Comparison of PCA and FDA for Monitoring of Coupled Liquid Tank System

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***Abstract*—This paper deals with the implementation of data driven techniques, Principal component analysis (PCA) and Fisher Discriminant analysis (FDA), for fault detection and identification in coupled liquid tank system (CLTS). A CLTS is used as a non-linear benchmark in control engineering. PCA transforms the higher dimensional data to a lower dimension, while FDA extracts the discriminant information for fault diagnosis. Actuator and sensor faults are detected. Multiple faults are detected simultaneously by using $T^2 - statistics$ and $Q - statistics(SPE)$. Component fault is not detected by common Fault detection and isolation (FDI) schemes, but data driven techniques are applied to detect such faults as well. PCA has limitations for fault identification purpose, while FDA performance for component fault detection and fault identification is significantly better then PCA.**

I. INTRODUCTION

in modern control systems results in more stringent and sophisticated designs for different processes. In such complex processes a large number of variables are involved to ensure the safety and security of operation [1], [2]. The main objective of modern industry is to produce high quality products with minimum rate of rejection. So, in order to ensure the reliability and safety of process we need to design a secure and reliable system which can detect and diagnose the fault as soon as it occurs in the system [3]. The traditional analytical and knowledge based and fault detection and isolation (FDI) techniques do not address this problem in such high dimensional processes [2]. Univariate statistical monitoring techniques can not be applied in higher dimensional systems, because most of these processes are correlated and when a fault occurs in the system, then multiple alarms are set off simultaneously which do not give a clear indication of responsible variables. Whereas multivariate statistical techniques consider the correlation among variables, hence efficient in fault detection [4]–[6].

Principal component analysis (PCA) projects the higher dimensional data into lower dimension while preserving the significant information. PCA was introduced by Jackson and Mudholker [7] in 1979. Since then it is widely used in pattern recognition, image processing and feature extraction techniques [8]. Hotteling's T^2 statistics [9] and squared prediction error (SPE) or Q statistics are commonly used for fault detection with PCA. T^2 statistics measures the score space of PCA while SPE considers the residual space.

Fisher discriminant analysis (FDA) is also a linear dimensionality reduction technique. It determines the set of vectors, which are arranged in such a way that they maximize the separation among classes while minimize the separation between classes [10], [11]. PCA is efficient in terms of detection of faults while FDA is optimal in terms of diagnosing the faults [12], [13].

Once a fault is detected, the next step is to determine the cause of occurrence of the fault. Fault identification finds the contribution of observation variables which are responsible for this fault. In this paper, fault identification algorithm is applied to determine the faulty instruments in Coupled liquid tank system (CLTS). CLTS is an important benchmark, which is used for simulation of chemical processes. It is similar to many plants in industry like petrochemical, waste-water treatment system, breweries and refineries [14]. Linearized model of CLTS is used for fault detection in [15] and [16]. But component fault problem is not addressed by analytical and knowledge based techniques.

In this paper, multi-variate statistical techniques are used for the detection of additive as well as multiplicative faults. Both PCA and FDA are applied for fault detection. In next step fault identification algorithm is implemented to find the faulty variables, which are used for fault diagnosis purpose. PCA and FDA performs better for the fault detection of additive faults. Since PCA does not consider the information among classes while calculating its transformation matrix, thus it performs poorly for fault identification. The performance of FDA is significantly better than PCA for the fault identification.

The rest of the paper is organized as follows: Section II and III describe the basic data-driven techniques, Principal component analysis (PCA) and Fisher discriminant analysis (FDA) for fault diagnosis. In Section IV, importance of coupled liquid tank system discussed in detail. In Section V, the data driven techniques are applied to Coupled Liquid Tank System and faults are detected and identified. Section VI concludes this paper.

