## LADR 2C 凝 (2)

P50 18题

Lemma: 任意 n (No), nept)维向量空间都可以分解为 n/m/维向量空间储和 proof: 设 V是 域F上的 n (nept)维向量空间,取V的一个基:

 $\alpha_1, \alpha_2, \cdots, \alpha_n$ . 则有: 向量组  $\alpha_1, \cdots, \alpha_n$  纸性无关且  $V=span(\alpha_1, \cdots, \alpha_n)$ .

 $i\hat{\chi}V_1 = Span(\alpha_1)$ ,  $V_2 = Span(\alpha_2)$ ,  $\cdots$ ,  $V_n = Span(\alpha_n)$ .

则有: 1/1,一,1/2是1/的个子空间。

·· 向量组 义,···, 义, 线性无关 ·· 义, 本0,···, 公, 本0

·· @量组《线性无关》…,向量组《线性无关

 $dim V_1 = 1, \cdots, dim V_n = 1$ 

:: V1, ..., Vn是V的叶/维子空间. :: V1+\*\*+Vn⊆V

zfVBEV.  $geV=span(\alpha_1,\dots,\alpha_n)$  :  $\exists k_1,\dots,k_n\in F$ , s.t.  $B=k_1\alpha_1+\dots+k_n\alpha_n$ .

 $k_1 \propto k_1 \in Span(\alpha_1) = V_1, \dots, k_n \propto k_1 \in Span(\alpha_n) = V_n$ 

 $\exists \beta \in V_1 + \dots + V_n \qquad \exists V \subseteq V_1 + \dots + V_n \qquad \exists V = V_1 + \dots + V_n$ 

没 0=V1+···+Vn, 其中V1∈V1,···, Vn∈Vn

 $: V_1 \in V_1 = Span(\alpha_1)$  :  $\exists l_1 \in \mathcal{F}, s.t. \ V_1 = l_1 \alpha_1$ 

..  $v_n \in V_n = Span(x_n)$  ..  $\exists h \in F, s.t. v_n = l_n x_n$ 

 $\cdots O = V_1 + \cdots + V_n = l_1 \times 1 + \cdots + l_n \times n \qquad : 何量组 \times 1, \cdots, \times n 线性无关$ 

 $v_1 = - v_2 = 0$   $v_1 = 0, - v_n = 0$ 

·· O表示式 Vi+···+Vn (其 Vi e Vi,····, Vn e Vn)的方式唯一,为 Vi=0,····, Vn=0

 $\therefore \bigvee = \bigvee_{i} \bigoplus_{j} \bigoplus_{i} \bigvee_{j} \bigvee_{j} \bigoplus_{i} \bigvee_{j} \bigvee_{j$ 

P50 20是反 Lemma (三个事有限维子空间的和的维数公式), V是域于上的向量空间, V1, V2, V3 是以的有限维子空间,则有:  $dim(V_1+V_2+V_3)=dvmV_1+dvmV_2+dvmV_3$ din(V1 1 1/2) + din(1/2 1 1/3) + din(1/3 1 1/1)  $= \frac{dim((V_1+V_2) \cap V_3) + dim((V_2+V_3) \cap V_1) + dim((V_3+V_1) \cap V_2)}{dim((V_1+V_2) \cap V_3) + dim((V_2+V_3) \cap V_1) + dim((V_3+V_1) \cap V_2)}$ proof::V1,V2,V3是V的有限维子空间 : Vi+V2+V2是V的有限维子空间,且有;  $V_1 + V_2 + V_3 = (V_1 + V_2) + V_3 = V_1 + (V_2 + V_3) = (V_2 + V_3) + V_1$  $= V_2 + (V_1 + V_1) = (V_2 + V_1) + V_2$ -. VI (1)2是V的有限维子空间  $V_1 \cap V_2 \subseteq V_1$   $V_1 \cap V_2 是 V_1 的 子空间$ ··VANS是V的有限维子空间

"从八以。三公 "从八以是收的子空间

· V3 () 以是V的有限维子空间 ·以介以是以的子空间  $V_3 \cap V_1 \subseteq V_3$ 

· (V1+V2) (V3 = V3 · (V1+V2) (V3 是以的产空间)

:(1+1/2)(1)3是1的有限维子空间

 $(V_2+V_3)\cap V_1 \subseteq V_1$   $(V_2+V_3)\cap V_1$  是VI的产室间

·(½+1/3)()/是1/的有限维子空间

 $: (\sqrt{3} + \sqrt{1}) \cap \sqrt{2} \subseteq \sqrt{2} : (\sqrt{3} + \sqrt{1}) \cap \sqrt{2} \neq \sqrt{2} \text{ for } \neq 2$ 

:-(以十八)八亿是人的有限维子空间

```
·· V, V2, V3是V的有限维子空间
:: 1/1+1/2, 1/2+1/3, 1/3+1/2 是1/的有限维子空间
dim(V_1 + V_2 + V_3) = dim((V_1 + V_2) + V_3) = dim(V_1 + V_2) + dim(V_1 + V_2) + dim(V_1 + V_2) \cap V_3
  = dim V_1 + dim V_2 - dim (V_1 \cap V_2) + dim V_3 - dim ((V_1 + V_2) \cap V_3)
  = \dim V_1 + \dim V_2 + \dim V_3 - \dim (V_1 \cap V_2) - \dim (V_1 + V_2) \cap V_3)
 din(V_1+V_2+V_3) = din((V_2+V_3)+V_1) = din(V_2+V_3) + din(V_1+V_2+V_3) + din((V_2+V_3))
   = dim V_2 + dim V_3 - dim (V_2 \cap V_3) + dim V_1 - dim (V_2 + V_3) \cap V_1)
   = dim V_1 + dim V_2 + dim V_3 - dim (V_2 \cap V_3) - dim ((V_2 + V_3) \cap V_1)
dim (V_1 + V_2 + V_3) = dim ((V_3 + V_1) + V_2) = dim (V_3 + V_1) + dim V_2 - dim ((V_3 + V_1) \cap V_2)
   = dim V_3 + dim V_1 - dim \left(V_3 / V_1\right) + dim V_2 - dim \left(\left(V_3 + V_1\right) / V_2\right)
    = dimV_1 + dimV_2 + dimV_3 - dim(V_3 \cap V_1) - dim(V_3 + V_1) \cap V_2)
dim(V_1+V_2+V_3)=dimV_1+dimV_2+dimV_3
                         = \frac{\dim(V_1 \cap V_2) + \dim(V_2 \cap V_3) + \dim(V_3 \cap V_1)}{3}
                         = \frac{dim(V_1+V_2)(V_3) + dim((V_2+V_3)(V_1) + dim((V_3+V_1)(V_2))}{3}
```