query
    while(cin >> op){
        if(op == "DONE") break;
        else if(op == "CHANGE"){
            cin >> i >> ti; i--;
            if(dep[edge[i].u] < dep[edge[i].v])
                swap(edge[i].u, edge[i].v);
            i = edge[i].u;
            update(root[i], 1, 0, len[root[i]]-1,
                dep[i]-dep[root[i]], ti);
        }
        else if(op == "QUERY"){
            cin >> a >> b;
            int ans = -INF;
            while(root[a] != root[b]){ //不同鍊
                if(dep[root[a]] < dep[root[b]])
                    swap(a, b);
                // 深鍊的最大邊
                ans = max(ans, query(root[a], 1, 0, len[root[a]]-1,
                    0, dep[a]-dep[root[a]]));
                a = fa[root[a]]; //跳鍊
            }
            if(a != b){ //不同節點
                int mn = min(dep[a], dep[b]) - dep[root[a]] + 1;
                int mx = max(dep[a], dep[b]) - dep[root[a]];
                //所在節點區間 mn, mx
                ans = max(ans, query(root[a], 1, 0, len[root[a]]-1, mn, mx));
            }
            cout << ans << "\n";
        }
    }
}

```

6.8 Scc 強連通分量 (轉 DAG)

```

#define MXN 200005
#define FZ(x) memset(x, 0, sizeof(x))
// 用於 DAG 2-SAT
struct Scc { // 強連通分量 0-base
    int n, nScc, vst[MXN], bln[MXN]; // 最後每個點所屬的
    // 連通分量存在 bln 陣列
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n) { // 先初始化點的數量
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v) { // 加有向邊
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u) {
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u) {
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve() { // 跑 kosaraju
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]) {
                rDFS(v); nScc++;
            }
    }
} scc;

```

6.9 2-SAT

```

// build
scc.init(2*m);
for(i=0; i<m; i++){ // 所有點比較
    for(j=i+1; j<m; j++){
        if(graph[i].x == graph[j].x) { // 同一行
            // m 中的 2 個控制
            // (x or y) x->!y, y->!x -> !x = x+m
            if(abs(graph[i].y - graph[j].y) <= 2*r) { // light
                // x or y
                scc.addEdge(i, j+m);
                scc.addEdge(j, i+m);
            }
        }
        else if(graph[i].y == graph[j].y) { // 同一列
            if(abs(graph[i].x - graph[j].x) <= 2*r) { // light
                // !x or !y
                scc.addEdge(i+m, j);
                scc.addEdge(j+m, i);
            }
        }
    }
}
// solve
scc.solve();
for(i=0; i<m; i++){
    if(scc.bln[i] == scc.bln[i+m]) {
        cout << "NO\n";
        return 0;
    }
}
cout << "YES\n";

```

6.10 BccVertex 點雙連通分量 (關節點、橋)

```

#define MXN 100005
#define FZ(x) memset(x, 0, sizeof(x))
#define REP(i, b) for(int i = 0; i < (b); ++i)
// 找關節點(分量重疊) 與 橋(只有兩點)
struct BccVertex { // 點雙連通分量(無向) 0-base
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) { // 初始化點的數量
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v) // 加無向邊
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v : E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else {
                low[u] = min(low[u], dfn[v]);
            }
        }
    }
    vector<vector<int>> solve() { // 跑 Tarjan
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i, i);
            }
        REP(i, nScc) res.PB(sccv[i]);
        return res;
    }
} graph;

```


7 DP

7.1 背包問題

```
// n：第0種到第n種物品要放進背包內。
// w：背包耐重限制。
// c(n, w)：只有第0種到第n種物品
// 耐重限制為w，此時的背包問題答案。
// weight[n]：第n種物品的重量。
// cost[n]：第n種物品的價值。
// number[n]：第n種物品的數量。

// 0/1背包滾動
// 每種物品只會放進背包零個或一個。
const int N = 500, W = 2000000; // N個物品，耐重W
int cost[N], weight[N];
int c[W + 1];
void knapsack(int n, int w){
    c[0] = 0;
    for (int i = 0; i < n; ++i)
        for (int j = w; j - weight[i] >= 0; --j)
            c[j] = max(c[j], c[j - weight[i]] + cost[i]);
    cout << c[w];
}