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II. Introduction

Advancement in modern control systems results in more stringent and sophisticated designs for different processes. In such complex processes a large number of variables are involved to ensure the safety and security of operation [1], [2]. The main objective of modern industry is to produce high quality products with minimum rate of rejection. So, in order to ensure the reliability and safety of process we need to design a secure and reliable system which can detect and diagnose the fault as soon as it occurs in the system [3]. The traditional analytical and knowledge based and fault detection and isolation (FDI) techniques do not address this problem in such high dimensional processes [2]. Univariate statistical monitoring techniques can not be applied in higher dimensional systems, because most of these processes are correlated and when a fault occurs in the system, then multiple alarms are set off simultaneously which do not give a clear indication of responsible variables. Whereas multivariate statistical techniques consider the correlation among variables, hence efficient in fault detection [4]–[6].

Principal component analysis (PCA) is a dimensionality reduction technique which projects higher dimension data into a lower dimension, while it preserves the significant information in the original input data set [17]. Because of dimensionality reduction ability of PCA, it is widely studied and applied in industrial process [4] in last few decades. Since PCA is optimal in terms of extracting the significant information, it is widely studied and used as a data-driven fault detection technique [2].

Consider a process with input data set matrix $X \in \Re^{n\times m}$, where m represents process variables and n is the number of observations for each variable. The standard PCA algorithm is briefly described as follows:

- *Step 1*: Normalize the columns of X by subtracting mean of each column and divide it with the standard deviation of each column in such a way that mean and variance of each column becomes equal to 0 and 1, respectively.
- *Step 2*: Find the covariance matrix C by following expression
$$C = \frac{1}{n-1} X^T X \tag{1}$$
- *Step 3*: In order to find Loading vector perform eigenvalue decomposition(EVD) on covariance matrix C
$$C = \frac{1}{n-1} X^T X = V\Lambda V^T \tag{2}$$
where Λ=diag$(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \geq 0)$
- *Step 4*: Find the number of principal components a by parallel analysis method and divide V into score and residual space
$$\Lambda = \begin{bmatrix} \Lambda_{pc} & 0 \\ 0 & \Lambda res \end{bmatrix}, \Lambda_{pc} = diag(\lambda_1, \lambda_2, \cdots, \lambda a)$$
$$\Lambda_{res} = diag(\lambda_{a+1}, \lambda_{a+2}, \cdots, \lambda_m)$$
$$V = \begin{bmatrix} V_{pc} & V res \end{bmatrix}, V_{pc} \in \Re^{m\times a}, V_{res} \in \Re^{m\times(m-a)}$$
Where V_{pc} and V_{res} represents score and residual spaces, respectively.
The loading matrix L is given as
$$L = V_{pc}$$
- *Step 5*: Determine the transformation matrix T
$$T = XL$$

A. Fault detection using PCA

In multivariate process, fault can be detected by using T^2 statistics and squared prediction error (SPE) or Q-statistics. Hotteling's T^2-statistics can be computed as [7]

$$T^2 = x^T L \Lambda_{pc}^{-1} L^T x \tag{3}$$

where L consists the loading vectors which contains significant amount of variance (vectors related with large singular variance). If a is the total number of vectors which are retained in Λ_{pc}, threshold can be computed by using [7]

$$J_{th,T^2} = \frac{a(n-1)(n+1)}{n(n-1)} F_\alpha(a, n-a) \tag{4}$$

where $F_{\alpha(a,n-a)}$ is the F-distribution with a and $(n-a)$ degrees of freedom. A fault is detected if T^2 given by equation (3) exceed the limit defined by equation (4).

As T^2 statistics is modelled on the basis of loading vectors which are related with large singular values, so it does suffer from over-sensitivities to inaccuracies [7] in the lower singular values. This problem is handled by Q statistics, which considers residual space rather than score space.

$$Q = x^T V_{res} V_{res}^T x \tag{5}$$

Threshold for Q-statistics can be computed by

$$J_{th,SPE} = \theta_1 \left(\frac{h_0 c_\alpha \sqrt{2\theta_2}}{\theta_1} + 1 + \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2} \right)^{1/h_0} \tag{6}$$

where

$$\theta_i = \sum_{j=a+1}^{n} \lambda_j^{2i}$$

and

$$h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}$$

and c_α is the deviation of normal distribution corresponding to the $(1-\alpha)$ percentile. α, is the level of significance, the threshold for the Q statistic can be computed using (6) and be used to detect faults.

