Contents

1 Basic

1.1 Default cod

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define EPS 1e-6
#define MOD 1e9+7
#define INF 0x3f3f3f3f
#define LINF 0x3f3f3f3f3f3f3f3f
#define PI acos(-1)
```

1.2 .vimrc

```
set nocp nu rnu cul ru ai cin si sta
set sc sta1=2 ls=2 so=5 ts=4 sw=4 sts=4 et
set hls sm is ic scs wrap lbr bg=dark
set encoding=utf8
set mouse=a
filetype plugin indent on
syntax enable
no ; :
no <C-l> :nohl<CR>
au filetype c,cpp ino <F9> <ESC>:w<CR>:!~/run '%'<CR>
au InsertLeave *.cpp write
let leader = '\'
function! Tg()
    s,^\(\s*\)\?,\1// ,e
    s,^\(\s*\)\(// \?\)\{2},\1,e
endfunc
au filetype c,cpp no <leader><leader> :call Tg()<CR>
```

1.3 Run

```
set -e
g++ ac.cpp -o ac
g++ wa.cpp -o wa
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

1.4 Generate

```
# sample
from random import *
t = randint(1,10)
print(t)
for i in range(t):
    n = randint(1,5)
    print(n)
    for j in range(n):
        print(randint(1,n),end=" ")
    print()
# rand
ch = chr(randint(ord('a'), ord('z')))
choiceSet = sample(s, 4)              # s choice 4
choiceSet = sample(range(1, n+1), 4) # 1~n choice 4
shuffle(arr) # permutation
# tree
for i in range(2, n+1):
    print(i, randint(1, i-1))
# cpp
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }
```

1.5 PBDS

```
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx; //rope
using pq_t = __gnu_pbds::priority_queue<int>;
using set_t = tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update>;

// hash_map
cc_hash_table<int, int> mp1;
gp_hash_table<int, int> mp2;

// priority_queue
pq_t pq;
pq.join(PQ);

// Tree(Treap,rank)
// 0-base,O(logn)
set_t Tree;
Tree.insert(val);
Tree.erase(val);
Tree.join(tree);                // 插入tree
Tree.split(v, tree);            // Tree(<=v),tree(>v)
rank = Tree.order_of_key(val); // val的排名
it   = Tree.find_by_order(rk); // 第rk小的it
it   = Tree.lower_bound(val);
it   = Tree.upper_bound(val);
// Replacing treap and rank tree
// Cannot be repeated (like set)

// rope (可持久化)
// string
rope<char> s[2];
s[1] = s[0] //O(1)複製 可持久化
s[0].insert(pos,str.c_str());
s[0].erase(pos.str.c_str());
s[0].size();
s[0].substr(pos, x);
s[0].copy(pos, x, s);
cout << s[0] + s[1] <<"\n";
// int[]
rope<int> x;
x.insert(0,12);
x.push_back(val);
for(auto val:x) cout << val <<"\n";
```

1.6 DS

```
unordered_map<type,type> u_map; //O(1)
unordered_set<int> u_set;
multiset<int> st;
priority_queue<T, vector<T>, less<T>> pq;
priority_queue<T, vector<T>, greater<T>> pq;
priority_queue<T, vector<T>, cmp> pq;
struct cmp {
    bool operator()(node a, node b) {
    //priority_queue優先判定為!cmp
  //，所以「由大排到小」需「反向」定義
    //實現「最小值優先」
        return a.x < b.x;
    }
};
// erase操作
iterator erase (it position);//刪除指定元素
iterator erase (it first, it last);//刪除指定範圍
// 刪除指定數值
auto it = find(p.begin(), p.end(), val);
p.erase(it);
// 刪除所有指定數值
p.erase(remove(p.begin(), p.end(), val), p.end());
```

1.7 IO

```
// file
freopen("intel.in","r",stdin);    // 讀 file.in 檔
freopen("intel.out","w",stdout);  // 寫入 file.out 檔
// output
cout << fixed << setprecision(10);
cout << setw(n) << setfill(c);
```

1.8 Math

```
abs(x);
pow(x);
sqrt(x);
__gcd(x, y);//c++14
lcm(x,y);   //c++17
__lg(x)     //以2為底數
log(x)      //以e為底數
log10(x)    //以10為底數
do {        //排列
    cout << s << "\n";
} while (next_permutation(s.begin(), s.end()));
prev_permutation(s.begin, s.end()) //大到小
set_intersection(v1.begin(), v1.end(), v2.begin(), v2.
    end(), v.begin());
set_union(v1.begin(), v1.end(), v2.begin(), v2.end(), v
    .begin());
set_difference(v1.begin(), v1.end(), v2.begin(), v2.end
    (), v.begin());
```

1.9 Bit

```
// bitset
bitset<30>(string)
b.set();
b.reset();
s = b.to_string();
unsigned long x = b.to_ullong();
b.any();//判別是否有 '1'
b.none();//判別是否沒 '1'
cnt = b.count();// 判別 '1' 之個數
cnt = b.size() - b.count();//判別 '0' 之個數
__builtin_popcount(n)   -> int
__builtin_popcountl(n)  -> long int
__builtin_popcountll(n) -> long long
__builtin_popcountll  //換成二進位有幾個1
__builtin_clzll       //返回左起第一個1之前0的個數
__builtin_parityll    //返回1的個數的奇偶性
__builtin_mul_overflow(a,b,&h) //回傳a*b是否溢位
```

2 Math

2.1 卡塔蘭數 Catalan number

```
// 1, 1, 2, 5, 14, 42
ll c[N];
c[0] = 1;
for(int i=1;i<n;i++)
    c[i]=2*(2*(i-1)+1)*c[i-1]/(i+1);
```

2.2 錯排公式 Derangements

```
// 1, 0, 1, 2, 9, 44, 265
ll d[n]; //d[0]=1
d[1]=0, d[2]=1;
for(int i=3;i<n;i++){
    d[i]=(i-1)*(d[i-1]+d[i-2]);
}
```

2.3 Euler function

```
ll phi(ll n){   // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n;    // O(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
    }   }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

2.4 Euler function(建表)

```
void phi_table(int n,int *phi){
    memset(phi,0,sizeof(phi)); //初始化
    phi[1] = 1;
    for(int i=2;i<=n;i++){
        if(phi[i]==0){
            for(int j=i;j<=n;j+=i){
                if(phi[j]==0) phi[j] = j;
                phi[j] = phi[j] / i * (i - 1);
}}}}
```

2.5 Sieve Prime

```
const int N = 20000000;//質數表大小
bool sieve[N];
vector<int> prime;
void linear_sieve(){
    for (int i = 2; i < N; i++){
        if (!sieve[i]) prime.push_back(i);
        for (int p : prime){
            if (i * p >= N) break;
            sieve[i * p] = true;
            if (i % p == 0) break;
}}}
```

2.6 josephusl

```
/*約瑟夫問題：n個人圍成一桌，數到m的人出列*/
int josephus(int n, int m) {  //n人每m次
    int ans = 0;
    for (int i = 1; i <= n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

2.7 快速冪

```
/*快速冪*/
ll FastPow(ll x, ll y, ll p) {
    long long ans = 1;
    while (y) {
        if (y & 1)    ans = ans * x % p; //prime
        x = x * x % p;
        y >>= 1;
    }
    return ans;
}
```

2.8 Cn 取 m 模反, Combination, inverse

```
#define MXN 1'000'005
#define N 1'000'000
long long fac[MXN], inv[MXN];
fac[0] = 1; // 0! = 1
for(long long i = 1; i <= N; i++)
    fac[i] = fac[i-1] * i % MOD; // 階乘
inv[N] = FastPow(fac[N], MOD-2); // 模逆元
for(ll i = N-1; i >=0; i--)
    inv[i] = inv[i+1] * (i+1) % MOD;
ll c(ll n, ll m){return fac[n]*inv[m]*inv[n-m];}
```

2.9 質因數分解 Factorizationn

```
int factor[MXN];
for(ll i = 2; i <= N; i++){
    if(factor[i])    continue;
    for(ll j = i*i; j <= N; j+=i){
        factor[j] = i;
    }
}
map<int, int> factorization(int x){
    map<int, int> prime;
    while(factor[x]){
        prime[factor[x]]++;
        x /= factor[x];
    }
    prime[x]++;
    return prime;
}
```

2.10 乘法取餘 Multiplication

```
/*大數乘法取餘數*/
ll mul(ll x, ll y, ll mod) {
    ll ret = x * y - (ll)((long double)x / mod * y) *
        mod;
    // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
    return ret < 0 ? ret + mod : ret;
}
```

2.11 ax+by=gcd(a,b)

```
/*ax+by=gcd(a,b) 一組解*/
ll a, b, x, y;
ll exgcd(ll a, ll b, ll& x, ll& y) {
    if (b) {
        ll d = exgcd(b, a % b, y, x);
        return y -= a / b * x, d;
    }
    return x = 1, y = 0, a;
}
```

2.12 Miller Rabinn

```
/*Miller_Rabin 質數判定*/
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
ll magic[N] = {};
bool witness(ll a, ll n, ll u, int t) {
    if (!a) return 0;
    ll x = mypow(a, u, n);  //快速冪
    for (int i = 0; i < t; i++) {
        ll nx = mul(x, x, n);   //快速乘
        if (nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(ll n) {
    int s = (magic number size);
        // iterate s times of witness on n
        if (n < 2) return 0;
    if (!(n & 1)) return n == 2;
    ll u = n - 1; int t = 0;
    // n-1 = u*2^t
    while (!(u & 1)) u >>= 1, t++;
    while (s--) {
        ll a = magic[s] % n;
        if (witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

2.13 快速乘法 karatsuba

