Learning Journal Unit 8
Math 1201 - College Algebra.

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QUESTION 1

Evaluate the cube root of $z = 27\text{cis}(240°)$
Then raise them to the cube(i.e the roots)
Solution

$$z = 27\text{cis}(240°)$$

Using the nth root theorem,

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right]$$

Finding the cube roots of z

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}}\left[\cos\left(\frac{240}{3} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{240}{3} + \frac{2k\pi}{3}\right)\right]$$

k = 0, 1, 2.
When k = 0

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}}\left[\cos\left(\frac{240}{3} + \frac{2(0)\pi}{3}\right) + i\sin\left(\frac{240}{3} + \frac{2(0)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 3\left[\cos(80) + i\sin(80)\right]$$

Therefore,

$$z^{\frac{1}{3}} = 3cis(80)$$

When k = 1

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}}\left[\cos\left(\frac{240}{3} + \frac{2(1)\pi}{3}\right) + i\sin\left(\frac{240}{3} + \frac{2(1)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 3\left[\cos(80 + 120) + i\sin(80 + 120)\right]$$

$$z^{\frac{1}{3}} = 3\left[\cos(200) + i\sin(200)\right]$$

Therefore,

$$z^{\frac{1}{3}} = 3cis(200)$$

When k = 2

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}}\left[\cos\left(\frac{240}{3} + \frac{2(2)\pi}{3}\right) + i\sin\left(\frac{240}{3} + \frac{2(2)\pi}{3}\right)\right]$$

$$z^{\frac{1}{3}} = 3\left[\cos\left(80 + 240\right) + i\sin\left(80 + 240\right)\right]$$

$$z^{\frac{1}{3}} = 3\left[\cos\left(320\right) + i\sin\left(320\right)\right]$$

Therefore,

$$z^{\frac{1}{3}} = 3cis(320)$$

The 3 roots of the equation are given as $3cis(80)$, $3cis(200)$, and $3cis(320)$

From the question we are required to raise the roots to the cube
Raising the roots of the equation to the cube using De Moivre's formula,

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

Cubing First root

$$3cis(80)$$

$$Root_1 = 3^3(\cos(80 \times 3) + i\sin(80 \times 3))$$

$$Root_1 = 27(\cos(240) + i\sin(240))$$

$$Root_1 = 27cis(240) \text{ or } Root_1 = 27cis\left(\frac{4\pi}{3}\right)$$

Cubing Second root

$$3cis(200)$$

$$Root_2 = 3^3(\cos(200 \times 3) + i\sin(200 \times 3))$$

$$Root_2 = 27(\cos(600) + i\sin(600))$$

$$Root_2 = 27cis(600) \text{ or } Root_2 = 27cis\left(\frac{10\pi}{3}\right)$$

Cubing Third root

$$3cis(320)$$

$$Root_3 = 3^3(\cos(320 \times 3) + i\sin(320 \times 3))$$

$$Root_3 = 27(\cos(960) + i\sin(960))$$

$$Root_3 = 27cis(960) \text{ or } Root_3 = 27cis\left(\frac{16\pi}{3}\right)$$

QUESTION 2

Evaluate

$$\left[\sqrt[3]{3}\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\right]^{10}$$

Solution
To solve this equation i will be making use of the De Moivre's theorem formula.

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

But we must find the value of θ first. Rearranging the given equation in De Moivre's formula we have

$$z^{10} = (\sqrt[5]{3})^{10}\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$$

If we compare it to the original equation, we can see that $\cos(\theta) = \frac{\sqrt{3}}{2}$ and $\sin(\theta) = \frac{1}{2}$ Solving for theta we get,

$$\cos(\theta) = \frac{\sqrt{3}}{2}$$

$$\theta = cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \frac{\pi}{6}$$

Also For sine we have,

$$\sin(\theta) = \frac{1}{2}$$

$$\theta = sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{6}$$

Rewriting De Moivre's formula with the value of θ, we have,

$$z^{10} = (\sqrt[5]{3})^{10}\left(\cos\left(\frac{\pi}{6}\times 10\right) + i\sin\left(\frac{\pi}{6}\times 10\right)\right)$$

Evaluating,

$$z^{10} = 3^{\frac{10}{5}}\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$$

$$z^{10} = 3^2\left(\left(\frac{1}{2}\right) + \left(-i\frac{\sqrt{3}}{2}\right)\right)$$

$$z^{10} = 9\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

Therefore, the final answer is

$$z^{10} = 9\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

QUESTION 3

Find $\frac{z_1}{z_2}$ in polar form:
$z_1 = 21\text{cis}(135°)$ $z_2 = 3\text{cis}(75°)$
Solution
Using the formula

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$$

Substituting

$$\frac{z_1}{z_2} = \frac{21}{3}(\cos(135 - 75) + i\sin(135 - 75))$$

$$\frac{z_1}{z_2} = 7(\cos(60) + i\sin(60))$$

$$\frac{z_1}{z_2} = 7\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

Therefore,

$$\frac{z_1}{z_2} = \frac{7}{2} + i\frac{7\sqrt{3}}{2}$$

References

Abramson, J. (2017). *Algebra and trigonometry*. OpenStax, TX: Rice University. Retrieved from https://openstax.org/details/books/algebra-and-trigonometry