B. Fault Identification

Fault detection is prerequisite of fault identification. Fault identification determine the contribution of variables for the occurrence of fault. Variables with large contribution index are responsible for fault. The general algorithm for fault identification is given as follows:

- *Step 1*: For each observation, find the out of control status for absolute values of observations, i.e calculate $(t_i/\sigma_i)^2$, where t is the loading vector of transformation matrix T.

- *Step 2*: Find the contribution of all variables by the following expression

$$cont_{i,j} = \frac{t_i}{\sigma_i^2} L_{i,j}(x_j - \mu_j)$$

where j represent the j^{th} variable and $L_{i,j}$ is the $(i,j)^{th}$ element of matrix L.
- *Step 3*: Only positive values of $cont_{i,j}$ are considered for contribution , all negative values of $cont_{i,j}$ will be set to zero.
- *Step 4*: The total contribution of j^{th} variable x_j will be

$$CONT_j = \sum_{i=1}^{m} (cont_{i,j})$$

- *Step 5*: Plot the contribution of all variables on a single graph. The variables which have larger contribution on plot, are responsible for the occurrence of fault. So these variables can be used for fault diagnosis.

III. Fisher discriminant analysis

PCA is a well known dimensionality reduction technique, but it does not consider the information between different classes of faults, while calculating its transformation matrix T [12]. This problem is incorporated by Fisher discriminant analysis (FDA). The transformation matrix of FDA contains vectors which maximize the separation between different classes of faults, while minimizing the within class scatter.

Consider a process with m variables and n observations for each variable, p is the number of classes under different faults and n_j number of observations in j^{th} class are stacked in a matrix $X \in \Re^{n \times m}$. Let x_i represents the transpose of i^{th} row of stacked matrix X. The standard FDA algorithm is briefly formulated as follows:

- *Step 1:* Calculate total-scatter matrix S_t

$$S_t = \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})^T \tag{7}$$

where $\bar{x}$ represents the the total mean vector.
- *Step 2:* Calculate within-class scatter matrix S_w

$$S_w = \sum_{j=1}^{p} S_j \tag{8}$$

where

$$S_j = \sum_{x_i \in \chi_j}^{n} (x_i - \bar{x}_j)(x_i - \bar{x}_j)^T \tag{9}$$

$\bar{x_j}$ is the mean of j^{th} class.
- *Step 3:* The between-class-scatter matrix is

$$S_b = \sum_{i=1}^{p} (\bar{x}_j - \bar{x})(\bar{x}_j - \bar{x})^T \tag{10}$$

$$S_t = S_b + S_w \tag{11}$$

- *Step 4:* Find FDA vectors by solving the generalized eigenvalue problem by solving following equation.

$$S_b w_k = \lambda_k S_w w_k \tag{12}$$

- *Step 5:* Since rank$(S_b) < p$, there exist at most $p-1$ eigenvectors which are associated with non-zero eigenvalues [2]. Let a represents the number of non-zero eigenvalues, then

$$W_a = \begin{bmatrix} w_1 & w_2 & \cdots & w_a \end{bmatrix}$$

- *Step 6:* FDA transformation vectors are calculated as

$$z_i = W_a^T x_i \tag{13}$$

FDA considers the information among different classes while computing the transformation vectors as it is evident from equation (13), while PCA does not. So FDA transformation vectors have more discriminant power than that of PCA. As a result FDA performance for fault detection and identification is far better than PCA.