```
/*karatsuba 快速乘法*/
// Get size of the numbers
int getSize(ll num){
    int count = 0;
    while (num > 0)
    {
        count++;
        num /= 10;
    }
    return count;
}
ll karatsuba(ll X, ll Y){
    // Base Case
    if (X < 10 && Y < 10)
        return X * Y;
    // determine the size of X and Y
    int size = fmax(getSize(X), getSize(Y));
    // Split X and Y
    int n = (int)ceil(size / 2.0);
    ll p = (ll)pow(10, n);
    ll a = (ll)floor(X / (double)p);
    ll b = X % p;
    ll c = (ll)floor(Y / (double)p);
    ll d = Y % p;
    // Recur until base case
    ll ac = karatsuba(a, c);
    ll bd = karatsuba(b, d);
    ll e = karatsuba(a + b, c + d) - ac - bd;
    // return the equation
    return (ll)(pow(10 * 1L, 2 * n) * ac + pow(10 * 1L,
         n) * e + bd);
}
```

2.14 矩陣快速冪

```
LL len,mod;
vector<vector<LL>> operator*(vector<vector<LL>> x,
    vector<vector<LL>> y){
    vector<vector<LL>> ret(len,vector<LL>(len,0));
    for(int i=0;i<len;i++){
        for(int j=0;j<len;j++){
            for(int k=0;k<len;k++){
                ret[i][j]=(ret[i][j]+x[i][k]*y[k][j])%
                    mod;
    }   }   }
    return ret;
}
struct Martix_fast_pow{ //O(len^3 lg k)
    LL init(int _len,LL m=9223372036854775783LL){
        len=_len, mod=m;
    }   // mfp.solve(k,{0, 1}, {1, 1}) k'th fib {值,係
        數} // 0-base
    LL solve(LL n,vector<vector<LL>> poly){
        if(n<len)   return poly[n][0];
        vector<vector<LL>> mar(len,vector<LL>(len,0)),x
            (len,vector<LL>(len,0));
        for(int i=0;i<len;i++)    mar[i][i]=1;
        for(int i=0;i+1<len;i++)  x[i][i+1]=1;
        for(int i=0;i<len;i++)    x[len-1][i]=poly[i
            ][1];
        while(n){
            if(n&1) mar=mar*x;
            n>>=1, x=x*x;
        }
        LL ans=0;
        for(int i=0;i<len;i++)   ans=(ans+mar[len-1][i
            ]*poly[i][0]%mod)%mod;
        return ans;
    }
}mfp;
```

2.15 GaussElimination

```
/*GaussElimination*/
// by bcw_codebook
const int MAXN = 300;
const double EPS = 1e-8;
int n;
double A[MAXN][MAXN];
void Gauss() {
  for(int i = 0; i < n; i++) {
    bool ok = 0;
    for(int j = i; j < n; j++) {
      if(fabs(A[j][i]) > EPS) {
        swap(A[j], A[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;
    double fs = A[i][i];
    for(int j = i+1; j < n; j++) {
      double r = A[j][i] / fs;
      for(int k = i; k < n; k++) {
        A[j][k] -= A[i][k] * r;
      }
    }
  }
}
```

2.16 大數 Big number

```
/*大數(Big Number)*/
void add(int a[100], int b[100], int c[100]){
    int i = 0, carry = 0;
    for (i = 0; i < 100; ++i) {
        c[i] = a[i] + b[i] + carry;
        carry = c[i] / 10;
        c[i] %= 10;
    }
}
void sub(int a[100], int b[100], int c[100]){
    int i = 0, borrow = 0;
    for (i = 0; i < 100; ++i) {
        c[i] = a[i] - b[i] - borrow;
        if (c[i] < 0) {
            borrow = 1;
            c[i] += 10;
        }
        else
            borrow = 0;
    }
}
void mul(int a[100], int b[100], int c[100]){
    int i = 0, j = 0,carry = 0;
    for (i = 0; i < 100; ++i) {
        if (a[i] == 0) continue;
        for (j = 0; j < MAX; ++j)
            c[i + j] += a[i] * b[i];
    }
    for (i = 0; i < MAX; ++i) {
        carry = c[i] / 10;
        c[i] %= 10;
    }
}
void div(int a[100], int b[100], int c[100]){
    int t[100];
    for (i = 100 - 1; i >= 0; i--) {
        for (int k = 9; k > 0; k--) // 嘗試商數
        {
            mul(b + i, k, t);
            if (largerthan(a + i, t))
            {
                sub(a + i, t, c + i);
                break;
            }
        }
    }
}
```

3 Geometry

3.1 Struct

```
struct Pt{
    ld x, y;
};
struct Line{
    Pt st, ed;
};
struct Circle{
    Pt o; // 圓心
    ld r; // 半徑
};
struct poly{
    int n; // n 邊形
    vector<Pt> pts;
};
struct Pt {
  ll x, y; // ll or ld
  Pt(){}
  Pt(ll _x,ll _y){ x=_x, y=_y; }
  Pt operator+(const Pt &a) const{
    return Pt(x+a.x, y+a.y);
  }
  Pt operator-(const Pt &a) const{
    return Pt(x-a.x, y-a.y);
  }
  Pt operator*(const ld &a) const{
    return Pt(x*a, y*a);
  }
  Pt operator/(const ld &a) const{
    return Pt(x/a, y/a);
  }
  ll operator*(const Pt &a) const{ //內積
    return x*a.x + y*a.y;
  }
  ll operator^(const Pt &a) const{ //外積
    return x*a.y - y*a.x;
  }
  bool operator<(const Pt &a) const{
    return x < a.x || (x == a.x && y < a.y);
  }
  friend int cross(const Pt& o,const Pt& a,const Pt& b)
      { //向量外積
    Pt lhs = o-a, rhs = o-b;
    return lhs.x*rhs.y - lhs.y*rhs.x;
  }
  friend bool operator ==(const Pt& lhs,const Pt& rhs){
    return (rhs.x==lhs.x && rhs.y==lhs.y);
  }
};
```

3.2 Function

```
//是否三點共線
bool collinearity(const Pt& a, const Pt& b, const Pt& c
    ){
    return (b-a)^(c-a) < EPS; //外積為0: ==0
}
//判斷點是否在線段上
bool inLine(const Pt& p, const Line& li){
    return collinearity(li.st, li.ed, p) && (li.st-p)*(
        li.ed-p) < EPS; //內積為負: <=0
}
//兩點距離
ld dis(Pt i,Pt j){ // (int i,int j)
    Pt vt = i - j; // Pt vt = p[i]-p[j];
    return sqrt((ld)(vt.x)*(vt.x)+(ld)(vt.y)*(vt.y));
}
//平行四邊形面積
ll areaPt(Pt i, Pt j, Pt k){
    return abs(cross(i, j, k));
}
//多邊形面積
ld area(vector<Pt> pt){
    ld sum=0;
    for(int i=0;i<pt.size();i++){
        Pt a=pt[i], b=pt[(i+1)%pt.size()];
        sum+=(a^b);
    }
    return abs(sum/2.0);
}
// 輸出
round(double x); //四捨五入至整數
cout << fixed << setprecision(11); //精度
void print(ll x){ //兩倍面積輸出判斷
    if(x&1)
        cout << (x>>1) << ".5\n";
    else
        cout << (x>>1) <<"\n";
}
```

3.3 凸包 Convex hull

```
vector<Pt> convex_hull(vector<Pt> hull){ //凸包
    sort(hull.begin(),hull.end());
    int top=0;
    vector<Pt> stk;
    for(int i=0;i<hull.size();i++){
        while(top>=2&&cross(stk[top-2],stk[top-1],hull[
            i])<=0) //或<0
            stk.pop_back(),top--;
        stk.push_back(hull[i]);
        top++;
    }
    for(int i=hull.size()-2,t=top+1;i>=0;i--){
        while(top>=t&&cross(stk[top-2],stk[top-1],hull[
            i])<=0) //或<0
            stk.pop_back(),top--;
        stk.push_back(hull[i]);
        top++;
    }
    stk.pop_back();
    return stk;
}
```

3.4 旋轉卡尺

```
// 最遠兩點配對
double FarthestPair(vector<Pt> arr){
  double ret = 0;
  for (int i = 0, j = i + 1; i < arr.size(); i++){
    while (distance(i, j) < distance(i, (j + 1) % arr.
        size())){
      j = (j + 1) % arr.size();
    }
    ret = max(ret, distance(i, j));
  }
  return ret;
}
// 點是否凸包內
bool in_hull(vector<Pt> &hull, Pt p){
  int flag = 0;
  for (int i = 0; i < hull.size(); i++){
    if (cross(p, hull[i], hull[(i + 1) % hull.size()])
        > 0){
      //與凸包上前後兩點作外積，如果夾角在0~180度內，則
          在內側。
      flag++;
    }
  }
  if (flag == hull.size())
    return true;
  else
    return false;
}

// 線找點最短距離
ld rot(vector<Pt> hull){
  ld mn = 2.0 * r;
  int n = hull.size();
  hull.pb(hull[0]);
  for (int i = 0, j = i + 2; i < n; i++){
    //平行四邊形面積 除以 底 = 高
    ld area = abs(cross(hull[i], hull[i + 1], hull[j]))
        ;
    ld base = abs(dis(hull[i], hull[i + 1]));
    while (area <= abs(cross(hull[i], hull[i + 1], hull
        [(j + 1) % n]))){
      j = (j + 1) % n;
      area = abs(cross(hull[i], hull[i + 1], hull[j]));
    }
    mn = min(mn, area / base);
  }
  return mn;
}
```