// 無限背包
// 物品有許多種類，每一種物品都無限量供應的背包問題。
void knapsack(int n, int w){
    memset(c, 0, sizeof(c));
    for (int i=0; i<n; ++i)
        for (int j = weight[i]; j <= w; ++j)
            c[j] = max(c[j], c[j - weight[i]] + cost[i]);
    cout << "最高的價值為" << c[w];
}

// 有限背包
// 物品有許多種類，每一種物品都是限量供應的背包問題。
int cost[N], weight[N], number[N];
// number[n]：第n種物品的數量。
void knapsack(int n, int w){
    for (int i = 0; i < n; ++i){
        int num = min(number[i], w / weight[i]);
        for (int k = 1; num > 0; k *= 2){
            if (k > num) k = num;
            num -= k;
            for (int j = w; j >= weight[i] * k; --j)
                c[j] = max(c[j], c[j - weight[i] * k] + cost[i] * k);
        }
    }
    cout << "最高的價值為" << c[w];
}
```

7.2 找零問題

```
// n：用第0種到第n種錢幣來湊得價位。
// m：欲湊得的價值。
// c(n, m)：用第0種到第n種錢幣湊得價位m的湊法數目。
// price[n]：第n種錢幣的面額大小。

// 這些面額湊不湊得到價位m，無限
int price[5] = {5, 2, 6, 11, 17}; // 錢幣面額
bool c[1000 + 1];
void change(int m){
    memset(c, false, sizeof(c));
    c[0] = true;
    // 依序加入各種面額
    for (int i = 0; i < 5; ++i)
        // 由低價位逐步到高價位
        for (int j = price[i]; j <= m; ++j)
            c[j] |= c[j - price[i]];
    if (c[m]) cout << "yes";
    else cout << "no";
}
```

```
}
// 湊得某個價位的湊法總共幾種
void change(int m){
    memset(c, 0, sizeof(c));
    c[0] = 1;
    for (int i = 0; i < 5; ++i)
        for (int j = price[i]; j <= m; ++j)
            c[j] += c[j - price[i]];
    cout << "湊得價位" << m;
    cout << "湊法總共" << c[m] << "種";
}

// 湊得某個價位的最少錢幣用量
// c(n, m)：用第0種到第n種錢幣湊得價位m，最少所需要的錢幣數量。
void change(int m){
    memset(c, 0x7f, sizeof(c));
    c[0] = 0;
    for (int i = 0; i < 5; ++i)
        for (int j = price[i]; j <= m; ++j)
            c[j] = min(c[j], c[j - price[i]] + 1);
    cout << "湊得價位" << m;
    cout << "最少需（只）要" << c[m] << "個錢幣";
}

// 湊得某個價位的錢幣用量，有哪幾種可能性。
void change(int m){
    memset(c, 0, sizeof(c));
    c[0] = 1;
    for (int i = 0; i < 5; ++i)
        for (int j = price[i]; j <= m; ++j)
            // 錢幣數量加一，每一種可能性都加一。
            c[j] |= c[j - price[i]] << 1;
    for (int i = 1; i <= 63; ++i)
        if (c[m] & (1 << i))
            cout << "用" << i << "個錢幣可湊得價位" << m;
}

// 能否湊得某個價位，限量
int price[5] = {5, 2, 6, 11, 17};
int number[5] = {4, 5, 5, 3, 2}; // 各種錢幣的供應數量
bool c[1000 + 1];
void change(int m){
    memset(c, 0, sizeof(c));
    c[0] = true;
    for (int i = 0; i < 5; ++i)
        // 各種餘數分開處理
        for (int k = 0; k < price[i]; ++k){
            int left = number[i]; // 補充彈藥
            // 由低價位到高價位
            for (int j = k; j <= m; j += price[i])
                // 先前的面額已能湊得，當前面額可以省著用。
                if (c[j])
                    left = number[i]; // 補充彈藥
            // 過去都無法湊得，一定要用目前面額硬湊。
            else if (left > 0){
                left--; // 用掉一個錢幣
                c[j] = true;
            }
        }
    if (c[m]) cout << "yes";
    else cout << "no";
}