A. Fault detection

Hotteling's T^2 statistics is used for fault detection. T^2 statistics of j^{th} class is computed as

$$T_j^2 = x^T W_{\bar{a}} (W_{\bar{a}}^T S_j W_{\bar{a}})^{-1} W_{\bar{a}}^T x \tag{14}$$

Choose largest value $\bar{a} \leq a$ in such a way that $(W_{\bar{a}}^T S_j W_{\bar{a}})$ should be a full rank matrix with

$$W_{\bar{a}} = \begin{bmatrix} w_1 & w_2 & \cdots w_{\bar{a}} \end{bmatrix}$$

Threshold for T^2 statistics of j^{th} class for a given significant level α is computed as

$$J_{th,T^2}^{j} = \frac{\bar{a}(n-1)(n+1)}{n(n-1)} F_\alpha(\bar{a}, n-\bar{a}) \tag{15}$$

A fault will be detected in j^{th} class if following condition is satisfied

$$T_j^2 \geq J_{th,T^2}^{j}$$

IV. System description and background results

A. System description

In modern Process and manufacturing industries like petrochemical, paper making and water treatment industries, liquid level and flow control among tanks is one of the basic problems. Liquid in tanks is needed to be pumped, stored and transferred to other tanks. Coupled liquid tank system is an important benchmark for chemical processes. It is similar to plants which are used in process industry. Most tanks are coupled in such a way that liquid level and flow interact, and this must also be controlled.

Coupled liquid tank system is a benchmark to test control and FDI algorithms for such chemical and industrial processes. A schematic of coupled liquid tank system (CLTS) is shown in Figure 1. It consists of three cylindrical tanks (two of them are placed vertically while one in horizontal position). The horizontal tank produces non-linearities in the dynamics of the system. Each tank has a capacity of 5 litres while water level varies from 0-20 cm. A computer controlled pump is used to pump the water from tank T1 to tank T3, while water flow from tank T3 to T2 and T2 to T1 is due to gravity. This

TABLE I: System parameters and their values

Notation	Description	Values
H_1, H_2 H_3	Level of liquid in Tanks 1,2 and 3	variable
C_2, C_3	Orifice discharge coefficient	3.44
q_i	Flow rate of pump	variable
r	radius of cylinder of tank T2	9.5 cm
L	Length of cylinder of tank T2	18.5 cm
A_1, A_3	Cross-sectional area of T1 and T3	283.5 cm^2

pump acts as actuator in this system. Water level is measured with the help of sensor (plastic float and potentiometer). Every tank is equipped with a sensor.

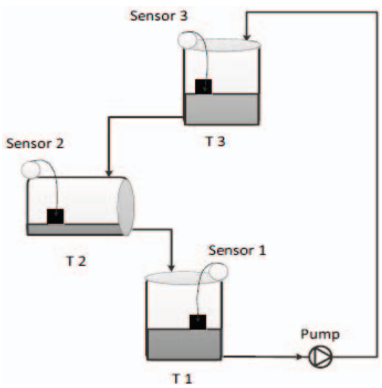

Fig. 1: Schematic of Coupled Liquid Three Tank System

B. Mathematical modelling

Utilizing the mass balance equations and Torricelli's Law, the nonlinear dynamics of the system are obtained as [16]:

$$\frac{dH_1}{dt} = \frac{C_2\sqrt{H_2} - q_i}{A_1} \tag{16}$$

$$\frac{dH_2}{dt} = \frac{C_3\sqrt{H_3} - C_2\sqrt{H_2}}{2L\sqrt{r^2 - r(r-H_2)^2}} \tag{17}$$

$$\frac{dH_3}{dt} = \frac{q_i - C_3\sqrt{H_3}}{A_3} \tag{18}$$

The parameters of the system and their numerical values are described in Table I..

C. Possible faults in CLTS

The common faults which can occur in the coupled liquid tank system are as follows:

1) *Actuator Fault:* Pump is the only actuator in this system. There is a fault in the pump if it does not provide the desired flow rate. The flow rate in the presence of a fault can be given as

$$FR = FR_{controlled} \pm FR_{fault}$$

2) *Sensor Faults:* A combination of float and rotary potentiometer is used to measure the level of water in tanks. Fault occurs in the system when position of float or potentiometer is displaced from their normal positions or float is broken.

