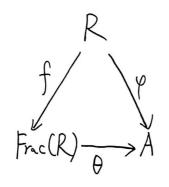
L_{emm} (整环上的分式域的泛性质) R是一个CCR的整环,已经证明了 Frac(R) 是域, $f: R \longrightarrow Frac(R)$ 是单同态 见有: $x \longmapsto [x,1]$

对V交换环A, V环同态 $\varphi: R \to A$ 满起 $\varphi(R \setminus \{0\}) \subseteq A^X$, 存在唯一的环同态 $\theta: F_{\text{rec}}(R) \longrightarrow A$, s:t. 下图交换:



 $Prof: x t V 交换环A, V 环同态 <math>\varphi: R \longrightarrow A 满足 \varphi(R \setminus \{0\}) \subseteq A^{\times}$,

:: Ratio $(R) = \{(x,y) \in \mathbb{R}^2 : y \neq 0\}$

·· R是整环 ·· R是非零环 ·· |_R ≠ O_R ·· (O_R , |_K) ∈ Ratio (R)

: Ratus (R) \$\$

已经证明了: Ratio (R)上的=元关系~: 对 (x_1,y_1) , $(x_2,y_2) \in Ratio$ (R), $(x_1,y_1) \sim (x_2,y_2) \iff x_1y_2 = x_2y_1$

是Ratio(R)上的等价关系

: A是交换环 : A是非空集合.

: 定义映射 α : Ratio (R) \longrightarrow A (x, y) $\longmapsto \varphi(x)(\varphi(y))^{-1}$

下面证明映射《的良定性.

```
xf∀(x,y)∈Ratio(R),有:x∈R且y∈R且y+0
 :: x∈R :: φ(x)∈A :: y∈RAy≠0 :: y∈R\ξ0) :: φ(y)∈Ax
 \therefore (\varphi(y))^{-1} \in A^{x} \qquad \therefore (\varphi(y))^{-1} \in A
 \therefore \  \  \, \varkappa \left( (x,y) \right) = \varphi(x) \left( \varphi(y) \right)^{-1} \in A
                                           A \propto (R_{atrio}(R)) \subseteq A
xtV(x1,y1),(x2,y2)∈ Ratio(R), 描(x1,y1)=(x2,y2),则有:
 X1=X2且Y1=Y2
:: x_1 = x_2 \in \mathbb{R}, \quad \varphi: \mathbb{R} \longrightarrow A 是映射 \quad :: \, \varphi(x_1) = \varphi(x_2)
y_1 = y_2 \in \mathbb{R}, \varphi: \mathbb{R} \to A是映射 \varphi(y_1) = \varphi(y_2)
: y_1 = y_2, y_1 \neq 0, y_2 \neq 0 : \varphi(y_1) \in A^{\times}, \varphi(y_2) \in A^{\times}
: \varphi(y_1) = \varphi(y_2) : (\varphi(y_1))^{-1} = (\varphi(y_2))^{-1}
.. 以: Ratio(R) →A是映射.
xt∀(x1,y1),(x2,y2)∈Ratio(R),若(x1,y1)~(x2,y2),则有:
x_1 y_2 = x_2 y_1.
\therefore x_1 y_2 = x_2 y_1 \in \mathcal{K} \qquad \therefore \qquad \varphi(x_1 y_2) = \varphi(x_2 y_1)
\varphi(x_1) \varphi(y_2) = \varphi(x_2) \varphi(y_1)
 : \varphi(x_1) \varphi(y_2) (\varphi(y_2))^{-1} (\varphi(y_1))^{-1} = \varphi(x_2) \varphi(y_1) (\varphi(y_1))^{-1} (\varphi(y_2))^{-1}
```

:
$$\varphi(x_1)(\varphi(y_1))^{-1} = \varphi(x_2)(\varphi(y_2))^{-1}$$

:: 存在唯一的映射
$$\theta$$
: $Ratio(R)/\sim \longrightarrow A$ 使得

$$f: R \longrightarrow Fnc(R)$$
 是 单同态 $O: Fnc(R) \longrightarrow A$ 是映射 $x \longmapsto [x,1]$

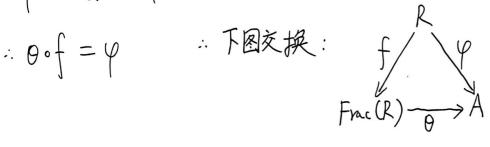
$$: \Theta \circ f : R \rightarrow A$$
 是映射 $P : R \rightarrow A$ 是环同态,

$$\forall x \in \mathbb{R}$$
, $\forall x \in \mathbb{R}$,

$$=\left(\theta\circ \gamma\right)\left(\left(x_{1}\right)\right)=\alpha\left(\left(x_{1}\right)\right)=\varphi(x)\left(\varphi(1)\right)^{-1}=\varphi(x)\cdot l_{A}^{-1}$$

$$= \varphi(x) \cdot |_{A} = \varphi(x)$$

$$\therefore \Theta \circ f = \varphi$$



$$\Theta\left(\left[X_{1},y_{1}\right]+\left[X_{2},y_{2}\right]\right)=\Theta\left(\left[X_{1}y_{2}+X_{2}y_{1},y_{1}y_{2}\right]\right)$$