```
// 最大三角形
ld solve(vector<Pt> arr){
  ld ret = 0;
  for (int i = 0; i < arr.size(); i++)
    for (int j = i + 1; j < arr.size(); j++)
      for (int k = j + 1; k < arr.size(); k++)
        ret = max(ret, area(i, j, k));
  return ret;
}
// 最大四邊形
ll solve(){//兩倍面積
  ll ret = 0;
  for (int i = 0; i < hull.size(); i++){
    int k = (i + 1) % hull.size(), k2 = (i + 3) % hull.
        size();
    for (int j = i + 2; j <= i + hull.size() - 2; j++){
      while (area(i, j % hull.size(), k) < area(i, j %
          hull.size(), (k + 1) % hull.size())){
        k = (k + 1) % hull.size();
      }
      while (area(i, j % hull.size(), k2) < area(i, j %
          hull.size(), (k2 + 1) % hull.size())){
        k2 = (k2 + 1) % hull.size();
      }
      ret = max(ret, area(i, j % hull.size(), k) + area
          (i, j % hull.size(), k2));
    }
  }
  return ret;
}
```

4 Algorithm

4.1 Brute Force

```
#define MAXN 1<<18+5 //雙倍空間
/*折半枚舉 與 二進制枚舉*/
int main() {
    int n, m, i, temp;
    ll mod, mod_max = 0;
    vector<ll> arr, ans(MAXN,0), ans2(MAXN,0);
    cin >> n >> m;
    for(i=0;i<n;i++){
        cin >> temp;
        arr.push_back(temp%m);
    }
    //折半枚舉
    for(int i=0;i<(1<<(n/2));i++){ //2^(n/2)
        for(int j=0;j<n/2;j++){
            if(i>>j&1) //二進制枚舉(選或不選)
                ans[i] = (ans[i] + arr[j]) % m; //前半
                    枚舉
        }
    }
    for(int i=0;i<(1<<(n-n/2));i++){ //2^(n-n/2)
        for(int j=0;j<(n-n/2);j++){
            if(i>>j&1) ans2[i] = (ans2[i] + arr[n/2+j])
                % m; //後半枚舉
        }
    }
    //二分維護
    temp = 1<<(n-n/2);
    sort(ans2.begin(), ans2.begin() + temp);
    for(auto i:ans){
        mod_max = max(mod_max, i + *(upper_bound(ans2.
            begin(), ans2.begin() + temp, m-1-i)-1));
        //mod最大為m-1，配對另一半最優解
    }
    cout << mod_max <<"\n";
}
```

4.2 Divide and Conquer

```
/*最近點對*/
double dist(pair<double, double> a, pair<double, double
    > b)
  return sqrt(pow((a.first - b.first), 2) + pow((a.
      second - b.second), 2));
double closest(int l, int r) {
  if (l >= r)
    return 10000;
  int mid = (l + r) / 2;
  double radl = closest(l, mid);
  double radr = closest(mid + 1, r);
  double range = min(radl, radr), middle = (point[mid].
      first + point[mid + 1].first) / 2, minimum;
  minimum = range;
  for (int i = mid + 1; i <= r && point[i].first <
      middle + range; i++) {
    for (int j = mid; j >= l && point[j].first > middle
         - range; j--) {
      minimum = min(minimum, dist(point[i], point[j]));
    }
  }
  return minimum;
}
void quicksort(int arr[], int left, int right){
    if (left >= right) return;
    int i = left, j = right;
    int k = (left + right) / 2;
    int pivot = arr[k];
    while (true){
        while (arr[i] < pivot) i++;
        while (arr[j] > pivot) j--;
        if (i >= j) return;
        swap(arr[i], arr[j]);
        i++; j--;
    }
    quicksort(arr, left, j);
    quicksort(arr, i, right);
}
void Merge(vector<int> &Arr, int front, int mid, int
    end) {
    vector<int> Left(Arr.begin() + front, Arr.begin() +
         mid + 1);
    vector<int> Right(Arr.begin() + mid + 1, Arr.begin
        () + end + 1);
    int idxLeft = 0, idxRight = 0;
    Left.insert(Left.end(), INT_MAX);
    Right.insert(Right.end(), INT_MAX);
    for (int i = front; i <= end; i++) {
        if (Left[idxLeft] < Right[idxRight]) {
            Arr[i] = Left[idxLeft];
            idxLeft++;
        } else {
            Arr[i] = Right[idxRight];
            idxRight++;
        }
    }
}
void MergeSort(vector<int> &Arr, int front, int end) {
    if (front >= end)
        return;
    int mid = front + (end - front) / 2;
    MergeSort(Arr, front, mid);
    MergeSort(Arr, mid + 1, end);
    Merge(Arr, front, mid, end);
}
```

5 Data Structure

5.1 SQRT

```
// build  O(n)
// update O(√n)
// query  O(√n)
//分塊結構
//假設要求區間總和
struct blk{
    vector<int> local;    //每塊的全部元素
    int global;           //儲存每塊的總和
    int tag;              //儲存整塊一起更新的值
    blk(){                //初始化
        local.clear();    //清空區間元素
        tag = global = 0; //將區間總和先設為0
    }
};
vector<blk> b;
void build(){
    int len=sqrt(n),num=(n+len-1)/len;
    for(int i=0;i<n;i++){    //第i個元素分在第 i/len 塊
        cin>>x;
        //存入區間中
        b[i/len].local.push_back(x);
        //更新區間總和
        b[i/len].global += x;
    }
}
void update(int ql,int qr,int v){
    int blk_l=ql/len,blk_r=qr/len,ret=0;
    if(blk_l == blk_r){
        //如果都在同一塊直接一個一個跑過去就好
        for(int i=ql;i<=qr;i++)
            b[blk_l].local[i%len]+=v;
        b[blk_l].global+=(qr-ql+1)*v;
        return;
    }
    for(int i=ql;i<(blk_l+1)*len;i++){  //最左的那一塊
        b[blk_l].local[i%len]+=v;
        b[blk_l].global+=v;
    }
    for(int i=blk_l+1;i<blk_r;i++){  //中間每塊
        b[i].tag+=v;
        b[i].global+=v*len;
    }
    for(int i=blk_r*len;i<=qr;i++){  //最右的那一塊
        b[blk_r].local[i%len]+=v;
        b[blk_r].global+=v;
    }
}
int query(int ql,int qr){
    int blk_l=ql/len,blk_r=qr/len,ret=0;
    if(blk_l == blk_r){
        //如果都在同一塊直接一個一個跑過去就好
        for(int i=ql;i<=qr;i++)
            ret+=b[blk_l].local[i%len]+b[blk_l].tag;
        return ret;
    }
    for(int i=ql;i<(blk_l+1)*len;i++)    //最左的那一塊
        ret+=b[blk_l].local[i%len]+b[blk_l].tag;
    for(int i=blk_l+1;i<blk_r;i++)    //中間每塊的總和
        ret+=b[i].global;
    for(int i=blk_r*len;i<=qr;i++)    //最右的那一塊
        ret+=b[blk_r].local[i%len]+b[blk_r].tag;
    return ret;
}
```

5.2 Mo's Algorithm

```
// n為序列總長度，q為詢問比數，p為移動一格的複雜度
// O(p(q+n)√n)
int n,k = sqrt(n);    //每塊大小為k
struct query{
    int l,r,id;       //詢問的左界右界 以及 第幾筆詢問
    friend bool operator<(const query& lhs,const query&
         rhs){
        return lhs.l/k==rhs.l/k ? lhs.r<rhs.r : lhs.l<
            rhs.l;
    }    //先判斷是不是在同一塊
         //不同塊的話就比較塊的順序，否則比較右界r
};
int num = 0;
int cnt[1000005], ans[30005];
vector<query> q;
void add(int x){    //新增元素到區間內
    ++cnt[x];
    if(cnt[x] == 1)    ++num;
}
void sub(int x){    //從區間內移除元素
    --cnt[x];
    if(cnt[x] == 0)    --num;
}
void solve(){
    sort(q.begin(),q.end());
    for(int i=0,l=-1,r=0;i<n;i++){
        while(l>q[i].l) add(--l);
        while(r<q[i].r) add(++r);//記得要先做新增元素的
        while(l<q[i].l) sub(l++);//再做移除元素的
        while(r>q[i].r) sub(r--);
        ans[q[i].id] = num;      //移到區間後儲存答案
}    }
```

5.3 Treap

```
struct Treap{
    int key,pri,sz;     //key,priority,size
    Treap *l, *r;       //左右子樹
    Treap(){}
    Treap(int _key){
        key = _key;
        pri = rand();   //隨機的數維持樹的平衡
        sz  = 1;
        l = r = nullptr;
    }
};
Treap *root;
int Size(Treap* x){ return x ? x->sz : 0 ; }
void pull(Treap *x)
{ x->sz = Size(x->l) + Size(x->r) + 1;}
Treap* merge(Treap *a,Treap *b){
    //其中一個子樹為空則回傳另一個
    if(!a || !b)    return a ? a : b;
    if(a->pri > b->pri){//如果a的pri比較大則a比較上面
        a->r = merge(a->r,b);//將a的右子樹跟b合併
        pull(a);
        return a;
    }
    else{  //如果b的pri比較大則b比較上面
        b->l = merge(a,b->l);//將b的左子樹根a合併
        pull(b);
        return b;
    }
}
void splitByKth(Treap *x,int k,Treap*& a,Treap*& b){
    if(!x){  a = b = nullptr;  }
    else if(Size(x->l) + 1 <= k){
        a = x;
        splitByKth(x->r, k - Size(x->l) - 1, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitByKth(x->l, k, a, b->l);
        pull(b);
    }
}
void splitByKey(Treap *x,int k,Treap*& a,Treap*& b){
    if(!x){  a = b = nullptr;  }
    else if(x->key<=k){
        a = x;
        splitByKey(x->r, k, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitByKey(x->l, k, a, b->l);
        pull(b);
    }
}
void insert(int val){           //新增一個值為val的元素
    Treap *x = new Treap(val);  //設一個treap節點
    Treap *l,*r;
    splitByKey(root, val, l, r);//找到新節點要放的位置
    root = merge(merge(l,x),r); //合併到原本的treap裡
}
void erase(int val){            //移除所有值為val的元素
    Treap *l,*mid,*r;
    splitByKey(root, val, l, r);//把小於等於val的丟到l
    splitByKey(l, val-1, l, mid);
    //小於val的丟到l,等於val的就會在mid裡
    root = merge(l,r);          //將除了val以外的值合併
}
int findVal(int val){ //小於等於val的size
    int size = -1;
    Treap *l, *r;
    splitByKey(root, val, l, r); //把小於等於val的丟到l
    size = Size(l);
    root = merge(l,r);
    return size;
}
void interval(Treap *&o, int l, int r) {// [l,r]區間
    Treap *a, *b, *c;
    splitByKey(o, l - 1, a, b), splitByKey(b, r, b, c);
    // operate
    o = merge(a, merge(b, c));
}
void inOrderTraverse(Treap* o, int print) {// 中序
    if (o != NULL){
        push(o);
        inOrderTraverse(o->l, print);
        // print
        if(print) cout << o->val <<"  ";
        inOrderTraverse(o->r, print);
}}
// Kth(k)：查找第k小的元素
int kth(Treap* o, int k){
    if(o == NULL || k > o -> sz || k <= 0)   return 0;
    int s = (o -> l == NULL ? 0 : o -> l -> sz);
    if(k == s + 1)  return o -> key;
    else if(k <= s) return kth(o -> l, k);
    else            return kth(o -> r, k - s - 1);
}
// Rank(x)：x的名次，即x是第幾小的元素
int rank(Node* o, int x){
    if(o == NULL) return 0;
    int res = 0;
    int s = (o -> l == NULL ? 0 : o -> l -> sz);
    if(x <= o -> key){
        res += rank(o -> l, x);
        res += x == o -> key;
    }
    else{
        res += s + 1;
        res += rank(o -> r, x);
    }
    return res;}
```

5.4 BIT