// 買東西找回最少硬幣。
int price[5] = {50, 20, 10, 4, 2}; // 面額由大到小排列
void cashier(int n){ // n 是總共要找的錢。
    int c = 0;
    for (int i = 0; i < 5; ++i)
        while (n >= price[i]){
            n -= price[i]; // 找了 price[i] 元
            c++;
        }
    if (n != 0) cout << "找不出來";
    else cout << "找了" << c << "個錢幣";
}
```

7.3 LCS 與 DP 回溯

```
n=str.size(); m=str2.size();
vector<vector<int>> dp(n+1,vector<int>(m+1,0));
// LCS
for(i=1;i<=n;i++){
    for(j=1;j<=m;j++){
        if(str[i-1]==str2[j-1]){
            dp[i][j] = dp[i-1][j-1] + 1;
        }else{
            dp[i][j] = max(dp[i][j-1],dp[i-1][j]);
        }
    }
}
// DP回溯
string ans="";
i=n; j=m;
while(i>0 && j>0){
    if(str[i-1]==str2[j-1]){
        ans = str[i-1] + ans;
        i--; j--;
    }
    else if(dp[i][j-1]>dp[i-1][j]) j--;
    else i--;
}
cout << dp[n][m] << "\n";
cout << ans << "\n";
```

7.4 LIS

```
/*LIS 最長遞增子序列*/
int LIS(int n, int s[], int length[]) {
    for (int i = 0; i < n; i++) length[i] = 1;
    for (int j = 0; j < n; j++)
        for (int i = j + 1; i < n; i++)
            if (s[j] < s[i]) length[i] =
                max(length[i], length[j] + 1);
    return *max_element(length, length+n);
}
```

8 DP on tree

8.1 全點對距離 Tree Distance

```
int dp[MAXN]={0};
void dfs_sz(int x,int f){
    sz[x] = 1, fa[x] = f;
    for(int i:edge[x]){
        if(i == f) continue;
        dfs1(i, x); // 先計算完子節點的答案再算自己的
        sz[x]+=sz[i];
        dp[x]+=(dp[i]+sz[i]);
    }
}
void dfs_dp(int x,int f,ll sum){
    ans += sum + dp[x]; //所有點到結點x距離總和為父節點
    //方向距離總和 + 子樹到自己距離總和
    for(int i:edge[x]){
        if(i == f) continue;
        //tmp 為從父節點x到子節點i的距離總和
        ll tmp = sum //x的父節點總和 sum 到結點x的距離
            + dp[x] - (dp[i]+sz[i])
            //加上x的子樹(除了i方向)到x的距離總和
            + (n - sz[i]);
        //加上從節點x到節點i的距離
        dfs2(i, x, tmp);
    }
}
```

8.2 最大獨立集 Independent set

```
int dp[MAXN][2]; //此點，選或不選
void dfs(int x,int f){
    dp[x][1] = 1; // 狀態[1] 計算自己數量 +1
    for(int i:edge[x]){
        if(i == f) continue;
        dfs(i, x); // 先計算完子節點的答案再算自己的
        dp[x][0] += max(dp[i][0], dp[i][1]);
        dp[x][1] += dp[i][0];
    }
}
```

8.3 最小點覆蓋 Vertex Cover

```
int dp[MAXN][2]; //此點，選或不選
void dfs(int x,int f){
    dp[x][1] = 1; // 狀態[1] 計算自己數量 +1
    for(int i:edge[x]){
        if(i == f) continue;
        dfs(i, x); // 先計算完子節點的答案再算自己的
        dp[x][0] += dp[i][1];
        dp[x][1] += min(dp[i][0],dp[i][1]);
    }
}
```

8.4 最小支配集 Dominating Set

```
//狀態
dp[i][0]: 點i屬於支配集，並且以點i為根的子樹都被覆蓋了
的情況下，支配集中包含的最少點數。
dp[i][1]: 點i不屬於支配集，且以i為根的子樹都被覆蓋，且i
被其中不少於1個子結點覆蓋的情況下，支配集包含的最少
點數。
dp[i][2]: 點i不屬於支配集，且以i為根的子樹都被覆蓋，且i
沒被子結點覆蓋的情況下，支配集包含的最少點數。
// 狀態轉移
dp[i][0] = 1 + Σmin( dp[u][0], dp[u][1], dp[u][2] )
if(i沒有子結點) dp[i][1] = INF
else dp[i][1] = Σmin( dp[u][0], dp[u][1] )
dp[i][2] = Σdp[u][1]
```

9 String

9.1 Trie

