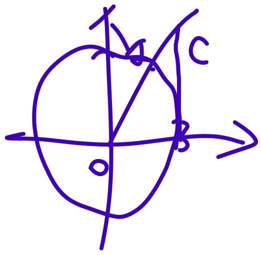
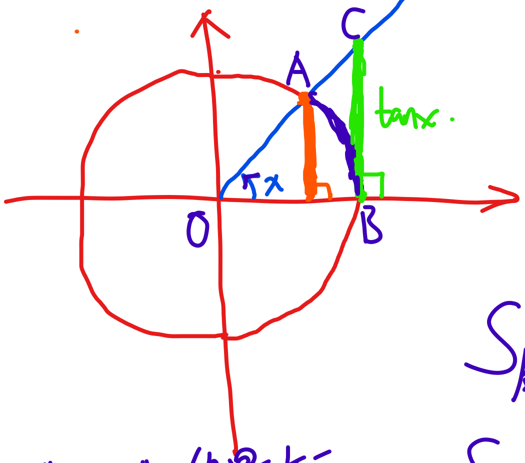
当 $x\in(0,\frac{\pi}{2})$ 时，求证：$0<\sin x<\boxed{x<\tan x}$

证明：在单位圆中考虑三角函数线.





$\tan x=\dfrac{对边}{邻边}$

$对边=\tan x\cdot 邻边$

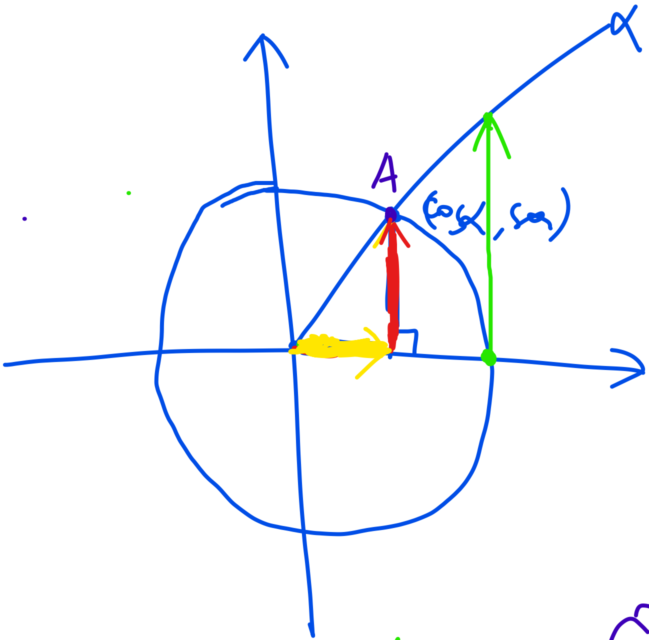
$S_{扇形OAB}=\frac{1}{2}lr=\frac{1}{2}x\cdot 1=\frac{1}{2}x.$

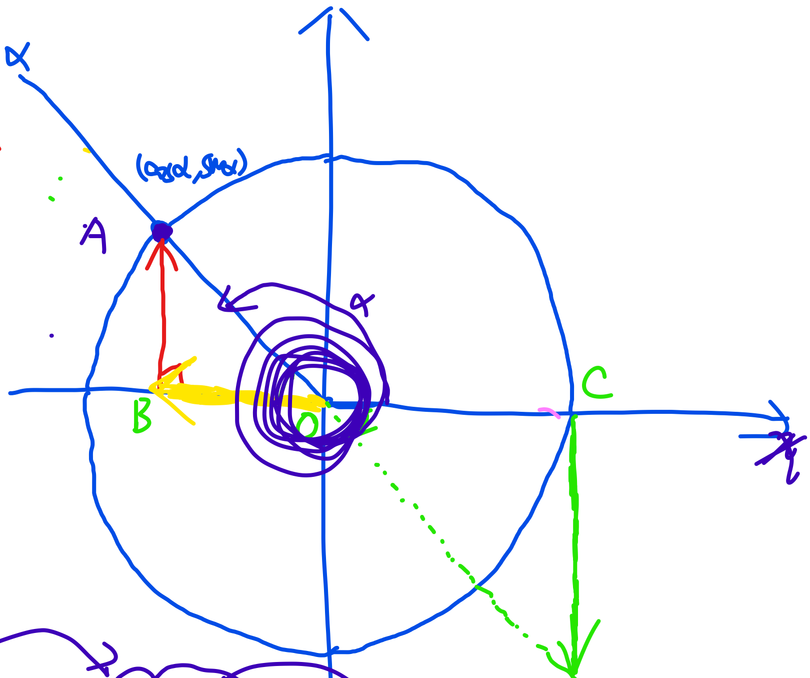
$S_{\triangle OBC}=\frac{1}{2}OB\cdot BC=\frac{1}{2}r\cdot\tan x=\frac{1}{2}\tan x.$

∵ 垂线段最短.

