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定理(排序不载). 没 x1, x2, ···, xn, y1, y2, ···, yn 是任意实数,满足:
                   x_1 \leq x_2 \leq \cdots \leq x_n, y_1 \leq y_2 \leq \cdots \leq y_n,
で: {1,2,···,n}→ {1,2,···,n}是一个双射(意即:0是个n级置换). 则有:
             X1 4n + x2 4n-1 + ... + Xn-1 /2 + Xny,
x_{1}y_{n} + x_{2}y_{n-1} + \cdots + x_{n-1}y_{2} + x_{n}y_{1} \leq x_{1}y_{0+1} + x_{2}y_{0+2} + \cdots + x_{n}y_{0+n}) \stackrel{\bullet}{\sim} x_{1}y_{1} + x_{2}y_{2} + \cdots + x_{n}y_{n}
Proof:先证明右边的产等式. 利用数学归纳法
                                   \therefore \times_1 y_{\sigma(1)} = \times_1 y_1 \leq \times_1 y_1.
 当们日本,
               v : [1] → [1]
                    当n=2at,
               o: [1,2] -> [1,2]
                                        *, you) + *2 you) = x,y, + x2 /2 < x, y, + x2 /2
                     11->1
                    21->2
              o: [1,2] -> [1,2]
                                    : y2-y1>0 x2-x1>0
                  1 1-> 2
                                     : (y2-y1)(x2-x1)>0
                   21->1
                                     .. 1 x2/2+x1/1-X1/2-X2/1/0
                                    · x1/2+ x2/1 € x1/1+ x2/2
       X_{1} y_{0(1)} + x_{2} y_{0(2)} = X_{1} y_{2} + x_{2} y_{1} \leq X_{1} y_{1} + x_{2} y_{2}
假设对于n=k(kept, k>2)时,对缝的置换。: [1,2,--, N)->[1,2,--,N],有:
                      X1 you) + X2 yor2) +...+ Xk york) € X1 y1 + X2 y2 +...+ Xkyk.
则对于一时时,分两种情况讨论:
①. o(k+1)=k+1. 则: o(1), o(2), ···, o(k)是1, 2, ···, k的一个排列, 由归纳假设, 有:
      X1 JOH) + X2 JOH2) + ... + XkJock) + Xk41 JOKH) = X1 JOH1) + X2 JOH2) + ... + XkJOK) + Xk41 JEH1
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② $\sigma(k+1) \neq k+1$. 则 $\sigma(k+1) \in \{1/2, \cdots, k\}$. 设 $t \in \{1/2, \cdots, k\}$ s.t. $\sigma(t) = k+1$ 则: $\sigma(1)$, \cdots , $\sigma(t-1)$, $\sigma(t+1)$, \cdots , $\varepsilon(k)$, $\sigma(k)$, $\varepsilon(k)$, $\varepsilon(k)$, $\varepsilon(k)$ 是 $1,2,\cdots$, $\varepsilon(k)$ 一个排列。 (这即: $\{\sigma(i)\}_{i=1}^{k+1}$ 是 $1,2,\cdots$, $\varepsilon(k)$ 一个排列). 计:

< XIYI + XZYZ + "+ XKYK + XKHI YKHI.

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: y (Xk+1-Xt) (Yk+1-Yo(k+1))>0
 :. Xp+1 yp+1 + xt yo-(p+1) > Xt yp+1 + Xp+1 yo-(p+1) = Xt yo-(t) + Xp+1 yo-(p+1)
 =. X1 you) + ... + X+1 yout) + X+ yout) + X+1 youth) + ... + Xk youk) + Xk+1 youk)
< x1 you) + · · + Xty you) + Xty you + Xty you + Xty you + Xty ykn
< XIYI + ··· + XRYR + XATI YRTI
   : 2 t Yne Nt, 有: X1 You) +··· + Xn You) < X1 Y1 +··· + Xn Yn.
 y_1 \in y_2 \in \dots \in y_n : -y_1 = -y_2 = \dots = -y_n = 
 \therefore \  \, \forall_1 \in \forall_2 \in \dots \in \forall_n, \quad -y_n \in -y_{n-1} \in \dots \in -y_2 \in -y_1 \; .
  = X1 (- Your) + X2 (- Your) + ... + Xn-1 (- Your) + Xn (- Your)
  = x_{1}(-y_{n}) + x_{n}(-y_{n-1}) + \cdots + x_{n-1}(-y_{n}) + x_{n}(-y_{n}) + x_{n}(-y_{n})
   : x, (-yo1)) + x2 (-yo(2)) + ... + xn-1 (-yo(n-1)) + xn (-you))
     \leq x_1(-y_n) + x_2(-y_{n-1}) + \dots + x_{n-1}(-y_2) + x_n(-y_1)
    = (x_1 y_{\sigma(1)} + x_2 y_{\sigma(2)} + \dots + x_{n+1} y_{\sigma(n+1)} + x_n y_{\sigma(n)}) \leq -(x_1 y_n + x_2 y_{n-1} + \dots + x_{n-1} y_2 + x_n y_1)
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:. X1 yn + X2 yn-1 + ... + Xn+ y2 + Xn y1 = X1 y0-11) + X2 y0-12) + ... + Xn+ y0-11, + Xn y0-11)