```
/*BIT 樹狀數組(動態前綴和)*/
//BIT and Array start at 1
#define MAXN 100005 //最大區間<MAXN
vector <int> arr(MAXN); //原始陣列
vector <int> bit(MAXN); //BIT數組

//前綴和查詢
ll query(int i) { //index
    ll ret = 0;
    while(i > 0) ret += bit[i], i -= i & -i; // 1-base
        i-lowbit(i)
    return ret;
}

//單點增值
void modify(int i, int val) { //index,value
    while(i <= MAXN) bit[i] += val, i += i & -i; // i+
        lowbit(i)
}
```

5.5 Segment tree

```
/*Segment tree 線段樹(區間問題)*/
//segment tree and Array start at 1
// [l,r] 最大區間設為[1,n]
// [ql,qr] 目標區間
// pos,val 修改位置,修改值
#define MAXN 100005*4  //tree大小為4n
#define cl(x) (x*2)    //左子節點index
#define cr(x) (x*2+1)  //右子節點index
#define NO_TAG 0 //懶惰記號
vector <int> tag(MAXN);
vector <int> arr(MAXN);
vector <int> tree(MAXN);

void build(int i,int l,int r){
    if(l == r){ // 遞迴到區間大小為1
        tree[i] = arr[l];
        return;
    }
    int mid=(l+r)/2; //往兩邊遞迴
    build(cl(i),l,mid);
    build(cr(i),mid+1,r);
    tree[i] = max(tree[cl(i)], tree[cr(i)]);       //<-
        可修改條件
    //將節點的值設成左右子節點的最大值
}

int query(int i,int l,int r,int ql,int qr){
    if(ql <= l && r <= qr){
        //若當前區間在詢問區間內，直接回傳區間最大值
        return tree[i];
    }
    int mid=(l+r)/2, ret=0; //<-可修改條件
    if(ql<=mid) // 如果左子區間在詢問區間內
        ret = max(ret, query(cl(i),l,mid,ql,qr));  //<-
            可修改條件
    if(qr> mid) // 如果右子區間在詢問區間內
        ret = max(ret, query(cr(i),mid+1,r,ql,qr));//<-
            可修改條件
    return ret;
}
```

```
/*單點修改*/
void update(int i,int l,int r,int pos,int val){
    if(l == r){ // 修改 a[pos] 的值為 val
        tree[i] = val;
        return;
    }
    int mid=(l+r)/2;
    if(pos <= mid) // 如果修改位置在左子節點，往左遞迴
        update(cl(i),l,mid,pos,val);
    else // 否則往右遞迴
        update(cr(i),mid+1,r,pos,val);
    tree[i] = max(tree[cl(i)], tree[cr(i)]);      //<-
        可修改條件
}

/*區間修改*/
//將區間 [l, r] 的值都加 v
void push(int i,int l,int r){
    if(tag[i] != NO_TAG){ // 判斷是否有打標記,NO_TAG=0
        tree[i] += tag[i];  // 有的話就更新當前節點的值
        if(l != r){  // 如果有左右子節點把標記往下打
            tag[cl(i)] += tag[i];
            tag[cr(i)] += tag[i];
        }
        tag[i] = NO_TAG; // 更新後把標記消掉
    }
}

void pull(int i,int l,int r){
    int mid = (l+r)/2;
    push(cl(i),l,mid); push(cr(i),mid+1,r);
    tree[i] = max(tree[cl(i)], tree[cr(i)]);
}

void update(int i,int l,int r,int ql,int qr,int v){
    push(i,l,r);
    if(ql<=l && r<=qr){
        tag[i] += v; //將區間 [l, r] 的值都加 v
        return;
    }
    int mid=(l+r)/2;
    if(ql<=mid)  update(cl(i),l,mid,ql,qr,v);
    if(qr> mid)  update(cr(i),mid+1,r,ql,qr,v);
    pull(i,l,r);
}

/*動態開點*/
struct node{
    node *l, *r;
    int val,tag;
};

void update(node *x, int l, int r, int ql, int qr, int
    v){
    push(x, l, r);
    if(ql <= l &&  r <= qr){
        x->tag += v;
        return;
    }
    int mid=(l+r)>>1;
    if(ql <= mid){
        if(x->l == nullptr)//判斷是否有節點
            x->l = new node();
        update(x->l, l , mid, ql, qr, v);
    }
    if(qr >  mid){
        if(x->r == nullptr)//判斷是否有節點
            x->r = new node();
        update(x->r, mid+1, r, ql, qr, v);
    }
    pull(x, l, r);
}
```

5.6 Persistent Segment tree and DSU

```
#define push_back emplace_back
struct node{
    ll val;
    node *l, *r;
    node(){val = 0;}
};
ll n,idx=0;
vector<node *> version;
//用一個vector紀錄全部版本的根節點
node mem[MAXN*25];
node *newNode(){
    return &mem[idx++];
}
node *build(int l, int r){
    node *x = newNode();
    if(l == r) return x;
    int mid = (l+r)>>1;
    x->l = build(l, mid);
    x->r = build(mid+1, r);
    return x;
}
node *update_version(node *pre, ll l, ll r, ll pos, ll
    v){
    node *x = newNode(); //當前位置建立新節點
    if(l == r){
        x->val = v;
        return x;
    }
    int mid = (l+r)>>1;
    if(pos <= mid){ //更新左邊
    //左邊節點連向新節點
        x->l = update_version(pre->l, l, mid, pos, v);
        x->r = pre->r;   //右邊連到原本的右邊
    }
    else{ //更新右邊
    //右邊節點連向新節點
        x->l = pre->l;   //左邊連到原本的左邊
        x->r = update_version(pre->r, mid+1, r, pos, v)
            ;
    }
    x->val = min(x->l->val, x->r->val); //<-修改
    return x;
}
ll query(node *x,int ql,int qr,int v){ //bin search
    if(ql == qr) return qr;
    int mid=(ql+qr)>>1;
    if(x->l->val<v) // 如果左子區間在詢問區間內
        return query(x->l,ql,mid,v);
    else// 如果右子區間在詢問區間內
        return query(x->r,mid+1,qr,v);
}
void add_version(int x,int v){ //修改位置 x 的值為 v
    version.push_back(update_version(version.back(), 0,
         n-1, x, v));
    //前一個版本
}
int find(int x) {
  int fa = query(version.back(), 0, n - 1, x);
  if (fa == x)return x;
  return find(fa);
}
void merge(int a, int b) {
  int fa = find(a), fb = find(b);
  if (sz[fa] < sz[fb])
    swap(fa, fb);
  sz[fa] += sz[fb];
  add_version(fb, fa);
}
signed main(){
    io
    ll q,temp,i,l,r;
    cin >> n >> q;
    version.push_back(build(0,n-1));
    for(i=1;i<=n;i++){
        cin >> temp;
        add_version(temp,i);
    }
    for(i=0;i<q;i++){
        cin >> l >> r;
        cout << query(version[r],0,n-1,l) <<"\n";
    }
}
```

5.7 Monotonic Queue(單調隊列)

```
vector<int> a; //array
deque<pair<int,int>> mx,mn;    //index, value
vector<pair<int,int>> dp(MAXN);//min,max
pair<int,int> calc(){
    pair<int,int> temp;
    temp.first = mn.front().second; temp.second = mx.
        front().second;
    return temp;
}
if(k>n)k=n;// k = windows size, n = array size
for(i=0;i<n;i++){   // dp 求最大值
    while(!mn.empty() && mn.front().first <= i-k)
        mn.pop_front();  //判斷dq裡的東西有沒有過期
    while(!mn.empty() && mn.back().second >= a[i])
        mn.pop_back();
    while(!mx.empty() && mx.front().first <= i-k)
        mx.pop_front();  //判斷dq裡的東西有沒有過期
    while(!mx.empty() && mx.back().second <= a[i])
        mx.pop_back();
    mn.push_back(make_pair(i,a[i]));
    mx.push_back(make_pair(i,a[i]));
    if(i>=k-1) dp[i] = calc();
}
```

6 Graph(path)

6.1 Disjoint Set(Union-Find)