3) *Component Faults:* Leakages in tanks, blockage or leakage in valves results in faults which are categorized as process or component fault.

V. Application to Couple Liquid Tank System

Additive as well as multiplicative faults are detected in this section. Contribution plots for both PCA and FDA are used for identification of variables, which are responsible for the occurrence of these faults. The variables in CLTS are actuator, sensor 1, sensor 2 and sensor 3, these variables are represented by x_1, x_2, x_3 and x_4, respectively. We generated 50 observations for each variable under normal operating condition. This data represents class 1 data (fault free case). Then another 50 measurements are generated in the presence of actuator fault. This data represents class 2 data (actuator fault). After first 100 measurements, sensor fault is also introduced. Now two faults (actuator plus sensor) are present simultaneously. This data represents class 3 data (actuator plus sensor fault).

A. Fault detection using PCA

The PCA application is illustrated using input data set consists of three classes, with each class containing $m = 4$ variables and $n = 50$ observations. Class I data represents data under normal operating condition i.e. fault free case. Class II data represents actuator fault data, while actuator plus sensor fault data is placed in Class III data. PCA algorithm, discussed in section II, is applied to this data. The algorithm discussed in section II is applied on data

$$\Lambda = \begin{bmatrix} 1.9740 & 0 & 0 & 0 \\ 0 & 1.2690 & 0 & 0 \\ 0 & 0 & 0.6064 & 0 \\ 0 & 0 & 0 & 0.1506 \end{bmatrix} \tag{19}$$

$$V = \begin{bmatrix} -0.1813 & 0.2920 & 0.9391 & -0.0005 \\ -0.4967 & -0.6839 & 0.1170 & 0.5215 \\ -0.7059 & -0.0395 & -0.1244 & -0.6962 \\ -0.4714 & 0.6675 & -0.2982 & 0.4933 \end{bmatrix} \tag{20}$$

PCA model contains only two principal components as for $a = 2$, then $(3.243/4) = 81\%$ of total variance is captured by these two PCs. Thus the loading vector L contains score space corresponding to these two eigenvalues.

$$L = \begin{bmatrix} -0.1813 & 0.2920 \\ -0.4967 & -0.6839 \\ -0.7059 & -0.0395 \\ -0.4714 & 0.6675 \end{bmatrix} \tag{21}$$

Figure 2 and 3 shows the T^2 and SPE charts respectively, with upper control limits. It is clearly visible from Figure 2 that as soon as a fault is occurred in the system, it is detected. Actuator fault appear after first 50 samples, so considerable amount of variation is present in the T^2 chart at this instant, which shows a fault is occurred. This behaviour is more precisely visible even when sensor fault also appear after 100 samples.The SPE chart gives an indication that something went wrong after 50 samples. Both of these charts can detect