$$=\Theta\left(2\left(\left(x_{1}y_{2}+x_{2}y_{1},y_{1}y_{2}\right)\right)\right)=\left(\theta\cdot 2\right)\left(\left(x_{1}y_{2}+x_{2}y_{1},y_{1}y_{2}\right)\right)$$

$$= \left\langle \left((x_{1}y_{2} + x_{2}y_{1}, y_{1}y_{2}) \right) = \left\langle \left((x_{1}y_{2} + x_{2}y_{1}) \left((y_{1}y_{2}) \right)^{-1} \right) \right|$$

$$= \left(\left((x_{1}y_{2}) + (x_{2}y_{1}) \right) \left((y_{1}y_{1}) (y_{2}y_{2}) \right)^{-1} \right|$$

$$= \left((y_{1}x_{1}) (y_{1}y_{2}) + (y_{1}x_{2}) (y_{1}y_{1}) \right) \left((y_{1}y_{2}) \right)^{-1} \left((y_{1}y_{1}) \right)^{-1} \right|$$

$$= \left((x_{1}) (y_{1}y_{1}) + (x_{2}) (y_{1}y_{2}) \right)^{-1} \right|$$

$$= \left((x_{1}, y_{1}) + (x_{2}) (y_{1}y_{2}) \right)^{-1} \right|$$

$$= \left((x_{1}, y_{1}) + (x_{2}) (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, y_{1}) + (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, y_{1}) + (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, y_{1}) + (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, y_{1}) \cdot (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, y_{1}) \cdot (x_{2}, y_{2}) \right) \right|$$

$$= \left((x_{1}, x_{2}, y_{1}y_{2}) \right)^{-1} \right|$$

$$= \left((x_{1}, x_{2}) (y_{1}y_{2}) \right)^{-1} \right|$$

$$= \left((x_{1}, y_{1}) \cdot (x_{2}) \cdot (y_{1}y_{2}) \right)^{-1} \right|$$

$$= \left((x_{1}, y_{1}) \cdot (x_{2}) \cdot (y_{1}y_{2}) \right)^{-1} \right|$$

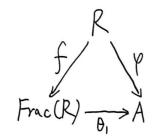
$$= \left((x_{1}, y_{1}) \cdot (x_{2}) \cdot (x_{2}, y_{2}) \right) = \left((x_{1}, y_{1}) \cdot (x_{2}, y_{2}) \right) \cdot \left((x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \right)$$

$$= \left((x_{1}, y_{1}) \cdot (x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \right) \cdot \left((x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \right) \cdot \left((x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \right) \cdot \left((x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \cdot (x_{2}, y_{2}) \right) \cdot \left((x_{2}, y_{2}) \cdot (x_{$$

$$\Theta([I,I]) = \Theta(\textbf{$Q((I,I))}) = \mathcal{A}((I,I))$$

= $\mathcal{Y}(I)(\mathcal{Y}(I))^{-1} = I_A \cdot I_A^{-1} = I_A \cdot I_A = I_A$
: $\Theta: Fac(R) \longrightarrow A$ 是环同态. :: 存在性得证.

假设存在环同态 $Θ_1: Fmc(R) \longrightarrow A$, s.t. 下图交换:



假设还存在环同态 $\Theta_2: F_{rac}(R) \longrightarrow A$, s.t. 下图交换;

Frac
$$(R) \xrightarrow{\theta_2} A$$

则有: $x \neq \forall y \in R \setminus E \circ Y$, 有: $\varphi(y) \in A^x$

$$= \Theta_{1}\left(\left[\left[y,1\right] \right) \cdot \Theta_{1}\left(\left[\left[1,y\right] \right) \right) = \Theta_{1}\left(\left[\left[\left(y,y\right] \right) \right) \cdot \Theta_{1}\left(\left[\left[1,y\right] \right) \right)$$

$$= \varphi(y) \cdot \theta_1([1,y]) \qquad \therefore \theta_1([1,y]) = (\varphi(y))^{-1}$$

$$\begin{aligned} & : \cdot \mid_{A} = \theta_{2} \left(\left[\left[1, 1 \right] \right) = \theta_{2} \left(\left[\left[y, y \right] \right) = \theta_{2} \left(\left[\left[y, 1 \right] \cdot \left[1, y \right] \right) \right) \\ & = \theta_{2} \left(\left[\left[y, 1 \right] \right) \cdot \theta_{2} \left(\left[\left[1, y \right] \right) = \theta_{2} \left(\left[\left[y, y \right] \right) \cdot \theta_{2} \left(\left[\left[1, y \right] \right) \right) \end{aligned}$$

$$=\varphi(y)\cdot\theta_{2}\left(\llbracket 1,y\rrbracket\right)\qquad \therefore\theta_{2}\left(\llbracket 1,y\rrbracket\right)=\left(\varphi(y)\right)^{-1}$$

$$=\theta_{1}\left(f(x)\right)\cdot\theta_{1}\left(\left[1,y\right]\right)=\varphi(x)\cdot\left(\varphi(y)\right)^{-1}$$

$$= \theta_2 \left(f(x) \right) \cdot \theta_2 \left(\left[1, y \right] \right) = \theta_2 \left(\left[x, 1 \right] \right) \cdot \theta_2 \left(\left[1, y \right] \right)$$

$$=\theta_{2}\left(\left[x,1\right]\cdot\left[1,y\right]\right)=\theta_{2}\left(\left[x,y\right]\right)$$