```
// insert O(|s|)
// query O(|s|)
struct trie{
    trie *nxt[26];
    int cnt; //紀錄有多少個字串以此節點結尾
    int sz; //有多少字串的前綴包括此節點
    set<int> cnt_idx, sz_idx;
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
trie *root = new trie();
void insert(string& s, int idx){
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        now->sz++; now->sz_idx.emplace(idx); //被誰經過
        if(now->nxt[i-'a'] == NULL){
            now->nxt[i-'a'] = new trie();
        }
        now = now->nxt[i-'a']; //走到下一個字母
    }
    now->cnt++; now->cnt_idx.emplace(idx); //以此點結尾
    now->sz++; now->sz_idx.emplace(idx); //被誰經過
}
//query
int query_prefix(string& s){ //查詢有多少前綴為 s
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->sz;
}
int query_count(string& s){ //查詢字串 s 出現次數
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->cnt;
}
//str有沒有在[l,r]的前綴中
bool query_ArrPrefix(string& s,int l,int r){
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] != NULL && now->nxt[i-'a']->sz > 0){ //存在
            now = now->nxt[i-'a'];
        }else return false; //不存在，無解
    }
    // 這個s的節點，[l,r]有沒有經過
    auto L = now->sz_idx.lower_bound(l);
    if(L<=*L && *L<=r) return true;
    else return false;
}
//[l,r]有沒有存在於str的前綴中
bool query_StrPrefix(string& s,int l,int r){
    trie *now = root; // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] != NULL && now->nxt[i-'a']->sz > 0){ //存在
            now = now->nxt[i-'a'];
        }else return false; //不存在，無解
    }
    // [l,r]存在於str的前綴中，代表有字串以str為結尾
    auto L = now->cnt_idx.lower_bound(l);
    if(L<=*L && *L<=r) return true;
    return false;
}
```

9.2 01Trie

```
// insert O(Lgx)
// query O(Lgx)
// 處理XOR問題
// struct
struct trie{
    trie *nxt[2]; // 差別
    int cnt; //紀錄有多少個數字以此節點結尾
    int sz; //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root = new trie();
void insert(int x){
    trie *now = root; // 每次從根結點出發
    for(int i=30;i>=0;i--){
        now->sz++;
        if(now->nxt[x>>i&1] == NULL){
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1]; //走到下一個字母
    }
    now->cnt++;
    now->sz++;
}
// in this set, the maximum value of bitwise XOR x
int query(int x){
    trie *now = root;
    int ans=0;
    for(int i=30;i>=0;i--){ // 不等於為1(0xr1=1,1xr0=1)
        if (now->nxt[!(x>>i&1)] != NULL && now->nxt[!(x>>i&1)]->sz > 0){ //下一個存在
            ans += 1<<i;
            now = now->nxt[!(x>>i&1)];
        }
        else now = now->nxt[x>>i&1];
    }
    return ans;
}
```

9.3 Hash

```
// build O(n)
// query O(1)
// double hash
// P = 53,97,193,49157,805306457,1610612741,1e9+9,1e9+7
const ll P1 = 75577;
const ll P2 = 12721; // 多一個質數 p2
const ll MOD = 998244353;
pair<ll,ll> Hash[MXN]; //Hash[i] 為字串 [0,i] 的hash值
void build(const string& s){
    pair<ll,ll> val = make_pair(0,0);
    for(int i=0; i<s.size(); i++){
        val.first = (val.first * P1 + s[i]) % MOD;
        val.second = (val.second * P2 + s[i]) % MOD;
        Hash[i] = val;
    }
}
// query:
// H[L,R] = Hr - H(L-1) * p^(r-L+1) %MOD + MOD %MOD
```

10 STL

10.1 常用 tool