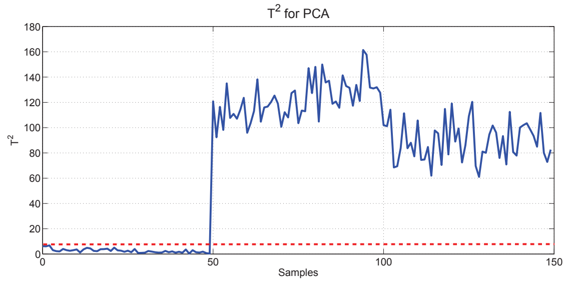

Fig. 2: T^2 plot for PCA

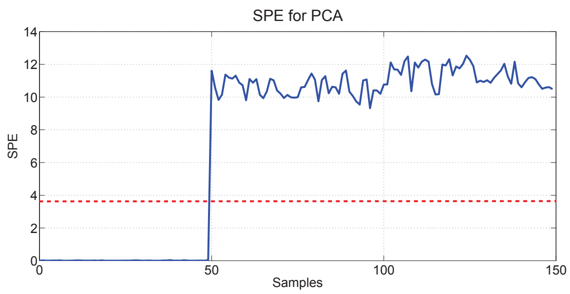

Fig. 3: SPE chart for PCA

the change in the process and clearly indicates the occurrence of fault after 50 samples. As soon as fault occur in the process, T^2 and SPE crosses the limits defined by equation (4) and equation (6), respectively. For a=2 and $\alpha = 98\%$, $J_{th,T^2} = 8.08$ and $J_{th,SPE} = 3.58$. So we can conclude that PCA is a quite useful for the detection of faults.

B. Fault detection using FDA

Fisher discriminant analysis can also be used for fault detection. FDA is applied on the same data. The within-class scatter and between class scatter matrices are computed as

$$S_w = \begin{bmatrix} 148.6159 & 13.9311 & 19.7390 & 13.2652 \\ 13.9311 & 72.2792 & 97.2769 & 60.9156 \\ 19.7390 & 97.2769 & 147.9363 & 106.0052 \\ 13.2652 & 60.9156 & 106.0052 & 85.2532 \end{bmatrix} \quad (22)$$

$$S_b = \begin{bmatrix} 0.3841 & -3.5781 & -0.5923 & 2.9700 \\ -3.5781 & 76.7208 & 8.0709 & -69.7322 \\ -0.5923 & 8.0709 & 1.0637 & -7.0551 \\ 2.9700 & -69.7322 & -7.0551 & 63.7468 \end{bmatrix} \quad (23)$$

Solving equation (12) we have only two non-zero eigenvalues, so the eigenvectors associated to these non-zero eigenvalues are

$$W_a = \begin{bmatrix} 0.0005 & 0.0069 \\ -0.7488 & -1.0000 \\ 1.0000 & 0.8828 \\ -0.7083 & -0.3214 \end{bmatrix} \quad (24)$$

and the corresponding eigenvalues are $\lambda_1 = 8.02$ and $\lambda_2 = 1.03$.

While Figure 4 shows the fault detection using FDA. The T^2 detect the fault after 50 samples, and this detection becomes more obvious after 100 samples as sensor fault also added with actuator fault. The threshold for T^2 is computed by equation (15). J_{th,T^2} for FDA becomes equal to 8.08. T^2 chart clearly indicate that something went wrong after 50 samples and other fault also occur after 100 samples.

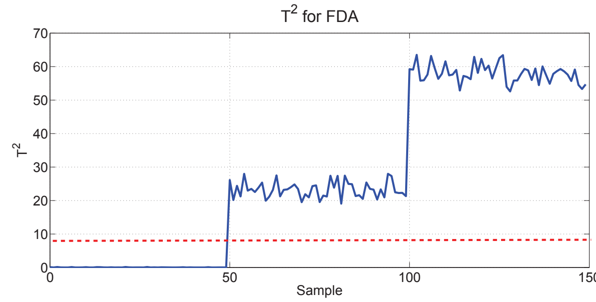

Fig. 4: T^2 chart for FDA

C. Component fault detection

Leakages in tanks results in faults which are termed as component fault. Component fault is categorized as multiplicative fault. Here the component fault appear after 50 samples. Component fault is not detected by commonly used analytical and knowledge based techniques [15], [16]. Here the data driven techniques are applied for fault detection.

Figure 5 shows the T^2 chart for PCA. It is evident that in the absence of fault the T^2 statistics for component remains under the defined thershold but as soon as fault appears after first 50 samples then T^2 exceeds the threshold limit $J_{th,T^2} = 8.08$, which gives the indication of fault occurrence.