```
int f[N];    // 宣告父節點陣列 f
int sz[N]; // 子樹大小
void init(int n) {
    for (int i = 0; i < n; i++){
        f[i] = i;
        sz[i] = 1;
    }
}
int find(int x) {
    return f[x] == x ? x : f[x] = find(f[x]);
}
void merge(int x, int y) {
    x = find(x), y = find(y);
    if (x != y){
        sz[x] += sz[y];
        f[y] = x;
    }
}
```

6.2 Kruskal’s algorithm 最小生成樹

```
/*Kruskal’s algorithm 最小生成樹*/
//搭配 Disjoint Set(Union-Find)
struct Edge {
    int u, v, w;  // 點 u 連到點 v 並且邊權為 w
    friend bool operator<(const Edge& lhs, const Edge&
        rhs) {
        return lhs.w > rhs.w;//兩條邊比較大小用邊權比較
    }
};
```

```
priority_queue<Edge> graph();// 宣告邊型態的陣列 graph
int kruskal(int m){
    int tot = 0;
    for (int i = 0; i < m ; i++) {
        if (find(graph.top().u) != find(graph.top().v))
             { // 如果兩點未聯通
            merge(graph.top().u, graph.top().v);
               // 將兩點設成同一個集合
            tot += graph.top().w;  // 權重加進答案
        }
        graph.pop();
    }
    return tot;
}
signed main() {
    cin >> n >> m; //node,edge
    init(n);
    for (int i = 0; i < m; i++) {
        cin >> u >> v >> w;
        graph.push(Edge{u,v,w});
    }
    cout << kruskal(m) << "\n";
}
```

6.3 Minimum Steiner Tree.cpp 最小斯坦因樹

```
// Minimum Steiner Tree 重要點的mst
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
  int n , dst[V][V] , dp[1 << T][V] , tdst[V];
  void init( int _n ){
    n = _n;
    for( int i = 0 ; i < n ; i ++ ){
      for( int j = 0 ; j < n ; j ++ )
        dst[ i ][ j ] = INF;
      dst[ i ][ i ] = 0;
  } }
  void add_edge( int ui , int vi , int wi ){
    dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
    dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
  }
  void shortest_path(){ // using spfa may faster
    for( int k = 0 ; k < n ; k ++ )
      for( int i = 0 ; i < n ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
          dst[ i ][ j ] = min( dst[ i ][ j ],
                dst[ i ][ k ] + dst[ k ][ j ] );
  }// call shorest_path before solve
  int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1 << t ) ; i ++ )
      for( int j = 0 ; j < n ; j ++ )
        dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
      dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
      if( msk == ( msk & (-msk) ) ){
        int who = __lg( msk );
        for( int i = 0 ; i < n ; i ++ )
          dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
        continue;
      }
      // Transfer starts at i=0, These points will be
          used as relay points
      for( int i = 0 ; i < n ; i ++ )
        for( int submsk = ( msk - 1 ) & msk ; submsk ;
                  submsk = ( submsk - 1 ) & msk )
          dp[ msk ][ i ] = min( dp[ msk ][ i ],
                               dp[ submsk ][ i ] +
                               dp[ msk ^ submsk ][ i ] );
      for( int i = 0 ; i < n ; i ++ ){
        tdst[ i ] = INF;
        for( int j = 0 ; j < n ; j ++ )
          tdst[ i ] = min( tdst[ i ],
                      dp[ msk ][ j ] + dst[ j ][ i ] );
      }
      for( int i = 0 ; i < n ; i ++ )
        dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
      ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
    return ans;
} }solver;
```

6.4 Dijkstra's algorithm

```
/*Dijkstra's algorithm 單源最短路徑*/
#define MAX_V 100
#define INF 10000
struct Edge {
  int idx,w;
};
bool operator>(const Edge& a, const Edge& b) {
  return a.w > b.w;
}
int dist[MAX_V];
vector<vector<Edge> > adj(MAX_V);
void dijkstra(int vn, int s) {
  vector <bool> vis(vn, false);
  fill(dist, dist + vn, INF); dist[s] = 0;
  priority_queue <Edge, vector<Edge>, greater<Edge> >
      pq;
  Edge node;
  node.idx = s; node.w = 0;
  pq.emplace(node);
  while (!pq.empty()) {
    int u = pq.top().idx; pq.pop();
    if (vis[u])continue;
    vis[u] = true;
    for (auto v : adj[u]) {
      if (dist[v.idx] > dist[u] + v.w) {
        dist[v.idx] = dist[u] + v.w;
        node.w = dist[v.idx];
        node.idx = v.idx;
        pq.emplace(node);
      }
    }
  }
}
signed main() {
    //從start連接到end的最短路徑
    cin >> start >> end;
    dijkstra(n, start);
    if(dist[end]==INF) cout << "NO\n";
    else cout << dist[end] <<"\n";
}
```

6.5 Floyd-Warshall

```
/*Floyd-Warshall 全點對最短路徑*/
void floyd(){
    //將每個點對距離設為INF
    memset(dist,0x3f3f3f3f,sizeof(dist));
    //dist[u][v]為點u到點v的最短路徑
    for(int i=0;i<n;i++)  dist[i][i]=0;
    for(int i=0;i<m;i++)  cin>>u>>v>>w,dist[u][v]=w;
    for(int i=0;i<n;i++)  //窮舉中繼點
        for(int j=0;j<n;j++) //j,k窮舉點對
            for(int k=0;k<n;k++)
                dist[j][k]=min(dist[j][k],dist[j][i]+
                    dist[i][k]);
}
```

6.6 SPFA

```cpp
/*SPFA 單源最短路徑(negative cycle)*/
struct Edge {
    int idx, w;
};
vector<Edge> adj[MAX_V]; //adjacency list
vector<bool> inp(MAX_V);
int dist[MAX_V];
//return true if negative cycle exists
bool spfa(int vn, int s) {
    fill(dist, dist + vn, INF); dist[s] = 0;
    vector<int> cnt(vn, 0);
    vector<bool> inq(vn, 0);
    queue<int> q; q.push(s); inq[s] = true;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        inq[u] = false;
        for (auto v : adj[u]) {
            if (dist[v.idx] > dist[u] + v.w) {
                if (++cnt[v.idx] >= vn)return true;
                dist[v.idx] = dist[u] + v.w;
                if (!inq[v.idx]) inq[v.idx] = true, q.
                    push(v.idx);
            }
        }
    }
    return false;
}
```

7 Graph(Tree)

7.1 DFS and BFS

```cpp
void dfs(int x){
    vis[x]=1;
    for(int i:adj[x])
        if(!vis[i]) dfs(i);
}
void bfs(int s){
    queue<int> q;
    q.push(s);
    vis[s]=1;
    while(!q.empty()){
        int x=q.front();q.pop();
        for(int i:adj[x])
            if(!vis[i])
                q.push(i),vis[i]=1;
    }
}
```

7.2 Topological Sort

```cpp
// O(N+M)
for(int i=0;i<m;i++){
    cin >> u >> v;    //點 u 連到點 v
    adj[u].push_back(v);
    ++deg[v];
}
// can solve DAG
void bfs(int n){ // Topological Sort
    queue<int> q;
    for(int i=0;i<n;i++)
        if(!deg[i]) q.push(i); //入度0先出
    while(!q.empty()){
        int u = q.front(); q.pop();
        for(int i:adj[u]){
            --deg[i];
            deg_sum[i] += deg_sum[u];
            if(deg[i] == 0) q.push(i);
}}}
```

7.3 Eulerian Path and Circuit

```cpp
// O(M)
//              歐拉迴路              歐拉路徑
// 無向圖/所有點的度數為偶數/度數為奇數的點數量不超過2
// 有向圖/所有點入度等於出度/全部點的入度出度一樣
//或剛好一個點出度-1=入度 另一點入度-1=出度，其他點入度
    等於出度
vector<int> path;
void dfs(int x){
    while(!edge[x].empty()){
        int u = edge[x].back();
        edge[x].pop_back();
        dfs(u);
    }
    path.push_back(x);}
int main(){
    build_graph();
    dfs(st); // 如果剛好一個點出度-1=入度 則為起點
    reverse(path.begin(),path.end());}
```

7.4 LCA

```cpp
// pre   O(NlgN)
// query O(lgN)
// 最近共同祖先
// 兩點間距離 / 兩點間最大邊 / 兩點間重合長度
// 時間戳記，判斷祖先關係
int ti = 0; // 當前時間
int tin[MAXN+5],tout[MAXN+5];
int dis[MAXN+5]; // 計算距離深度
int query[MAXN+5][lgN+5]; //點N的2^lgN祖先的最大邊
void dfs(int x,int f,int deep){
    fa[x] = f;
    tin[x] = ti++;
    dis[x] = deep;
    for(auto i:edge[x]){
        if(i.v == f){
            //query[x][0] = i.w;
            continue;
        }
        dfs(i.v, x, deep+i.w);
    }
    tout[x] = ti++;
}
bool isAncestor(int u, int v){
    return tin[u]<=tin[v] && tout[u] >= tout[v];
}
// LCA
int n,lgn;
int anc[MAXN+5][lgN+5]; //點N的2^lgN祖先
int getLca(int u, int v){
    if(isAncestor(u, v))    return u;
    if(isAncestor(v, u))    return v;
    for(int i=lgn;i>=0;i--){
        // 判斷 2^lgN, 2^(lgN-1),...2^1, 2^0 倍祖先
        if(!isAncestor(anc[u][i], v))
            u = anc[u][i]; // 則往上移動
    }
    return anc[u][0];
}
// 找出路徑最大邊
int max_cost(int u, int v){
    int max_cost = 0;
    if(u == v) return max_cost;
    for(int i=lgn;i>=0;i--){
        if(!isAncestor(anc[u][i], v)){
            max_cost = max(max_cost,query[u][i]);
            u = anc[u][i]; // 則往上移動
        }
    }
    return max(max_cost,query[u][0]);
} // max(max_cost(u,nodeLca), max_cost(v,nodeLca))
```