$\therefore 0<\sin x<x$.

∵ 扇形OAB包含在△OBC的内部. $\therefore S_{扇形OAB}<S_{\triangle OBC}$ $\therefore x<\tan x$





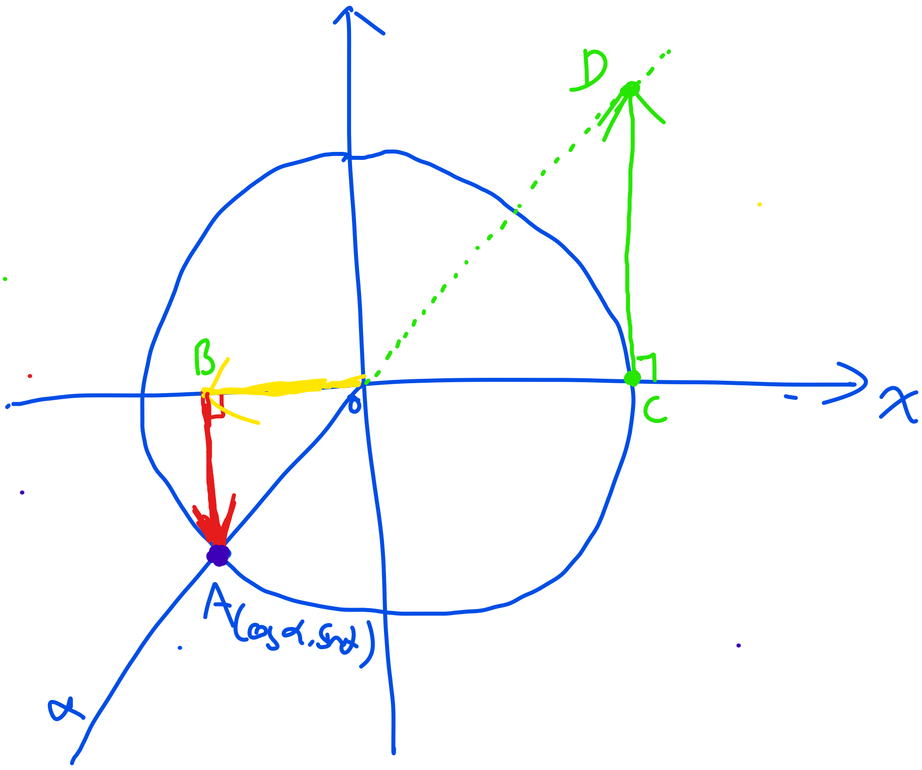
D $\cdot \alpha+\pi$

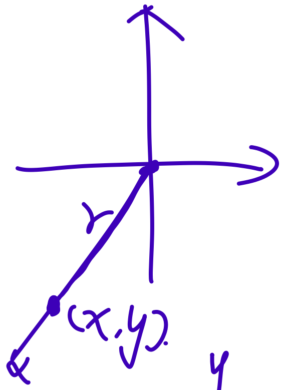
$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{AB}{-OB}=-\dfrac{AB}{OB}=-\tan\angle AOB=-\tan\angle DOC$

$=-\dfrac{CD}{OC}=-\dfrac{CD}{1}=-CD$

$\because \angle AOB=\angle DOC$ (对顶角相等)

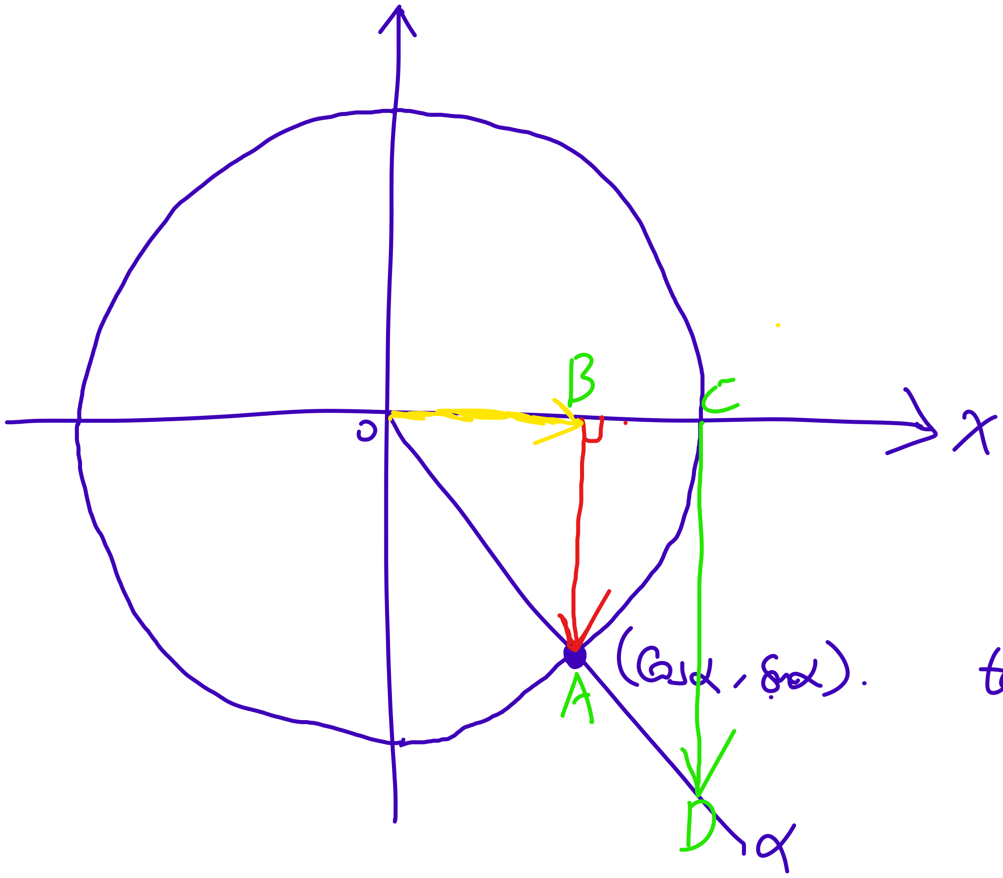
$\therefore \tan\angle AOB=\tan\angle DOC$