```
// freopen()
freopen("intel.in", "r", stdin); // 讀 file.in 檔
freopen("intel.out", "w", stdout); // 寫入 file.out 檔
//二進制"1"的個數
__builtin_popcount(n) -> int
__builtin_popcountl(n) -> long int
__builtin_popcountll(n) -> long long
//陣列處理
sort(arr, arr+n);
reverse(arr, arr+n);
*min_element(arr, arr+n); //value
min_element(arr, arr+n) - arr; //index
*lower_bound(arr, arr+4, c) << '\n'; //第一個大於等於c
*upper_bound(arr, arr+4, c) << '\n'; //第一個大於c
fill(arr, arr+3, 123);
//輸出
cout << fixed << setprecision(10);
cout << setw(n) << setfill(c);
//迭代器
T.begin()
T.end()
T.rbegin() //逆序迭代器
T.rend() //逆序迭代器
T.find() //可用於set, map的erase()。
```

10.2 Erase 操作

```
// erase
iterator erase (it position); //刪除指定元素
iterator erase (it first, it last); //刪除指定範圍
// 刪除指定數值
auto it = find(p.begin(), p.end(), val);
p.erase(it);
// 刪除所有指定數值
p.erase(remove(p.begin(), p.end(), val), p.end());
```

10.3 StringStream

```
stringstream ss;
getline(cin, str);
ss.str("");
ss.clear();
ss << oct << s; //以8進制讀入流中
ss << hex << s; //以16進制讀入流中
ss >> n; //10進制int型輸出
ss >> s; //x進制str型輸出
```

10.4 List

```
• push_back()
• pop_back()
• push_front()
• pop_front()
• back()
• front()
• insert(index, obj)
• erase()
//遍歷
for (auto iter = _list.begin(); iter != _list.end();
    iter++)
    cout << *iter << "\n";
```

10.5 Set

```
unordered_set<int> u_set;
multiset<int> st;
• insert(val)
• erase(l, r) //l與r皆為iterator
• erase(it)
• erase(val)
• count(val) //元素是否存在
it = st.find(val);
it = st.lower_bound(val);
it = st.upper_bound(val);
//遍歷
int mints[] = { 75, 23, 65, 42, 13, 75, 65 };
set<int> myset(myints, myints + 7);
for (auto it = myset.begin(); it != myset.end(); it++)
    cout << ' ' << *it;
```

10.6 Map

```
// 無序map O(1)
unordered_map<type, type> u_map;
//find
auto iter = mymap.find("a");
if (iter != mapStudent.end())
    cout << "Find, the value is" << iter->second << endl;
else
    cout << "Do not Find" << endl;
//erase
auto iter = mymap.find("a");
mymap.erase(iter);
//map遍歷
for (auto it = mymap.begin(); it != mymap.end(); it++)
    cout << it->first << ", " << it->second << endl;
```

10.7 Priority Queue

```
//預設由大排到小
priority_queue<T> pq
priority_queue<int, vector<int>, less<int> > pq;
//改成由小排到大
priority_queue<T, vector<T>, greater<T> > pq;
//自行定義 cmp 排序
priority_queue<T, vector<T>, cmp> pq;
struct cmp {
    bool operator()(node a, node b) {
        //priority_queue優先判定為!cmp
        //，所以「由大排到小」需「反向」定義
        //實現「最小值優先」
        return a.x < b.x;
    }
};
```

10.8 Bitset

```
//init
b.set(); //每個位元設 '1'
b.reset(); //每個位元設 '0'
b[pos] = 1;
//轉換
s = b.to_string();
unsigned long x = b.to_ulong();
//sum
b.any(); //判別是否有 '1'
b.none(); //判別是否沒 '1'
cnt = b.count(); //判別 '1' 之個數
cnt = b.size() - b.count(); //判別 '0' 之個數
```

11 Flow

11.2 Dinic

11.1 MinCostMaxFlow

