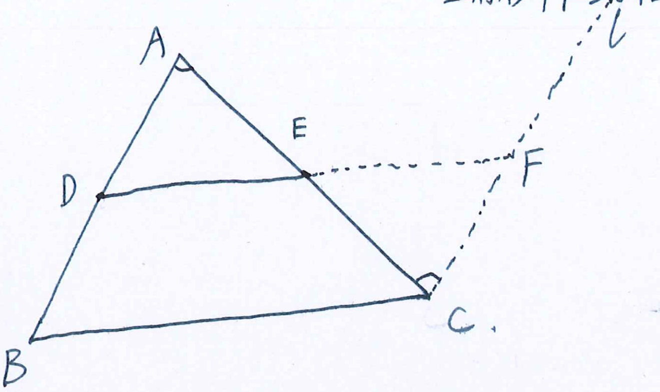
三角形中位线相关性质



已知 $\triangle ABC$，D是 AB中点，E是AC中点．求证：$DE \parallel BC$ 且 $DE = \frac{1}{2}BC$．

Proof：过点C作直线 $l \parallel AB$．延长DE交直线 l 于F．

$\because l \parallel AB$ $\therefore \angle DAE = \angle FCE$．

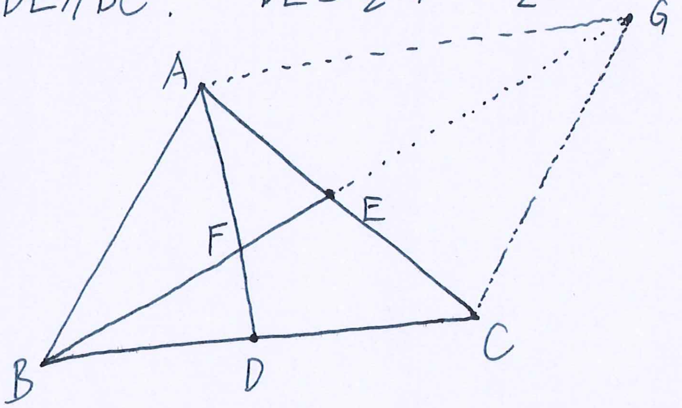
$\because$ E是AC中点 $\therefore AE = CE$

$\because \angle AED = \angle CEF$ $\therefore \triangle AED \cong \triangle CEF$ $\therefore DE = FE$． $\therefore$ E是DF中点

$\because \triangle AED \cong \triangle CEF$ $\therefore CF = AD$． $\because$ D是AB中点 $\therefore AD = DB$ $\therefore CF = AD = DB$．

$\because CF \parallel DB$ $\therefore$ 四边形 DFCB 是平行四边形（一组对边平行且相等的四边形是平行四边形）

$\therefore DE \parallel BC$， $DE = \frac{1}{2}DF = \frac{1}{2}BC$ □．



已知 $\triangle ABC$．D是BC中点，E是AC中点，F是AD和BE的交点．求证：$\frac{AF}{FD} = \frac{2}{1}$．

Proof：延长BE到G使 $BE = EG$．连结AG，CG．

$\because$ E是AC中点 $\therefore EA = EC$． $\because EB = EG$， $\angle AEG = \angle CEB$（对顶角相等）

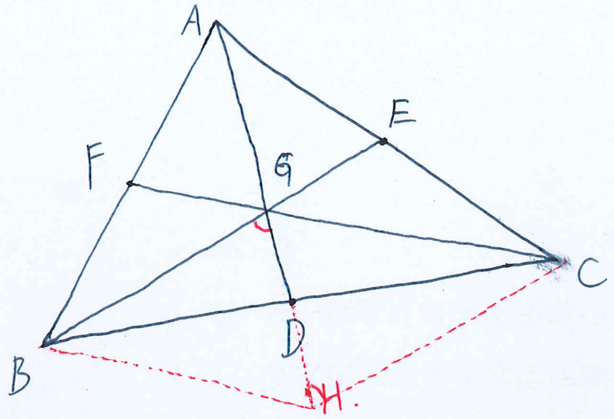
$\therefore \triangle AEG \cong \triangle CEB$． $\therefore \angle EGA = \angle EBC$ $\therefore AG \parallel BC$．

$\because \triangle AEG \cong \triangle CEB$ $\therefore AG = CB$ $\therefore AG \parallel CB$ 且 $AG = CB$ $\therefore$ 四边形AGCB是平行四边形．

$\because AG \parallel BC$ $\therefore \angle FAG = \angle FDB$， $\angle FGA = \angle FBD$ $\because \angle AFG = \angle DFB$

$\therefore \triangle AFG \sim \triangle DFB$ $\because$ ~~BD~~ D是BC中点 $\therefore BD = \frac{1}{2}BC = \frac{1}{2}AG = \frac{1}{2}GA$

$\therefore \frac{BD}{GA} = \frac{1}{2}$ $\therefore \frac{DF}{AF} = \frac{1}{2}$ $\therefore \frac{AF}{FD} = \frac{2}{1}$ □



已知△ABC，AD，BE，CF是三条中线，G是重心．求证：$\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$，$\overrightarrow{GD}+\overrightarrow{GE}+\overrightarrow{GF}=\vec{0}$．

Proof：延长GD到H，使DH＝GD．连结BH，CH．

∵D是BC中点 ∴DB＝DC． ∵DG＝DH，∠GDB＝∠HDC．（对顶角相等）

∴△GDB≌△HDC ∴GB＝HC 且∠BGD＝∠CHD ∴GB＝HC且GB∥HC

∴四边形GBHC是平行四边形．

由上一结论知，$\frac{AG}{GD}=\frac{2}{1}$． 又∵ $\frac{GD}{DH}=\frac{1}{1}$ ∴ $\cancel{\overrightarrow{GA}=}$ $\frac{AG}{GH}=\frac{1}{1}$

∴ $\overrightarrow{GA}+\overrightarrow{GH}=\vec{0}$ ∴ $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$．

∵ $\overrightarrow{GD}=\frac{1}{2}\overrightarrow{AG}=-\frac{1}{2}\overrightarrow{GA}$． $\overrightarrow{GE}=\frac{1}{2}\overrightarrow{BG}=-\frac{1}{2}\overrightarrow{GB}$，$\overrightarrow{GF}=\frac{1}{2}\overrightarrow{CG}=-\frac{1}{2}\overrightarrow{GC}$．

∴ $\overrightarrow{GD}+\overrightarrow{GE}+\overrightarrow{GF}=-\frac{1}{2}(\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC})=-\frac{1}{2}\cdot\vec{0}=\vec{0}$ □