$$\frac{y}{r} = \sin\alpha$$
$$\frac{x}{r} = \cos\alpha$$
$$\frac{y}{x} = \tan\alpha$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-AB}{-OB} = \frac{AB}{OB} = \tan\angle AOB = \tan\angle DOC = \frac{DC}{OC} = \frac{DC}{1} = DC$$



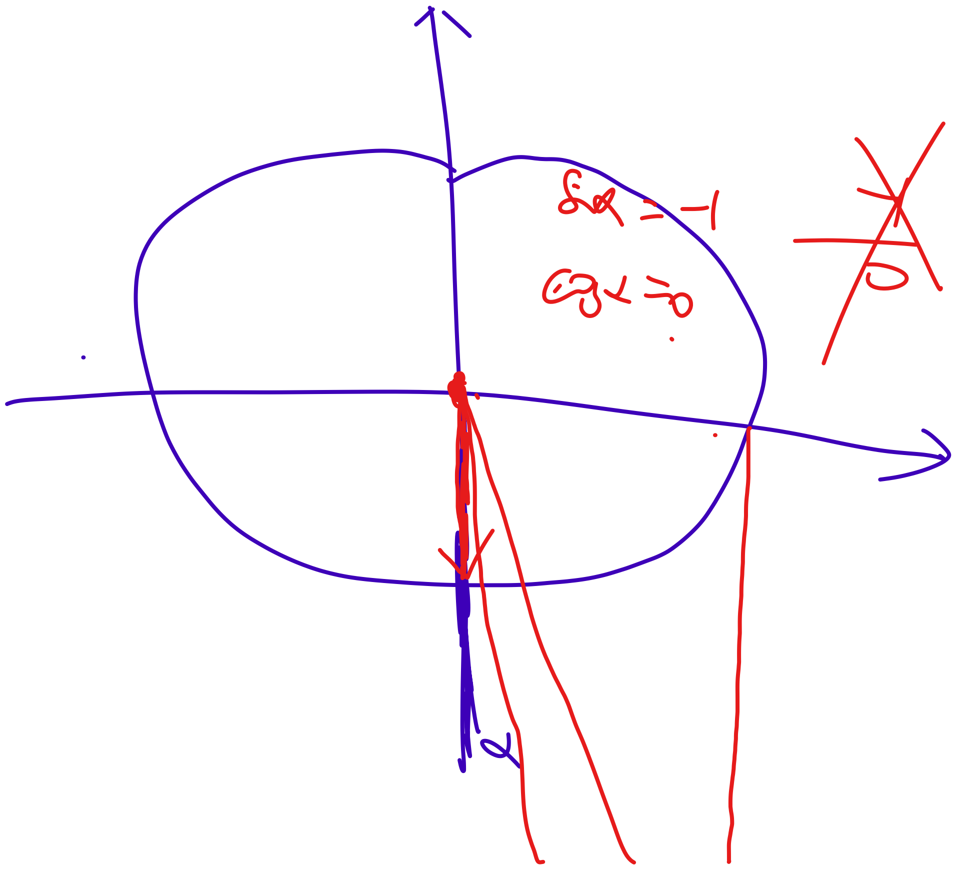
$\sin\alpha = -AB$

$\cos\alpha = OB$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-AB}{OB} = -\frac{AB}{OB} = -\tan\angle AOB = -\tan\angle DOC$$

$$= -\frac{DC}{OC} = -\frac{DC}{1} = -DC$$


sinα = -1
cosα = 0
α