```
// 兩點距離
int dist(int u, int v){
    //depth[X] + depth[Y] - 2 * depth[ancestor]
    return dis[u] + dis[v]-2*dis[find(v)];
}
// init 建表
for(s=1;s<=n;s++) anc[s][0] = fa[s];
for(i=1;i<=lgn;i++)
    for(s=1;s<=n;s++)
        anc[s][i] = anc[anc[s][i-1]][i-1];
// query[s][i] =
// max(query[s][i-1],query[anc[s][i-1]][i-1]);
```

7.5 DSU on Tree

```
void add(int v, int p, int x){
    cnt[ col[v] ] += x;
    // now you can insert test
    for(auto u: g[v])
        if(u != p && !big[u])
            add(u, v, x);
}
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p && sz[u] > mx)
          mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
        // run a dfs on small childs and clear them
            from cnt
            dfs(u, v, 0);

    if(bigChild != -1)
    // bigChild marked as big and not cleared from cnt
        dfs(bigChild, v, 1), big[bigChild] = 1;
    add(v, p, 1);
    ans[v] = sum;
    //now cnt[c] is the number of vertices in subtree
        of vertex v that has color c. You can answer
        the queries easily.
    if(bigChild != -1)
        big[bigChild] = 0;
    if(keep == 0){
        add(v, p, -1);
        // now you can init to 0
    }
}
```

7.6 樹上差分

```
#define MAX 3e5+5
int n;
vector<vector<int>>edge(MAX), fa(MAX, vector<int>(21,
    0));
vector<int>a(MAX), dep(MAX), cnt(MAX, 0);
void dfs(int rt,int f) {
  fa[rt][0] = f;
  dep[rt] = dep[f] + 1;
  for (int i = 1; i <= 20; i++) {
    fa[rt][i] = fa[fa[rt][i - 1]][i - 1];
  }
  for (auto i : edge[rt]) {
    if (i == f)continue;
    dfs(i, rt);
  }
}
int lca(int a, int b) {
  if (dep[a] < dep[b]) {
    swap(a, b);
  }
  for (int i = 20; i >= 0; i--) {
    if (dep[fa[a][i]] >= dep[b]) {
      a = fa[a][i];  //上跳
    }
  }
  if (a == b)
    return a;
  for (int i = 20; i >= 0; i--) {
    if (fa[a][i] != fa[b][i]) {
      a = fa[a][i];
      b = fa[b][i];
    }
  }
  return fa[a][0];
}
void dfssum(int rt,int f) {
  for (auto i : edge[rt]) {
    if (i == f)continue;
    dfssum(i, rt);
    cnt[rt] += cnt[i];
  }
}
void solve() {
  int u, v, cmnlca;
  for (int i = 0; i < n; i++) {
    cin >> a[i];
  }
  for (int i = 0; i < n - 1; i++) {
    cin >> u >> v;
    edge[u].push_back(v);
    edge[v].push_back(u);
  }
  dfs(1, 0);
  for (int i = 0; i < n - 1; i++) {
    cmnlca = lca(a[i], a[i + 1]);
    cnt[fa[cmnlca][0]]--;  //父節點 -v
    cnt[cmnlca]--;       //lca -v
    cnt[a[i]]++;       //兩端點 +v
    cnt[a[i + 1]]++;
  }
  dfssum(1, 0);
  for (int i = 1; i <= n; i++) {  //多加的減回去
    cnt[a[i]]--;
  }
  for (int i = 1; i <= n; i++) {
    cout << cnt[i] << "\n";
  }
}
int main() {
  cin >> n;
  solve();
}
```

7.7 HLD with Segment tree

```
#define MXN 10005
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
#define INF 1e9+5
int n;
int sz[MXN], fa[MXN], heavy[MXN], dep[MXN];
int root[MXN]; //鍊的根節點
int len[MXN]; //鍊長度
struct Edge {int u, v;};
struct node {int v, w;};
vector<Edge> edge;
vector<node> graph[MXN];
vector<int> tree[MXN];  // 第i個節點為根的線段樹
vector<int> val[MXN];   // 第i個節點為根的序列
// 子樹大小
void dfs_sz(int u, int f, int d){
  sz[u] = 1, fa[u] = f, dep[u] = d;
    for(auto v : graph[u]){
        if(v.v != f){
            dfs_sz(v.v, u, d+1);
            sz[u] += sz[v.v];
            if(sz[v.v] > sz[heavy[u]]) heavy[u] = v.v;
                //重兒子
        }
    }
}
// 樹鍊剖分
void dfs_hld(int u,int f){
    for(auto v : graph[u]){
        if(v.v != f){
            if(v.v == heavy[u])root[v.v] = root[u];
            //重兒子的根，重鍊的頭
            else root[v.v] = v.v; //輕兒子的根
            val[root[v.v]].push_back(v.w); //點權
            dfs_hld(v.v, u);
        }
    }
    len[root[u]]++; //鍊長度
}
// LCA
int getLca(int x,int y){
    while(root[x] != root[y]){
        if(dep[root[x]] > dep[root[y]])
            x = fa[root[x]]; //跳鍊
        else
            y = fa[root[y]];
    }
    return (dep[x] <= dep[y] ? x : y);
}
// 線段樹
void build(int ver,int i,int l,int r){
    if(l == r){
        tree[ver][i] = val[ver][l];
        return;
    }
    int mid=(l+r)>>1;
    build(ver, cl(i), l, mid);
    build(ver, cr(i), mid+1, r);
    tree[ver][i] = max(tree[ver][cl(i)], tree[ver][cr(i
        )]); //最大邊
}
void update(int ver,int i,int l,int r,int pos,int val){
    if(l == r){ // 修改 a[pos] 的值為 val
        tree[ver][i] = val; return;
    }
    int mid=(l+r)>>1;
    if(pos <= mid) update(ver,cl(i),l,mid,pos,val);
    else           update(ver,cr(i),mid+1,r,pos,val);
    tree[ver][i] = max(tree[ver][cl(i)], tree[ver][cr(i
        )]);
}
 // i 為當前節點index, l, r當前區間左右界，ql, qr詢問左
     右界
int query(int ver,int i,int l,int r,int ql,int qr){
    if(ql <= l && r <= qr){
        return tree[ver][i];
    }
    int mid=(l+r)>>1, ret=-INF;
    if(ql<=mid) ret =
        max(ret, query(ver,cl(i),l,mid,ql,qr));
    if(qr> mid) ret =
        max(ret, query(ver,cr(i),mid+1,r,ql,qr));
    return ret;
}
void init(){
    edge.clear(); edge.resize(n-1);
    for(int i=1;i<=n;i++){
        graph[i].clear();
        tree[i].clear();
        val[i].clear();
        heavy[i]=len[i]=0;
    }
}
signed main(){
    int i,t,a,b,w,ti;
    string op;
    cin >> n;
    init();
    for(i=0; i+1<n; i++){
        cin >> a >> b >> w;
        graph[a].push_back(node{b,w});
        graph[b].push_back(node{a,w});
        edge[i] = Edge{a,b};
    }
    val[1].push_back(-INF);
    root[1] = 1;
    dfs_sz(1, 1, 0);
    dfs_hld(1, 1);
    // build tree
    for(i=1;i<=n;i++){ // 第i個節點為根的線段樹
        if(root[i] == i){
            tree[i].resize(len[i]*4,0);
            build(i, 1, 0, len[i]-1);
        }
    }
    // query
    while(cin >> op){
        if(op == "DONE") break;
        else if(op == "CHANGE"){
            cin >> i >> ti; i--;
            if(dep[edge[i].u] < dep[edge[i].v])
                swap(edge[i].u,edge[i].v);
            i = edge[i].u;
            update(root[i], 1, 0, len[root[i]]-1,
            dep[i]-dep[root[i]], ti);
        }
        else if(op == "QUERY"){
            cin >> a >> b;
            int ans = -INF;
            while(root[a] != root[b]){ //不同鍊
                if(dep[root[a]] < dep[root[b]])
                    swap(a, b);
                // 深鍊的最大邊
                ans = max(ans, query(root[a], 1, 0, len
                    [root[a]]-1, 0, dep[a]-dep[root[a
                    ]]));
                a = fa[root[a]]; //跳鍊
            }
            if(a != b){ //不同節點
                int mn =
                    min(dep[a],dep[b])-dep[root[a]]+1;
                int mx =
                    max(dep[a],dep[b])-dep[root[a]];
                //所在節點區間 mn,mx
                ans = max(ans, query(root[a], 1, 0, len
                    [root[a]]-1, mn, mx));
            }
            cout << ans <<"\n";
}   }   }
```

7.8 Scc 強連通分量 (轉 DAG)

```
#define MXN 200005
#define FZ(x) memset(x, 0, sizeof(x))
// 用於 DAG 2-SAT
struct Scc{ // 強連通分量 0-base
  int n, nScc, vst[MXN], bln[MXN]; // 最後每個點所屬的
      連通分量存在bln陣列
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){ //先初始化點的數量
    n = _n;
    for (int i=0; i<MXN; i++)
      E[i].clear(), rE[i].clear();
  }
  void addEdge(int u, int v){  // 加有向邊
    E[u].PB(v); rE[v].PB(u);
  }
  void DFS(int u){
    vst[u]=1;
    for (auto v : E[u]) if (!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1; bln[u] = nScc;
    for (auto v : rE[u]) if (!vst[v]) rDFS(v);
  }
  void solve(){  // 跑 kosaraju
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++)
      if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec)
      if (!vst[v]){
        rDFS(v); nScc++;
      }
  }
}scc;
```

7.9 2-SAT

