COSC1003 ASM 470345744

Question 1
(1.1)

Observations	1	2	3	4	5	6	7	8	9	10	Average(days)
Result (days)	593	542	591	576	533	531	401	571	576	508	542.2

Table 1: Length of time until the last human gets infected

By running 10 experiments, the average length of time until the last human gets infected is 542.2 days.

(1.2)
Let

- I = the theoretical number of infections per day
- N = the total number of people
- P = zombie prevalence rate
- S = the side length of the world, then

$$I(P) = \frac{N^2P(1-P)}{S^2}$$

Which is the theoretical number of infections per day as a function of zombie prevalence. This number was calculated by multiplying

- $\frac{NP}{S^2}$, the probability that any given square is occupied by a zombie
- $N(1-P)$, the total number of healthy people remaining

And its derivative is

$$I'(p) = \frac{N^2}{S^2}(1-2p)$$

p	0-0.5	0.5	0.5-1
$\mathbf{I'(p)}$	>0	0	<0
$\mathbf{I(p)}$	Increasing	max	Decreasing

Table 2: Relationship between p, I'(p) and I(p)

Firstly, there is a clear correspondence between the zombie prevalence (p) and the days that the simulation has been running. The number of days required for the zombie prevalence to be equal to 1 is 508. Similarly, we can see that the peak of figures 1 and figures 2 occur at p=0.5 and around days 250 respectively, where there is an equal number of zombies and non-zombies. Matching these points, we can then compare the general shape of the two functions within these two domains and find that they both match the downwards parabola shape. It follows that the maximum number of infections per day peaks at around day 250 since that corresponds to when zombie prevalence is 0.5. Relating this to figure 3 we find that the slope corresponds to figure 1 where the total number of zombies initially increases at an increasing rate until around day 250, then at around day 250 a point of inflection occurs and then begins to increase at a decreasing rate and finally peaks at day 508 when zombie prevalence is 1.

The reason is that, before around Day 250 and p = 0.5, the increasing zombie prevalence resulted in an increasing collision between zombies and non-zombies. An increasing number of people are infected; hence the number of zombies increases in an increasing rate accordingly. After the number of zombies increases to the same number of non-zombies (p = 0.5) and collision decreases, the number of infections per day decreases and the number of zombies increases in a decreasing rate.

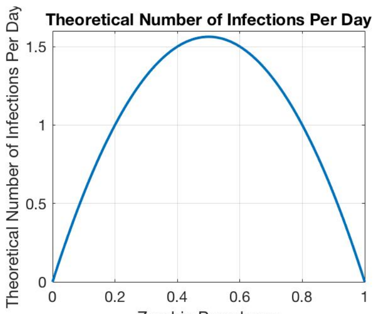

Figure 1

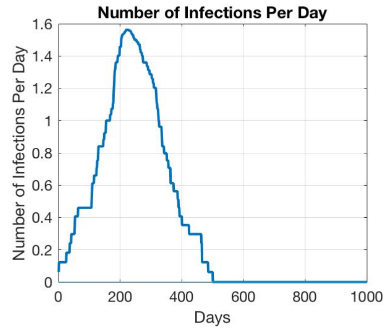

Figure 2

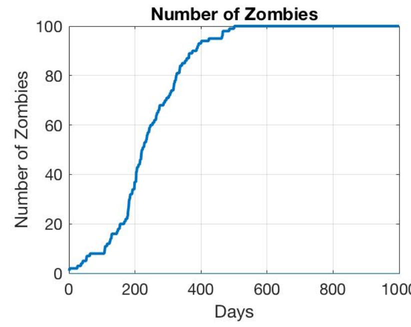

Figure 3

(1.3)

Observations	1	2	3	4	5	6	7	8	9	10	Average(days)
Result (days)	115	138	129	131	113	139	129	138	109	113	125.4

Table 3: Length of time until the last human gets infected when the side length is halved

By halving the side length to 20, the average length of time until the last human gets infected decreases to 125.4 days, which is roughly a quarter of the previous length (542.2 days).