```

struct MinCostMaxFlow{
typedef int Tcost;
static const int MAXV = 20010;
static const int INFF = 1000000;
static const Tcost INFC = 1e9;
struct Edge{
int v, cap;
Tcost w;
int rev;
Edge(){
Edge(int t2, int t3, Tcost t4, int t5)
: v(t2), cap(t3), w(t4), rev(t5) {}
};
int V, s, t;
vector<Edge> g[MAXV];
void init(int n, int _s, int _t){
V = n; s = _s; t = _t;
for(int i = 0; i <= V; i++) g[i].clear();
}
void addEdge(int a, int b, int cap, Tcost w){
g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
}
Tcost d[MAXV];
int id[MAXV], mom[MAXV];
bool inqu[MAXV];
queue<int> q;
pair<int,Tcost> solve(){
int mxf = 0; Tcost mnc = 0;
while(1){
fill(d, d+1+V, INFC);
fill(inqu, inqu+1+V, 0);
fill(mom, mom+1+V, -1);
mom[s] = s;
d[s] = 0;
q.push(s); inqu[s] = 1;
while(q.size()){
int u = q.front(); q.pop();
inqu[u] = 0;
for(int i = 0; i < (int) g[u].size(); i++){
Edge &e = g[u][i];
int v = e.v;
if(e.cap > 0 && d[v] > d[u]+e.w){
d[v] = d[u]+e.w;
mom[v] = u;
id[v] = i;
if(!inqu[v]) q.push(v), inqu[v] = 1;
} } }
if(mom[t] == -1) break ;
int df = INFF;
for(int u = t; u != s; u = mom[u])
df = min(df, g[mom[u]][id[u]].cap);
for(int u = t; u != s; u = mom[u]){
Edge &e = g[mom[u]][id[u]];
e.cap -= df;
g[e.v][e.rev].cap += df;
}
mxf += df;
mnc += df*d[t];
}
return {mxf,mnc};
} }flow;

```

```

struct Edge{ int v,f,re; };
int n,s,t,level[MXN];
vector<Edge> E[MXN];
void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
}
void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
}
bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
        int u = que.front(); que.pop();
        for (auto it : E[u]){
            if (it.f > 0 && level[it.v] == -1){
                level[it.v] = level[u]+1;
                que.push(it.v);
            } }
    }
    return level[t] != -1;
}
int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
        if (it.f > 0 && level[it.v] == level[u]+1){
            int tf = DFS(it.v, min(nf,it.f));
            res += tf; nf -= tf; it.f -= tf;
            E[it.v][it.re].f += tf;
            if (nf == 0) return res;
        } }
    if (!res) level[u] = -1;
    return res;
}
int flow(int res=0){
    while ( BFS() )
        res += DFS(s,2147483647);
    return res;
} }flow;

```

[illegible][illegible]

@
Hong~Long~Long~Long~

/ _ o \ _____ / | \ _ / \ / - AC | AC | NO BUG / == * - * * -
[^ ^] Chong~Chong~Chong~

11.3 KM

```

struct KM{ // max weight, for min negate the weights
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) { // 1-based, N個節點
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;} //左
        邊的集合節點x連邊右邊集合節點y權重為w
    void augment(int y) {
        for(int x, z; y; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x, sy[y]=t;
                } }
                ll cut = INF;
                for(int y=1; y<=n; ++y)
                    if(!vy[y]&&cut>sy[y]) cut=sy[y];
                for(int j=1; j<=n; ++j){
                    if(vx[j]) lx[j] -= cut;
                    if(vy[j]) ly[j] += cut;
                    else sy[j] -= cut;
                }
                for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                    if(!my[y]){augment(y);return;}
                    vy[y]=1, q.push(my[y]);
                } } }
    ll solve(){ // 回傳值為完美匹配下的最大總權重
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    } }graph;

```

12 Match

12.1 run

```

set -e
g++ ac.cpp -o ac
g++ wa.cpp -o wa
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done

```

12.2 generate

```

# sample
from random import *
t = randint(1,10)
print(t)
for i in range(t):
    n = randint(1,5)
    m = randint(1,3)
    print(n, m)
    for j in range(n):
        print(randint(1,n),end=" ")
    print()
    for j in range(m):
        print(randint(1,m),end=" ")
    print()
# rand
n = randint(1, 100)
ch = chr(randint(ord('a'), ord('z')))
choiceSet = sample(s, 4) # s choice 4
choiceSet = sample(range(1, n+1), 4) # 1~n choice 4
shuffle(arr) # permutation
# tree
for i in range(2, n+1):
    print(i, randint(1, i-1))
# cpp
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

```

13 Other

```

/*unroll-Loops*/
#pragma GCC optimize("00")//不優化(預設)
#pragma GCC optimize("01")//優化一點
#pragma GCC optimize("02")//優化更多
#pragma GCC optimize("03")//02優化再加上inline函式優化
#pragma GCC optimize("unroll-Loops")
/*常數宣告*/
#define MXN 1'000'005
#define EPS 1e-6
#define INF 0x3f3f3f3f
#define PI acos(-1)

```