```
// build
scc.init(2*m);
for(i=0;i<m;i++){ //所有點比較
  for(j=i+1;j<m;j++){
    if(graph[i].x == graph[j].x){ //同一行
      // m中的2個控制
      // (x or y) x->!y, y->!x -> !x = x+m
      if(abs(graph[i].y - graph[j].y)<=2*r){ // light
          // x or y
          scc.addEdge(i,j+m);
          scc.addEdge(j,i+m);
      }
    }
    else if(graph[i].y == graph[j].y){ //同一列
      if(abs(graph[i].x - graph[j].x)<=2*r){ // light
          // !x or !y
          scc.addEdge(i+m,j);
          scc.addEdge(j+m,i);
      }
    }
  }
}
// solve
scc.solve();
for(i=0;i<m;i++){
  if(scc.bln[i] == scc.bln[i+m]){
      cout << "NO\n";
      return 0;
  }
}
cout << "YES\n";
```

7.10 BccVertex 點雙連通分量 (關節點、橋)

```
#define MXN 100005
#define FZ(x) memset(x, 0, sizeof(x))
#define REP(i, b) for(int i = 0; i < (b); ++i)
// 找關節點(分量重疊) 與 橋(只有兩點)
struct BccVertex { // 點雙連通分量(無向) 0-base
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {  //初始化點的數量
    n = _n; nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void addEdge(int u, int v)  // 加無向邊
  { E[u].PB(v); E[v].PB(u); }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          int z;
          sccv[nScc].clear();
          do {
            z = stk[--top];
            sccv[nScc].PB(z);
          } while (z != v);
          sccv[nScc++].PB(u);
        }
      }else
        low[u] = min(low[u],dfn[v]);
  } }
  vector<vector<int>> solve() { // 跑 Tarjan
    vector<vector<int>> res;
    for (int i=0; i<n; i++)
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
      }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
  }
}graph;
```

8 DP

8.1 背包問題

```
// n：第0種到第n種物品要放進背包內。
// w：背包耐重限制。
// c(n, w)：只有第0種到第n種物品
// 耐重限制為w，此時的背包問題答案。
// weight[n]：第n種物品的重量。
// cost[n]：第n種物品的價值。
// number[n]：第n種物品的數量。

// 0/1背包滾動
// 每種物品只會放進背包零個或一個。
const int N = 500, W = 2000000; //N個物品,耐重W
int cost[N], weight[N];
int c[W + 1];
void knapsack(int n, int w){
  c[0] = 0;
  for (int i = 0; i < n; ++i)
    for (int j = w; j - weight[i] >= 0; --j)
      c[j] = max(c[j], c[j - weight[i]] + cost[i]);
  cout << c[w];
}
// 無限背包
// 物品有許多種類，每一種物品都無限量供應的背包問題。
void knapsack(int n, int w){
  memset(c, 0, sizeof(c));
  for (int i=0; i<n; ++i)
    for (int j = weight[i]; j <= w; ++j)
      c[j] = max(c[j], c[j - weight[i]] + cost[i]);
  cout << "最高的價值為" << c[w];
}
// 有限背包
// 物品有許多種類，每一種物品都是限量供應的背包問題。
int cost[N], weight[N], number[N];
// number[n]：第n種物品的數量。
void knapsack(int n, int w){
  for (int i = 0; i < n; ++i){
    int num = min(number[i], w / weight[i]);
    for (int k = 1; num > 0; k *= 2){
      if (k > num) k = num;
      num -= k;
      for (int j = w; j >= weight[i] * k; --j)
        c[j] = max(c[j], c[j - weight[i] * k] + cost[i]
             * k);
    }
  }
  cout << "最高的價值為" << c[w];
}
```

8.2 找零問題

```
// n：用第0種到第n種錢幣來湊得價位。
// m：欲湊得的價位值。
// c(n, m)：用第0種到第n種錢幣湊得價位m的湊法數目。
// price[n]：第n種錢幣的面額大小。

// 這些面額湊不湊得到價位m，無限
int price[5] = {5, 2, 6, 11, 17}; //錢幣面額
bool c[1000 + 1];
void change(int m){
  memset(c, false, sizeof(c));
  c[0] = true;
  // 依序加入各種面額
  for (int i = 0; i < 5; ++i)
    // 由低價位逐步到高價位
    for (int j = price[i]; j <= m; ++j)
      c[j] |= c[j - price[i]];
  if (c[m]) cout << "yes";
  else cout << "no";
}
// 湊得某個價位的湊法總共幾種
void change(int m){
  memset(c, 0, sizeof(c));
  c[0] = 1;
  for (int i = 0; i < 5; ++i)
    for (int j = price[i]; j <= m; ++j)
      c[j] += c[j - price[i]];
  cout << "湊得價位" << m;
  cout << "湊法總共" << c[m] << "種";
}
// 湊得某個價位的最少錢幣用量
// c(n, m)：用第0種到第n種錢幣湊得價位m，最少所需要的錢
    幣數量。
void change(int m){
  memset(c, 0x7f, sizeof(c));
  c[0] = 0;
  for (int i = 0; i < 5; ++i)
    for (int j = price[i]; j <= m; ++j)
      c[j] = min(c[j], c[j - price[i]] + 1);
  cout << "湊得價位" << m;
  cout << "最少需（只）要" << c[m] << "個錢幣";
}
// 湊得某個價位的錢幣用量，有哪幾種可能性。
void change(int m){
  memset(c, 0, sizeof(c));
  c[0] = 1;
  for (int i = 0; i < 5; ++i)
    for (int j = price[i]; j <= m; ++j)
      // 錢幣數量加一，每一種可能性都加一。
      c[j] |= c[j - price[i]] << 1;
  for (int i = 1; i <= 63; ++i)
    if (c[m] & (1 << i))
      cout << "用" << i << "個錢幣可湊得價位" << m;
}
// 能否湊得某個價位，限量
int price[5] = {5, 2, 6, 11, 17};
int number[5] = {4, 5, 5, 3, 2}; // 各種錢幣的供應數量
bool c[1000 + 1];
void change(int m){
  memset(c, 0, sizeof(c));
  c[0] = true;
  for (int i = 0; i < 5; ++i)
    // 各種餘數分開處理
    for (int k = 0; k < price[i]; ++k){
      int left = number[i]; // 補充彈藥
      // 由低價位到高價位
      for (int j = k; j <= m; j += price[i])
        // 先前的面額已能湊得，當前面額可以省著用。
        if (c[j])
          left = number[i]; // 補充彈藥
        // 過去都無法湊得，一定要用目前面額硬湊。
        else if (left > 0){
          left--; // 用掉一個錢幣
          c[j] = true;
        }
    }
  if (c[m]) cout << "yes";
  else cout << "no";
}
// 買東西找回最少硬幣。
int price[5] = {50, 20, 10, 4, 2}; // 面額由大到小排列
void cashier(int n){ // n 是總共要找的錢。
  int c = 0;
  for (int i = 0; i < 5; ++i)
    while (n >= price[i]){
      n -= price[i]; // 找了 price[i] 元
      c++;
    }
  if (n != 0) cout << "找不出來";
  else cout << "找了" << c << "個錢幣";
}
```

8.3 LCS 與 DP 回溯

```
    n=str.size(); m=str2.size();
    vector<vector<int>> dp(n+1,vector<int>(m+1,0));
    // LCS
    for(i=1;i<=n;i++){
        for(j=1;j<=m;j++){
            if(str[i-1]==str2[j-1]){
                dp[i][j] = dp[i-1][j-1] + 1;
            }else{
                dp[i][j] = max(dp[i][j-1],dp[i-1][j]);
            }
        }
    }
    // DP回溯
    string ans="";
    i=n; j=m;
    while(i>0 && j>0){
        if(str[i-1]==str2[j-1]){
            ans = str[i-1] + ans;
            i--; j--;
        }
        else if(dp[i][j-1]>dp[i-1][j]) j--;
        else i--;
    }
    cout << dp[n][m] <<"\n";
    cout << ans <<"\n";
```

8.4 LIS

```
/*LIS 最長遞增子序列*/
int LIS(int n, int s[], int length[]) {
  for (int i = 0; i < n; i++) length[i] = 1;
  for (int j = 0; j < n; j++)
    for (int i = j + 1; i < n; i++)
      if (s[j] < s[i]) length[i] =
        max(length[i], length[j] + 1);
  return *max_element(length, length+n);
```

9 DP on tree

9.1 全點對距離 Tree Distance

```
int dp[MAXN]={0};
void dfs_sz(int x,int f){
    sz[x] = 1, fa[x] = f;
    for(int i:edge[x]){
        if(i == f) continue;
        dfs1(i, x);  // 先計算完子節點的答案再算自己的
        sz[x]+=sz[i];
        dp[x]+=(dp[i]+sz[i]);
}   }
void dfs_dp(int x,int f,ll sum){
    ans += sum + dp[x]; //所有點到結點x距離總和為父節點
        方向距離總和 + 子樹到自己距離總和
    for(int i:edge[x]){
        if(i == f) continue;
        //tmp 為從父節點x到子節點i的距離總合為
        ll tmp = sum  //x的父節點總和 sum 到結點x的距離
                  + dp[x] - (dp[i]+sz[i])
                  //加上x的子樹(除了i方向)到x的距離總和
                  + (n - sz[i]);
                  //加上從節點x到節點i的距離
        dfs2(i, x, tmp);
}   }
```

9.2 最大獨立集 Independent set

```
int dp[MAXN][2]; //此點，選或不選
void dfs(int x,int f){
    dp[x][1] = 1; // 狀態[1] 計算自己數量 +1
    for(int i:edge[x]){
        if(i == f)    continue;
        dfs(i, x);    // 先計算完子節點的答案再算自己的
        dp[x][0] += max(dp[i][0], dp[i][1]);
        dp[x][1] += dp[i][0];
}   }
```

9.3 最小點覆蓋 Vertex Cover