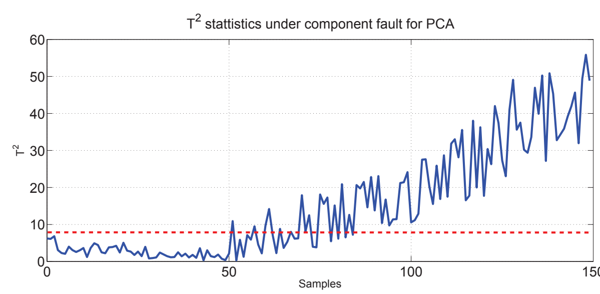

Fig. 5: T^2 chart under component fault for PCA

Figure 6 shows the output of same data applied for FDA. Here also T^2 exceeds the threshold limit $J_{th,T^2} = 8.08$ when the component fault appears after 50 samples. In the absence of fault, there are no such false alarms and there are no such violations of the defined threshold. Thus fault detection is done quite successfully.

D. Fault identification

Fault identification and its algorithm are discussed in detail in section III. Fault identification is an important step in fault diagnosis. The contribution plots tells the information about contribution of variables for the occurrence of fault. Figure 7 shows the contribution plot for PCA. As fault exists in sensor

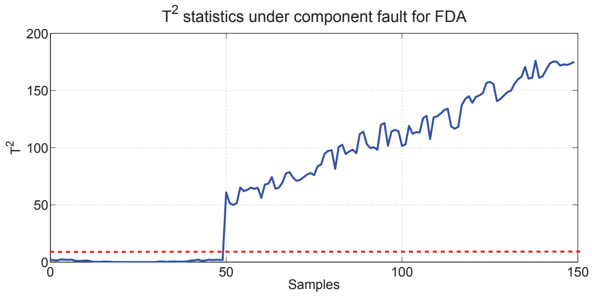

Fig. 6: T^2 chart under component fault for FDA

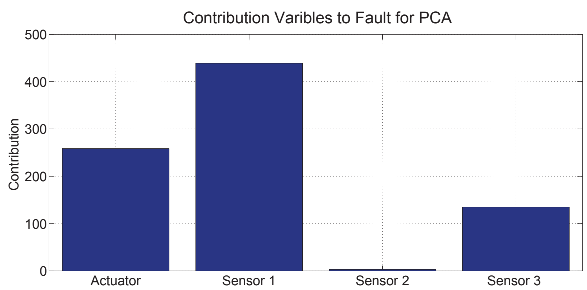

Fig. 7: Contribution Plots of variables using PCA

1 and actuator, so the contribution clearly gives the indication that these two variables have large contribution for fault.

The contribution plots for FDA are shown in Figure 8. As we have already discussed that FDA considers the information of different classes, so its contribution shows that the two variables, i.e. sensor 1 and actuator have larger contribution to fault. while contribution of other variables is small. Thus FDA is efficient in terms of fault identification then PCA.

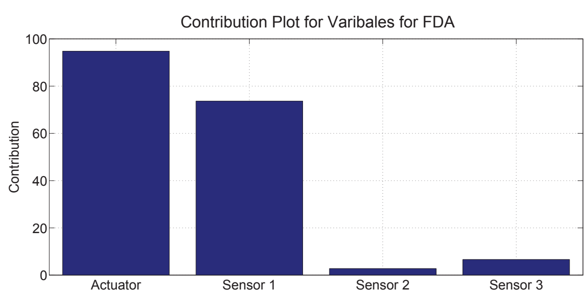

Fig. 8: Contribution Plots of variables using FDA

VI. Conclusion

In this paper, we demonstrate the importance of fault detection and variable identification for fault diagnosis in non-linear systems. The simulations are carried out with real time data of coupled liquid tank system (CLTS). The fault can be associated with process, sensors or equipments. The data driven techniques are capable for detecting all these faults.

The output plots show the efficiency of these techniques for fault detection and identification rather than using crude method of just defining thresholds for measurement and observing these variables. Principal component analysis substantially increased sensitivity of fault detection.

Although PCA is efficient in fault detection but it is not well suited for fault identification or diagnosis as it does not consider the information between different classes of data, while computing the principal components. Fisher discriminant analysis considers the information among different classes while computing the transformation matrix. Hence its performance for fault detection and identification is better then PCA. A comparison shows that FDA performance for fault identification is significantly better than PCA. The variables which are responsible for the occurrence of fault, are identified and these variable need to be diagnosed.