Hence it is likely to have the following relationship:

$$t \propto side\ length^2$$

In the case of tuberculosis, the time it takes to spread out in crowded hospitals or prisons can be very sensitive to the size of the area. Hence it would be effective for us to decrease the infection rate by increasing the areas, such as:

1. Distancing patients in the current hospital or prisons
2. Moving patients to larger institutions

(1.4)

Days	t = 1000										Average
#Zombies	76	0	69	0	79	66	70	72	64	0	35.7

Table 4: Number of zombies at t = 1000 when remission rate is 0.01

Changing the remission rate from 0 to 0.01 hugely changed the dynamics of the model. Among all 10 experiments we run, not a single time did all people are infected before the simulation finishes, while all people are infected in the zero-remission case.

For most experiments (7 out of 10, Table 4), the number of zombies increases and then keeps "wobbling" between 60 and 80 after around Day 670. In this period, new infections are almost cancelled out by remissions, which results in this steady number of zombies. (Figure 4)

Besides the steady state phenomenon, it is worth noticing that in 4 out of 10 experiments the number of zombies is zero. This is because the zombies are mostly remitted in the early stage (e.g. when there is the only zombie given in the beginning) before they can keep infecting more people.

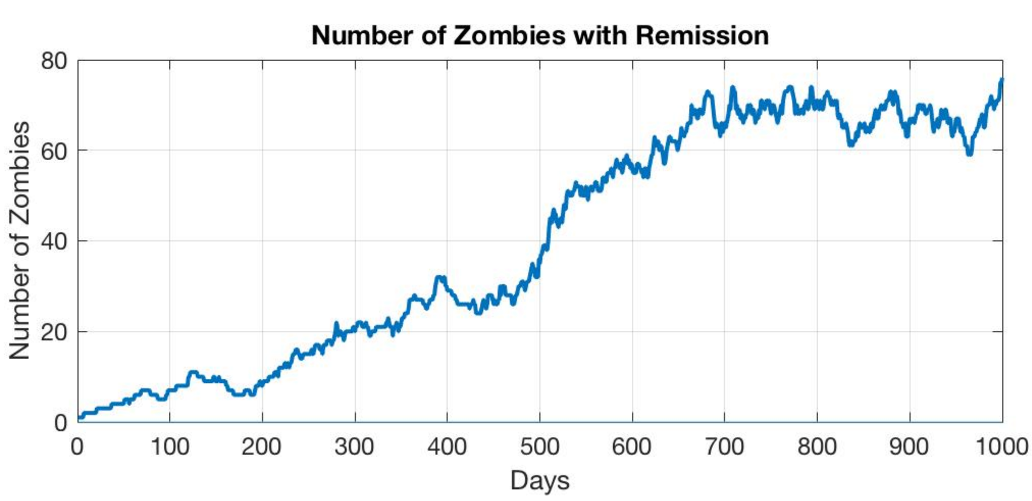

Figure 4: Number of Zombies with Remission

Question 2
(2.1)
Code

(2.2)

Choices	Stay	Switch
# Wins	3274	6672
Probability	32.74%	66.72%

Table 5: The probability of winning for “Stay” and “Switch” given 3 doors

By running the program for 10000 times, the contestant wins 3274 times (32.74%) if he/she stays, and 6672 times (66.72%) if he/she switches. The probability is almost doubled in the latter case therefore it is an optimal strategy for the contestant to switch the door.

(2.3)

Choices	Stay	Switch
# Wins	109	9930
Probability	1.09%	99.30%

Table 6: The probability of winning for “Stay” and “Switch” given 100 doors

When the number of doors increases to 100, there is an even larger difference in the result of stay and switch. The contestant is 99.3% likely to win if he/she switches, while only 1.09% likely to win if he/she chooses to stay. The reason behind it is that, initially, when the contestant randomly picks a door out of 100, the probability of winning is 1%. If he/she stays rather than switches, this probability will stay the same as 1%. However, after the host reveals the 98 goats, there are two doors left: the one picked initially and the other door. Since the contestant picks the door randomly from the very beginning, the probability for it to win is still 1%. However, if he/she switches, the probability will significantly increase as the other door is filtered by the host, which has the remaining probability 99% in theory. This analysis is also consistent with the simulation result.