```
int dp[MAXN][2]; //此點，選或不選
void dfs(int x,int f){
    dp[x][1] = 1; // 狀態[1] 計算自己數量 +1
    for(int i:edge[x]){
        if(i == f)    continue;
        dfs(i, x);    // 先計算完子節點的答案再算自己的
        dp[x][0] += dp[i][1];
        dp[x][1] += min(dp[i][0],dp[i][1]);
}   }
```

9.4 最小支配集 Dominating Set

```
void Dominating_Set(int u,int p){
  dp[u][2]=0;
  dp[u][0]=1;
  bool s=false;
  int sum=0,inc=INF;
  int k;
  for(k=head[u];k!=-1;k=edge[k].next){
    int to=edge[k].to;
    if(to==p)continue;
    DP(to,u);
    dp[u][0]+=min(dp[to][0],min(dp[u][1],dp[u][2]));
    if(dp[to][0]<=dp[to][1]){
      sum+=dp[to][0];
      s=true;
    }
    else{
      sum+=dp[to][1];
      inc=min(inc,dp[to][0]-dp[to][1]);
    }
    if(dp[to][1]!=INF&&dp[u][2]!=INF)dp[u][2]+=dp[to
        ][1];
    else dp[u][2]=INF;
  }
  if(inc==INF&&!s)dp[u][1]=INF;
  else{
    dp[u][1]=sum;
    if(!s)dp[u][1]+=inc;
  }
}
```

```
// 最大獨立集
二分圖中，選擇一些頂點，使這些點互不相連。
// 最小點覆蓋
二分圖中，選擇最少的點數，使這些點和所有的邊都有關聯
    （把所有的邊的覆蓋）。
// 最小支配集
指從所有頂點中，取盡量少的點組成一個集合，使得剩下的所
    有點都與取出來的點有邊相連。

最小點覆蓋 + 最大獨立點集 = 總頂點數
```

10 String

10.1 Trie

```
// insert O(|s|)
// query  O(|s|)
struct trie{
    trie *nxt[26];
    int cnt;    //紀錄有多少個字串以此節點結尾
    int sz;     //有多少字串的前綴包括此節點
    set<int> cnt_idx, sz_idx;
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
}   };
trie *root = new trie();
void insert(string& s, int idx){
    trie *now = root;  // 每次從根結點出發
    for(auto i:s){
        now->sz++; now->sz_idx.emplace(idx); //被誰經過
        if(now->nxt[i-'a'] == NULL){
            now->nxt[i-'a'] = new trie();
        }
        now = now->nxt[i-'a'];  //走到下一個字母
    }
    now->cnt++; now->cnt_idx.emplace(idx);//以此點結尾
    now->sz++;  now->sz_idx.emplace(idx); //被誰經過
}
//query
int query_prefix(string& s){  //查詢有多少前綴為 s
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->sz;
}
int query_count(string& s){  //查詢字串 s 出現次數
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] == NULL){
            return 0;
        }
        now = now->nxt[i-'a'];
    }
    return now->cnt;
}
//str有沒有在[l,r]的前綴中
bool query_ArrPrefix(string& s,int l,int r){
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] != NULL && now->nxt[i - 'a'
            ]->sz > 0 ){ //存在
            now = now->nxt[i-'a'];
        }else return false; //不存在，無解
    }
    // 這個s的節點，[l,r]有沒有經過
    auto L = now->sz_idx.lower_bound(l);
    if(l<=*L && *L<=r)return true;
    else return false;}
//[l,r]有沒有存在於str的前綴中
bool query_StrPrefix(string& s,int l,int r){
    trie *now = root;    // 每次從根結點出發
    for(auto i:s){
        if(now->nxt[i-'a'] != NULL && now->nxt[i - 'a'
            ]->sz > 0 ){ //存在
            now = now->nxt[i-'a'];
        }else return false; //不存在，無解
    // [l,r]存在於str的前綴中，代表有字串以str為結尾
        auto L = now->cnt_idx.lower_bound(l);
        if(l<=*L && *L<=r)return true;
    } return false;
}
```

10.2 01Trie

```
// insert O(lgx)
// query  O(lgx)
// 處理XOR問題
// struct
struct trie{
    trie *nxt[2];  // 差別
    int cnt;    //紀錄有多少個數字以此節點結尾
    int sz;     //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root = new trie();
void insert(int x){
    trie *now = root;  // 每次從根結點出發
    for(int i=30;i>=0;i--)
        now->sz++;
        if(now->nxt[x>>i&1] == NULL){
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1];  //走到下一個字母
    }
    now->cnt++;
    now->sz++;
}
// in this set, the maximum value of bitwise XOR x
int query(int x){
    trie *now = root;
    int ans=0;
    for(int i=30;i>=0;i--){// 不等於為1(0xr1=1,1xr0=1)
        if (now->nxt[!(x>>i&1)] != NULL && now->nxt[!(x
            >>i&1)]-> sz > 0){ //下一個存在
            ans += 1<<i;
            now = now->nxt[!(x>>i&1)];
        }
        else now = now->nxt[x>>i&1];
    }
    return ans;
}
```

10.3 Hash

```
// build O(n)
// query O(1)
// double hash
// P = 53,97,193,49157,805306457,1610612741,1e9+9,1e9+7
const ll P1 = 75577;
const ll P2 = 12721;
const ll MOD = 998244353;
pair<ll,ll> Hash[MXN];//Hash[i] 為字串 [0,i] 的hash值
void build(const string& s){
    pair<ll,ll> val = make_pair(0,0);
    for(int i=0; i<s.size(); i++){
        val.first = (val.first * P1 + s[i]) % MOD;
        val.second = (val.second * P2 + s[i]) % MOD;
        Hash[i] = val;
    }
}
pair<ll,ll> query(ll l,ll r){
    pair<ll,ll> q;
    q.first = (Hash[r].first - \
    Hash[l-1].first* \
    fastpow(P1,r-l+1,MOD)%MOD + MOD)%MOD;
    q.second = ( Hash[r].second - \
    Hash[l-1].second* \
    fastpow(P2,r-l+1,MOD2)%MOD2 + MOD2 )%MOD2;
    return q;
}
```

11 Flow

11.1 MinCostMaxFlow

```
struct MinCostMaxFlow{
typedef int Tcost;
  static const int MAXV = 20010;
  static const int INFf = 1000000;
  static const Tcost INFc  = 1e9;
  struct Edge{
    int v, cap;
    Tcost w;
    int rev;
    Edge(){}
    Edge(int t2, int t3, Tcost t4, int t5)
    : v(t2), cap(t3), w(t4), rev(t5) {}
  };
  int V, s, t;
  vector<Edge> g[MAXV];
  void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();
  }
  void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
  }
  Tcost d[MAXV];
  int id[MAXV], mom[MAXV];
  bool inqu[MAXV];
  queue<int> q;
  pair<int,Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
      fill(d, d+1+V, INFc);
      fill(inqu, inqu+1+V, 0);
      fill(mom, mom+1+V, -1);
      mom[s] = s;
      d[s] = 0;
      q.push(s); inqu[s] = 1;
      while(q.size()){
        int u = q.front(); q.pop();
        inqu[u] = 0;
        for(int i = 0; i < (int) g[u].size(); i++){
          Edge &e = g[u][i];
          int v = e.v;
          if(e.cap > 0 && d[v] > d[u]+e.w){
            d[v] = d[u]+e.w;
            mom[v] = u;
            id[v] = i;
            if(!inqu[v]) q.push(v), inqu[v] = 1;
      } } }
      if(mom[t] == -1) break ;
      int df = INFf;
      for(int u = t; u != s; u = mom[u])
        df = min(df, g[mom[u]][id[u]].cap);
      for(int u = t; u != s; u = mom[u]){
        Edge &e = g[mom[u]][id[u]];
        e.cap              -= df;
        g[e.v][e.rev].cap += df;
      }
      mxf += df;
      mnc += df*d[t];
    }
    return {mxf,mnc};
} }flow;
```

```
                                  @
              (__)    (__) ____/ Hong~Long~Long~Long~
           /| (oo) _  (oo)/----/_____     *
  _o\_____/_|\_\/_/_|__\/|____|//////== *- *  * -
 /__________  \   AC |  AC | NO BUG /== -*
[_____/^^\_____\_____|_____/^^\_____]     *- * -
       \__/                \__/      Chong~Chong~Chong~
```

11.2 Dinic

```
struct Dinic{
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  }
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
    } } }
    return level[t] != -1;
  }
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
        if (nf == 0) return res;
    } }
    if (!res) level[u] = -1;
    return res;
  }
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
} }flow;
```

```
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```

11.3 KM

```
struct KM{ // max weight, for min negate the weights
  int n, mx[MXN], my[MXN], pa[MXN];
  ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
  bool vx[MXN], vy[MXN];
  void init(int _n) { // 1-based, N個節點
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
  }
  void addEdge(int x, int y, ll w) {g[x][y] = w;} //左
      邊的集合節點x連邊右邊集合節點y權重為w
  void augment(int y) {
    for(int x, z; y; y = z)
      x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
  }
  void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
      while(q.size()) {
        int x=q.front(); q.pop(); vx[x]=1;
        for(int y=1; y<=n; ++y) if(!vy[y]){
          ll t = lx[x]+ly[y]-g[x][y];
          if(t==0){
            pa[y]=x;
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
          }else if(sy[y]>t) pa[y]=x,sy[y]=t;
      } }
      ll cut = INF;
      for(int y=1; y<=n; ++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
      for(int j=1; j<=n; ++j){
        if(vx[j]) lx[j] -= cut;
        if(vy[j]) ly[j] += cut;
        else sy[j] -= cut;
      }
      for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){augment(y);return;}
        vy[y]=1, q.push(my[y]);
  } } }
  ll solve(){ // 回傳值為完美匹配下的最大總權重
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
} }graph;
```