 $: \chi_{1}y_{1} + \chi_{2}y_{1-1} + ... + \chi_{n}y_{1} \leq \chi_{1}y_{2}y_{2} + ... + \chi_{n}y_{2}y_{2} + ... + \chi_{n}y_{n} \leq \chi_{1}y_{1} + ... + \chi_{n}y_{n} .$

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以下我们来试着深入讨论以上进证明过程中自6一些细节.
①当二3时,情况会处何?
0:[1,2,3] -> [1,2,3]
                           x1 yo11) + x2 yo12) + x3 yo13) = x1 y1 + x2 y2 + x3 y3
      1 1->1
                                                     < x1 /1 + x2 /2 + x3 /3.
      2 1->2
      ) m)
                         0:(x)-x2)(y3-y2)=0
o: [1,2,]] → [1,2,]]
                                                      x_2y_2 + x_3y_3 \geqslant x_2y_3 + x_3y_2.
   1 1->1
                         : X1 You) + X2 Your) +X3 You)
                                                    = \times_1 y_1 + \times_2 y_2 + \times_3 y_2
   21->3
                                                    < xy, + xzyz + xzyz
     31->2
o: [1/2,3] -> [1,2,]).
                           .. (x2-x1) (y2-y1)>0
                                                    :. x1y1+ x2/2 > x1/2+ x2/1
     11->2
                          : x, yo1) + x2 yo12) + x3 you) = x, y2 + x2 y, +x3 y3
      21-71
                                                     < x1/21 + x2/2 + x3/3
     31->3
\sigma: \{1,2,\}\} \longrightarrow \{1,2,\}\}
                           : (X3-X2)(3-1/1)>0
     11-2
                           : x2y1 + x3y3 > x2y3 + x3y1.
       2/0)
                      =. x1 yor1) + x2 yor2) + x3 yor3) = x1 y2 + x2 y3 + x3 y1 < x1 y2 + x2 y1 + x3 y8
       3 -> 1
                                                                     € xy+xy+xy
                         o: {1,2,3}→{1,2,3}
   1 1 3
                         :. X3 y3 + X1 y2 >> X1 y3 + X3 y2
      2 -> 1
                       : X, you) + X2 you) + X3 you) = X, y3 + X2 y, + X3 y2 < X, y2 + X2 y, + X3 y3
      ) m>2
                                                                    \leq x_1.y_1 + x_2y_2 + x_3y_3
                          : (x3-x1) [32-31)>0
o: [1,2,3] -> [1,2,3]
     1 \longrightarrow 3
                           = X, y, +x3 y3 > X, y3 + x3 y,
      2 1->2
                           - X1 you) + x2 you) + x3 you) = X1 y3 + X2 y2 + X3 y1
      3 ->1
                                                    < x1/1+ x2/2+ x3/3.
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②可否用数学归纳海直接证明《新念到后》和? proof: \$ n=1 Bf, 0: [1]→[1] $= x_1 y_1 = x_1 y_{\sigma(1)}$ $:= x_1 y_1 \leq x_1 y_{\sigma(1)}.$ 当n=2at, o: [1,2] -> [1,2] $(x_2-x_1)(y_2-y_1) > 0$: x2y2+x1y1 > X1y2+ x2y1 - ×1 y2+×241 € ×1 yo4) + ×2 yo(2) $\sigma: \{1, 2\} \longrightarrow \{1, 2\}.$:. $x_1y_2 + x_2y_1 \leq x_1 y_{\sigma(1)} + x_2 y_{\sigma(2)}$ 假设n=k (keth, k>2) 时楼式成之,即:对从上放置换o: [1,2,-,k]→[1,2,-,k], X, yk + xzyk+ +···+ xk+yz+ xky, ≤ X, yo(1)+ xzyo(2)+···+ xkyo(k). 见Jan=k+lat,分两种情况讨论: D. o(k+1) = 1, 21 0(1), 0(2), ..., o(k) ① o(1)=k+1. 则: o(2), o(), ···, o(k+1)是 1, 2, ···, k的一个排列. 由归纳假设, 有: X 1 1 + X 1 + X 1 + X 1 + X 1 = X 1 1 0 11) + X 1 + ··· + X 1 X, you) + X2 yo(2) + ... + Xk yo(k) + XkH yo(kH) = X1 yk+1 + X2 yo(2) + ... + Xk yo(k) + Xk+1 yo(k+1) / X, yk+1 + Xzyk + ... + Xkyz + Xp+1 y1. ① o(1) ≠ k+1. 则 o(k+1) e {1/2,···,k}. i& te {2,1,···,k,k+1}, s.t. o(t)=k+1 则: 0-(1), --, 0-(+1), 0-(+1), --, 0-(k), 0-(k+1)是 1,2,--, 166-1排列 $\therefore x_1 \leq x_t, \quad y_{k+1} \geqslant y_{\sigma(1)} \qquad \therefore (x_1 - x_t)(y_{k+1} - y_{\sigma(1)}) \leq 0$: X1 yk+1 + X+ yo11) < X1 yo11) + X+ yk+1 = X1 yo11) + X+ yott) : X1 yo11) + X2 yo(2) + ... + X+1 yo(+1) + X+ yo(+) + X+1 yo(+1) + ... + Xk+1 yo(k+1) > X1 yk+1 + X2 you) + ... + X+1 you+1) + X+ you) + X+4 youth) + ... + X+1 youk+1) = x1 9k+1 + x29k+...+ Xk+1 \$1

x, yn + x2 yn-1 + ... + xn y1 < x1 yo(1) + x2 yo(2) + ... + xn yo(n)

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