References

[1] M. Adil, M. Abid, A. Khan, G. Mustafa, and N. Ahmed, "Exponential discriminant analysis for fault diagnosis," *Neurocomputing*, vol. 171, pp. 1344 – 1353, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0925231215011078

[2] L. Chiang, R. Braatz, and E. Russell, *Fault Detection and Diagnosis in Industrial Systems*, ser. Advanced Textbooks in Control and Signal Processing. Springer London, 2001. [Online]. Available: http://books.google.com.pk/books?id=G71zWeHzg2QC

[3] S. Yin, S. Ding, X. Xie, and H. Luo, "A review on basic data-driven approaches for industrial process monitoring," *Industrial Electronics, IEEE Transactions on*, vol. 61, no. 11, pp. 6418–6428, Nov 2013.

[4] L. M. Elshenawy and T. A. M. Mohamed, "Fault detection of nonlinear processes using fuzzy c-means-based kernel pca," in *ICMLEME'2014*, Jan 2014, pp. 318–324.

[5] X. Dai and Z. Gao, "From model, signal to knowledge: A data-driven perspective of fault detection and diagnosis," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 4, pp. 2226–2238, Nov 2013.

[6] V. Venkatasubramanian, R. Rengaswamy, K. Yin, and S. N. Kavuri, "A review of process fault detection and diagnosis: Part i: Quantitative model-based methods," *Computers & chemical engineering*, vol. 27, no. 3, pp. 293–311, 2003.

[7] J. E. Jackson, *A user's guide to principal components*. John Wiley & Sons, 2005, vol. 587.

[8] S. Joe Qin, "Statistical process monitoring: basics and beyond," *Journal of chemometrics*, vol. 17, no. 8-9, pp. 480–502, 2003.

[9] H. Hotelling, "Analysis of a complex of statistical variables into principal components." *Journal of educational psychology*, vol. 24, no. 6, p. 417, 1933.

[10] Y.-Q. Cheng, Y.-M. Zhuang, and J.-Y. Yang, "Optimal Fisher discriminant analysis using the rank decomposition," *Pattern recognition*, vol. 25, no. 1, pp. 101–111, 1992.

[11] M. Misra, H. H. Yue, S. J. Qin, and C. Ling, "Multivariate process monitoring and fault diagnosis by multi-scale PCA," *Computers & Chemical Engineering*, vol. 26, no. 9, pp. 1281–1293, 2002.

[12] Q. P. He, S. J. Qin, and J. Wang, "A new fault diagnosis method using fault directions in Fisher discriminant analysis," *AIChE Journal*, vol. 51, no. 2, pp. 555–571, 2005.

[13] M. Fuente, D. Garcia-Alvarez, G. I. Sainz-Palmero, and T. Villegas, "Fault detection and identification method based on multivariate statistical techniques," in *Emerging Technologies Factory Automation, 2009. ETFA 2009. IEEE Conference on*, Sept 2009, pp. 1–6.

[14] "Laboratory software for the three-tank benchmark system: From PID to multi-agent fault-tolerant fractional control," *Procedia - Social and Behavioral Sciences*, vol. 46, no. 0, pp. 1919 – 1923, 2012.

[15] A. Hussain, A. Q. Khan, M. Abid, and M. Tufail, "On fault detection in coupled liquid three tank system using subspace aided data driven design," in *Emerging Technologies (ICET), 2012 International Conference on*, Oct 2012, pp. 1–6.

[16] F. Kousar, M. Abid, and A. Q. Khan, "Sensor fault detection in coupled liquid tanks system," in *FIT'12*, 2012, pp. 318–324.

[17] K. Pearson, "LIII. on lines and planes of closest fit to systems of points in space," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 2, no. 11, pp. 559–572, 1901.