Discussion Assignment Unit 8
Math 1201 - College Algebra.

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October 2021

PART 1

QUESTION
How can De Moivre's theorem be described? what is the scope of this theorem?
Solution
The De Moivre theorem is a theorem that is used for finding the powers and roots of complex numbers.
"De Moivre's theorem states that, for a positive integer n, z^n is found by raising the modulus to the nth power and multiplying the argument by n"(Abramson, 2017. p 821).
if $z = r\cos\theta + i\sin\theta$ is a complex number then,

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$z^n = r^n cis(n\theta)$$

where n is a positive integer.
The De moivre's theorem can also be modified to find the nth roots of a complex number.
"To find the nth root of a complex number in polar form, use the formula given as "(Abramson, 2017. p 822).

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right]$$

"where k = 0,1,2,3,...,n-1. we add $\frac{2k\pi}{n}$ to $\frac{\theta}{n}$ in order to obtain the periodic roots"(Abramson, 2017. p 822).
The scope of the De Moivre's theorem is roots of complex equations and the powers of complex equations(Abramson, 2017).

PART 2

Give two Examples of roots
ROOTS: Example 1

Find the four fourth roots of $16cis(120°)$
Solution
using the nth root theorem,

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{2k\pi}{n}\right)+i\sin\left(\frac{\theta}{n}+\frac{2k\pi}{n}\right)\right]$$

$$z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left[\cos\left(\frac{120}{4}+\frac{2k\pi}{4}\right)+i\sin\left(\frac{120}{4}+\frac{2k\pi}{4}\right)\right]$$

k = 0, 1, 2, 3.
when k = 0

$$z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left[\cos\left(\frac{120}{4}+\frac{2(0)\pi}{4}\right)+i\sin\left(\frac{120}{4}+\frac{2(0)\pi}{4}\right)\right]$$

$$z^{\frac{1}{4}} = 2[\cos(30)+i\sin(30)]$$

$$z^{\frac{1}{4}} = 2\left[\frac{\sqrt{3}}{2}+i\frac{1}{2}\right]$$

$$z^{\frac{1}{4}} = \sqrt{3}+i$$

when k = 1

$$z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left[\cos\left(\frac{120}{4}+\frac{2(1)\pi}{4}\right)+i\sin\left(\frac{120}{4}+\frac{2(1)\pi}{4}\right)\right]$$

$$z^{\frac{1}{4}} = 2[\cos(30+90)+i\sin(30+90)]$$

$$z^{\frac{1}{4}} = 2[\cos(120)+i\sin(120)]$$

$$z^{\frac{1}{4}} = 2\left[-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]$$

$$z^{\frac{1}{4}} = -1+i\sqrt{3}$$

when k = 2

$$z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left[\cos\left(\frac{120}{4}+\frac{2(2)\pi}{4}\right)+i\sin\left(\frac{120}{4}+\frac{2(2)\pi}{4}\right)\right]$$

$$z^{\frac{1}{4}} = 2[\cos(30+180)+i\sin(30+180)]$$

$$z^{\frac{1}{4}} = 2[\cos(210)+i\sin(210)]$$

$$z^{\frac{1}{4}} = 2\left[-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right]$$

$$z^{\frac{1}{4}} = -\sqrt{3}-i$$

when k = 3

$$z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left[\cos\left(\frac{120}{4} + \frac{2(3)\pi}{4}\right) + i\sin\left(\frac{120}{4} + \frac{2(3)\pi}{4}\right)\right]$$

$$z^{\frac{1}{4}} = 2[\cos(30+270) + i\sin(30+270)]$$

$$z^{\frac{1}{4}} = 2[\cos(300) + i\sin(300)]$$

$$z^{\frac{1}{4}} = 2\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$

$$z^{\frac{1}{4}} = 1 - i\sqrt{3}$$

Therefore the roots of the complex equation $16cis(120°)$ are $\sqrt{3}+i$, $-1+i\sqrt{3}$, $-\sqrt{3}-i$ and $1-i\sqrt{3}$.

ROOTS: Example 2
Evaluate the cube root of z when $z = 32cis(\frac{2\pi}{3})$
Solution
using the nth root theorem,

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{\frac{2\pi}{3}}{3} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\frac{2\pi}{3}}{3} + \frac{2k\pi}{3}\right)\right]$$

k = 0, 1, 2.
when k = 0

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9} + \frac{2(0)\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{2(0)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}cis\left(\frac{2\pi}{9}\right)$$

when k = 1

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9} + \frac{2(1)\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{2(1)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9} + \frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{2\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{8\pi}{9}\right) + i\sin\left(\frac{8\pi}{9}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}} cis\left(\frac{8\pi}{9}\right)$$

when k = 2

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9} + \frac{2(2)\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{2(2)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{2\pi}{9} + \frac{4\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{4\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}}\left[\cos\left(\frac{14\pi}{9}\right) + i\sin\left(\frac{14\pi}{9}\right)\right]$$

$$z^{\frac{1}{3}} = 32^{\frac{1}{3}} cis\left(\frac{14\pi}{9}\right)$$

But

$$\sqrt[3]{32} = \sqrt[3]{8 \times 4}$$

But

$$\sqrt[3]{32} = 2\sqrt[3]{4}$$

Therefore the three roots of the complex equation $z = 32cis(\frac{2\pi}{3})$ are $2\sqrt[3]{4}$ cis($\frac{2\pi}{9}$), $2\sqrt[3]{4}$ cis($\frac{8\pi}{9}$), and $2\sqrt[3]{4}$ cis($\frac{14\pi}{9}$).

PART 3

Give two Examples of powers
POWERS: Example 1
find z^3 when $z = 5cis(45°)$
Solution
Using De Moivre's Theorem

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$z^n = r^n cis(n\theta)$$

$$z^3 = 5^3(\cos(3 \times 45) + i\sin(3 \times 45))$$

$$z^3 = 125(\cos(135) + i\sin(135))$$

$$z^3 = 125 \text{ cis}(135°)$$

POWERS: Example 2
find z^2 when $z = 3cis(120°)$
Solution
Using De Moivre's Theorem

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$z^n = r^n cis(n\theta)$$
$$z^2 = 3^2(\cos(2 \times 120) + i \sin(2 \times 120))$$
$$z^2 = 9(\cos(240) + i \sin(240))$$
$$z^2 = 9 \text{ cis}(240^\circ)$$

References

Abramson, J. (2017). *Algebra and trigonometry.* OpenStax, TX: Rice University. Retrieved from https://openstax.org/details/books/algebra-and-trigonometry