Written Assignment Unit 5
Math 1201- College Algebra.

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Question 1

A retirement account is opened with an initial deposit of $8,500 and earns 8.12% interest compounded monthly. What will the account be worth in 20 years? What if the deposit were compounded monthly with simple interest? Could you see the situation in a graph? From what point one is better than the other?

SOLUTION

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where,

- A(t) is the value of the account including interest.
- P is the principal value or initial value = $8,500.
- r is the annual interest rate(8.12% or 0.0812 in decimal form).
- n is the frequency of the calculation in relation to t (12 months per year).
- t is the timespan of the calculation (20 years).

So, therefore

$$A = 8500\left(1 + \frac{0.0812}{12}\right)^{12\times 20}$$

$$A = 8500(1 + 0.00677)^{240}$$

$$A = 42,922.27$$

Calculation of Deposit using simple interest

$$A = P(1 + rt)$$

$$A = 8500(1 + 0.0812 \times 20)$$

$$A = 8500 \times 2.624$$

$$A = 22,304$$

Question 2

Graph the function $f(x) = 5(0.5)^{-x}$ and its reflection about the line $y = x$ on the same axis, and give the x-intercept of the reflection. Prove that $a^x = e^{xlna}$. [Suggestion: type $y = 5(0.5)^{-x}\{-7 < x < 2\}\{0 < y < 7\}$ in desmos, and then type its inverse function]

SOLUTION

given

$$a^x = e^{xlna}$$

$$y = 5(0.5)^{-x}$$

$$y = 5e^{-xln0.5}$$

Taking the natural log of both sides of the equation,

$$ln\frac{y}{5} = -xln0.5$$

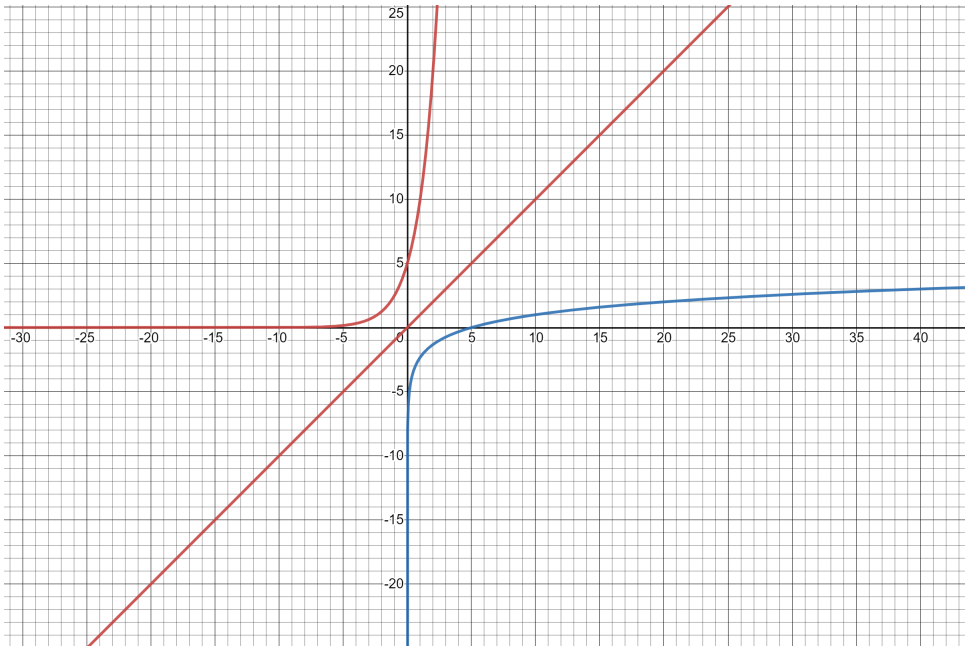
Making x the subject

$$x = -\frac{ln\frac{y}{5}}{ln0.5}$$

swapping y with x

$$y = -\frac{ln\frac{x}{5}}{ln0.5}$$

The graph is shown below



Question 3

How long will it take before twenty percent of our 1,000-gram sample of uranium-235 has decayed?
The decay equation is $A(t) = A_0e^{kt}$, where t is the time for the decay, and K is the characteristic of the material. Suppose T is the time it takes for half of the unstable material in a sample of a radioactive substance to decay, called its half-life. Prove that $K = \frac{ln0.5}{T}$. What is K for the uranium-235? show the steps of your reasoning.

SOLUTION
T= half life of uranium-235 = 703,800,000 years

$$800 = 1000e^{\frac{ln0.5}{703800000}t}$$

Dividing through by 1000

$$0.8 = e^{\frac{ln0.5}{703800000}t}$$

taking the natural log of both sides

$$ln0.8 = \frac{ln0.5}{703800000}t$$

Dividing through by $\frac{ln0.5}{703800000}$

$$t = 703800000 \times \frac{ln0.8}{ln0.5}$$

$$t = 703800000 \times 0.321928$$

$$t = 226,572,993$$

For the constant K

$$A(t) = A_0e^{Kt}$$

$$\frac{A_0}{2} = A_0e^{KT}$$

$$\frac{1}{2} = e^{KT}$$

$$ln0.5 = KT$$

$$K = \frac{ln0.5}{T}$$

For K in the Uranium-235, we have

$$K = \frac{ln0.5}{T}$$

$$K = \frac{ln0.5}{703,800,000}$$

$$K = \frac{1}{100,000,000} \times \frac{ln0.5}{7}$$

$$K = \frac{1}{100,000,000} \times -0.099021026$$

$$K = \frac{1}{100,000,000} \times -0.1$$

$$K = -10^9$$

REFERENCES

Abramson, J. (2017). *Algebra and trigonometry.* OpenStax, TX: Rice University. Retrieved from https://openstax.org/details/books/algebra-and-trigonometry